Visual Search Engine for Handwritten and Typeset Math in Lecture Videos and LaTeX Notes

Kenny Davila and Richard Zanibbi

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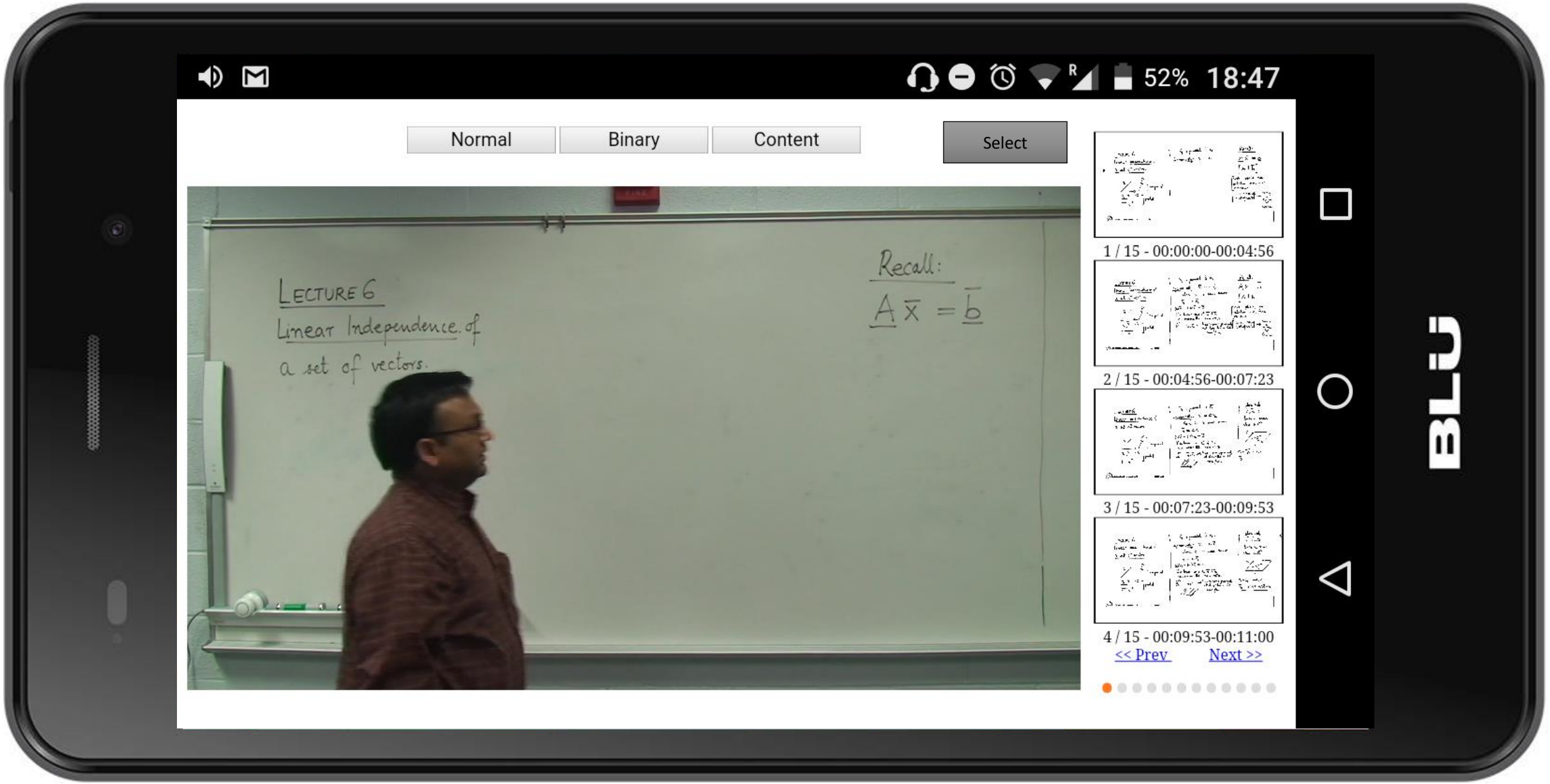
University at Buffalo

Center for Unified Biometrics and Sensors

R·I·T

dprl [DOCUMENT AND PATTERN RECOGNITION LAB]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Normal
Binary
Content
Select
LECTURE 6
Linear Independence of
a set of vectors.
Recall:
$A\bar{x} = \bar{b}$
1 / 15 - 00:00:00-00:04:56
2 / 15 - 00:04:56-00:07:23
3 / 15 - 00:07:23-00:09:53
4 / 15 - 00:09:53-00:11:00
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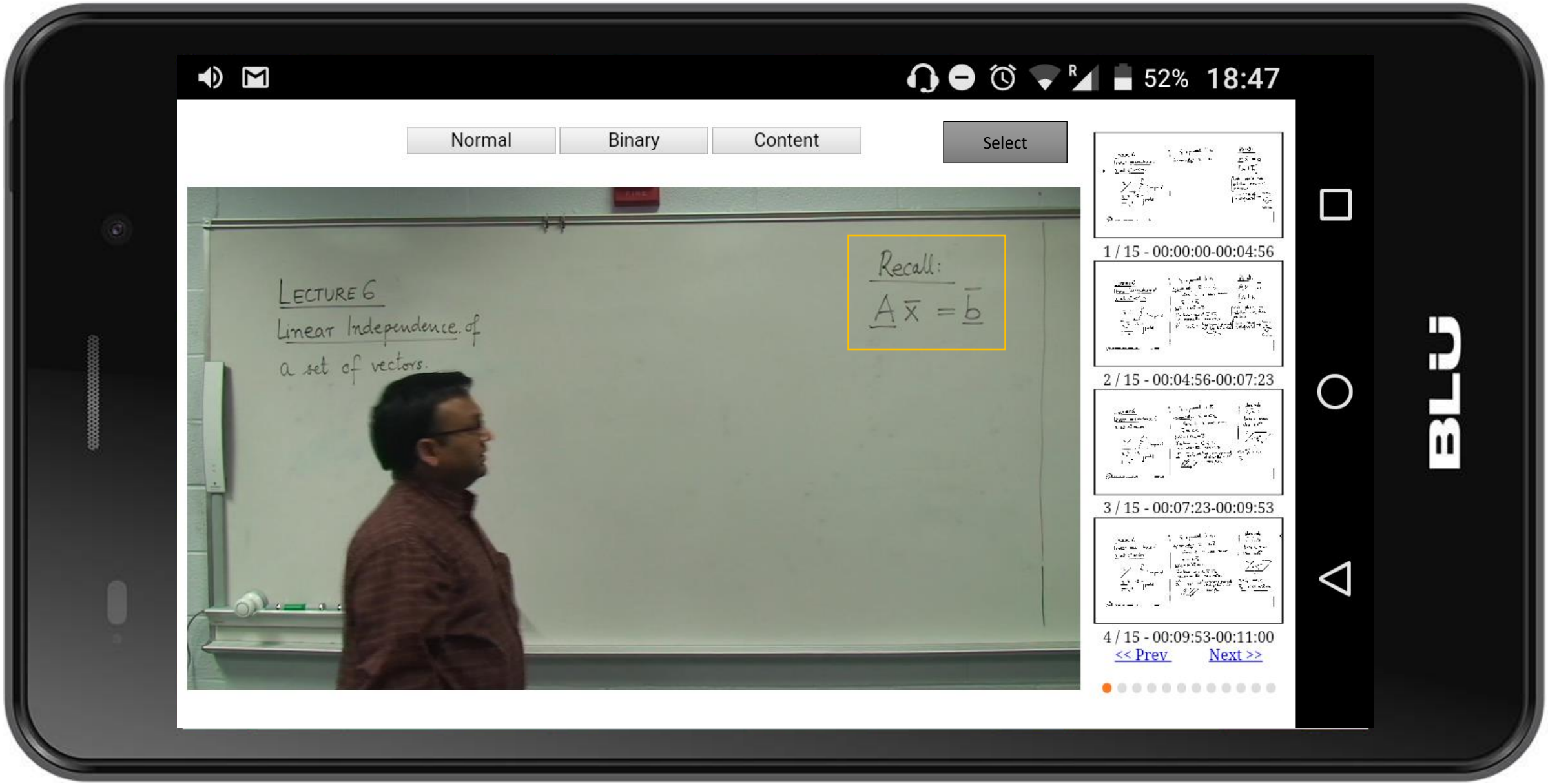
Next >>
BLU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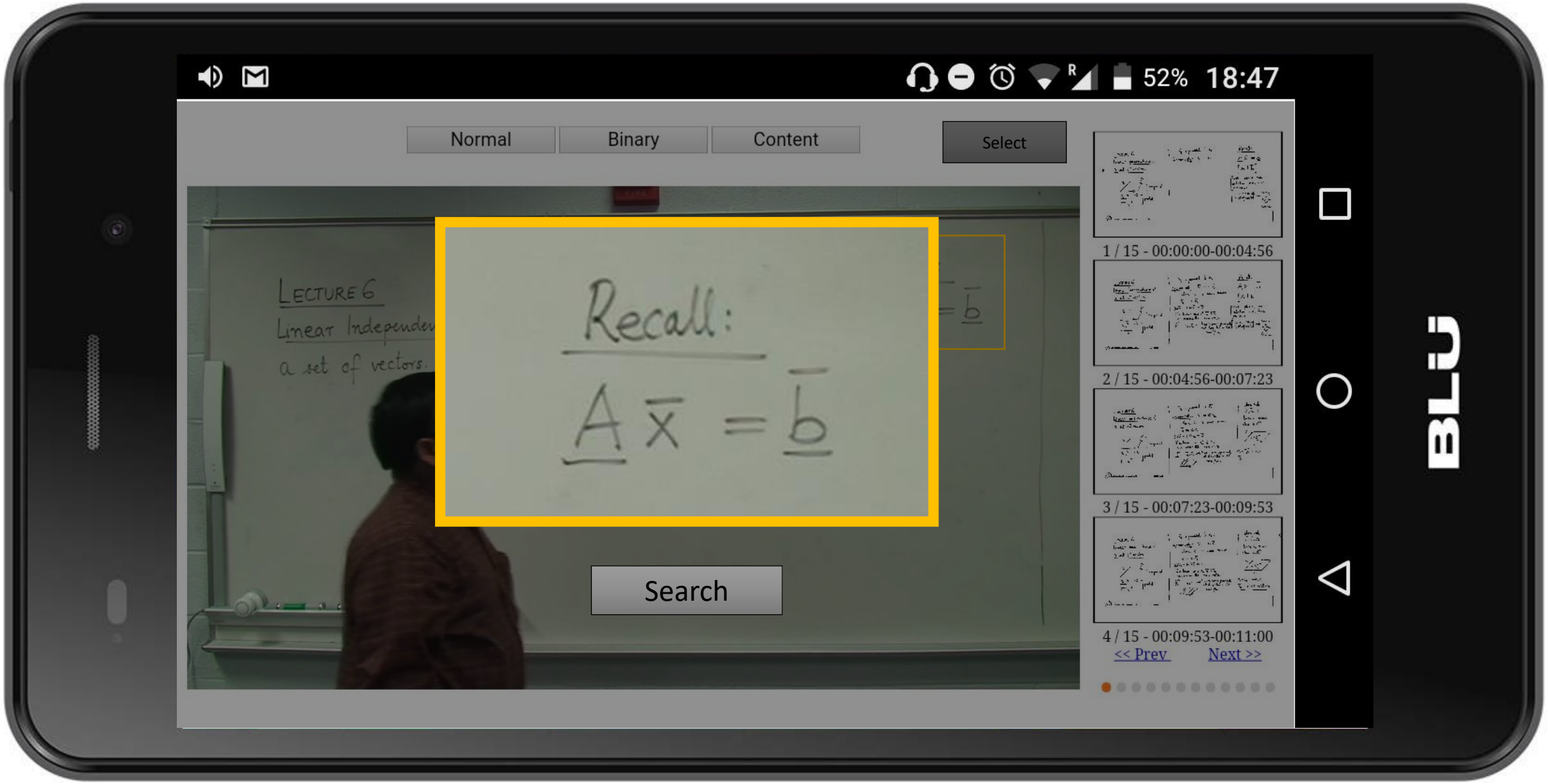
52% 18:47
Normal
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Recall:
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Search
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2 / 15 - 00:04:56-00:07:23
3 / 15 - 00:07:23-00:09:53
4 / 15 - 00:09:53-00:11:00
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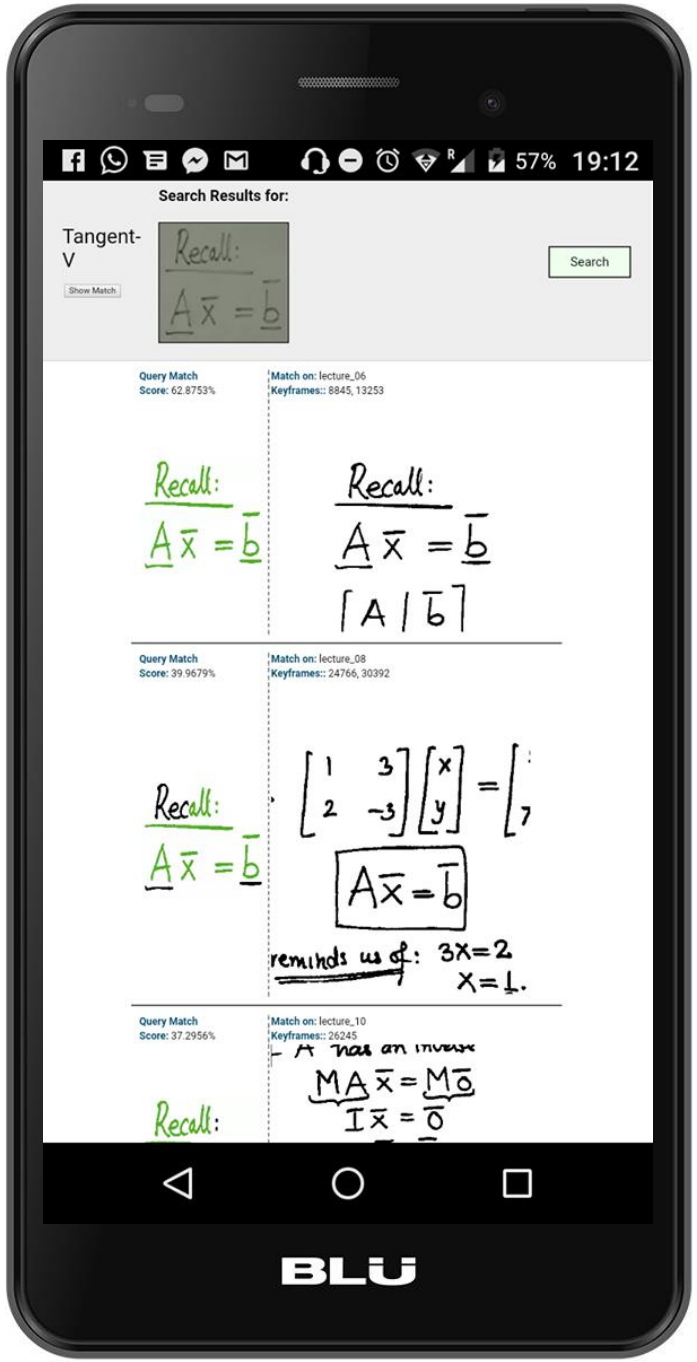


SEARCH RESULTS

Found in Lecture Videos

1. Linear Algebra – Lecture 06
2. Linear Algebra – Lecture 08
3. Linear Algebra – Lecture 10

…

Related Topics

1. Systems of Equations
2. Matrix Reduction
3. Linear Algebra

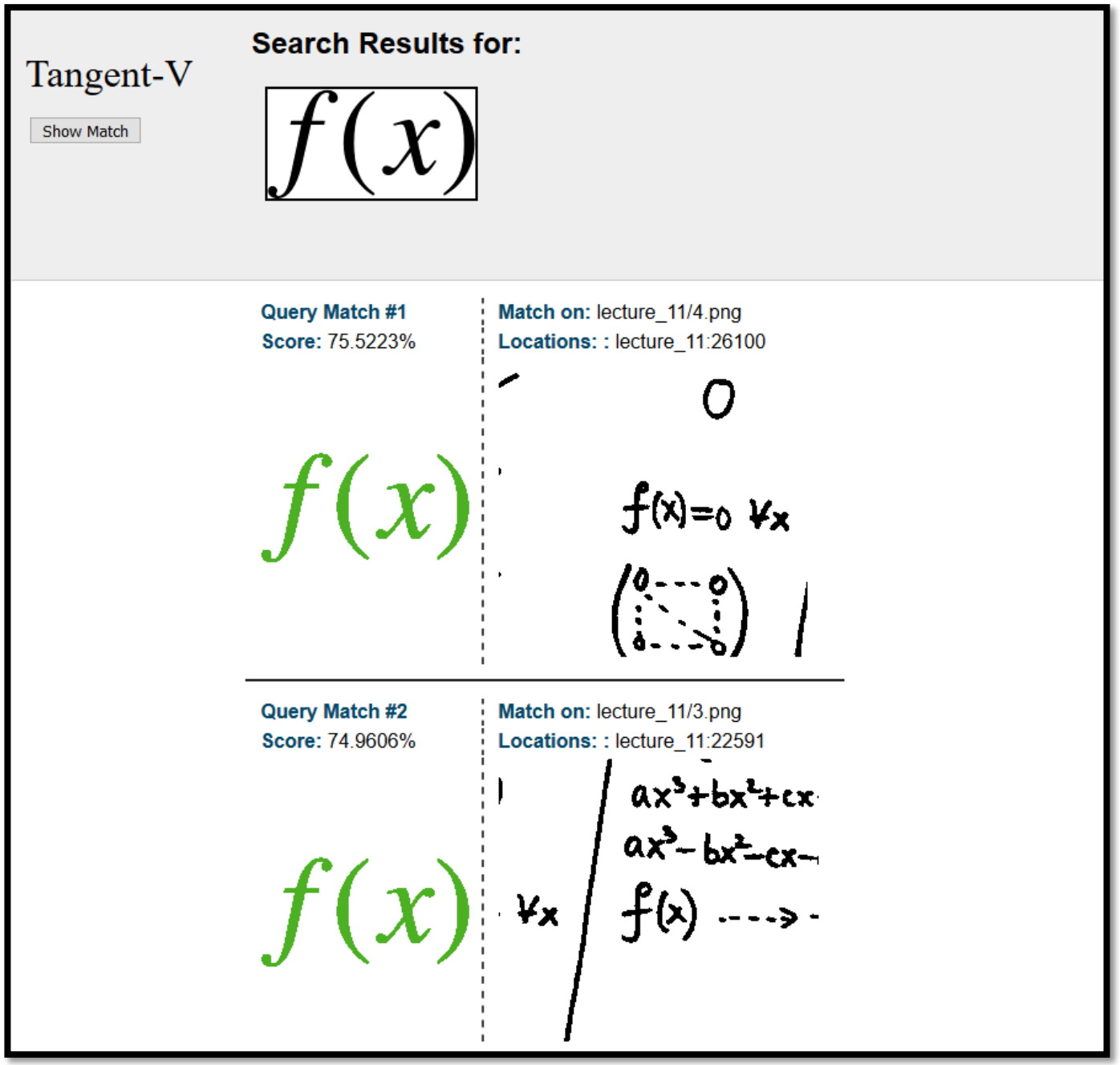
What about other
Mathematical Expressions?

Could I write my **queries** instead
of using Images?

What about other Mathematical Expressions?

Could I write my **queries** instead of using Images?

Yes, using LaTeX

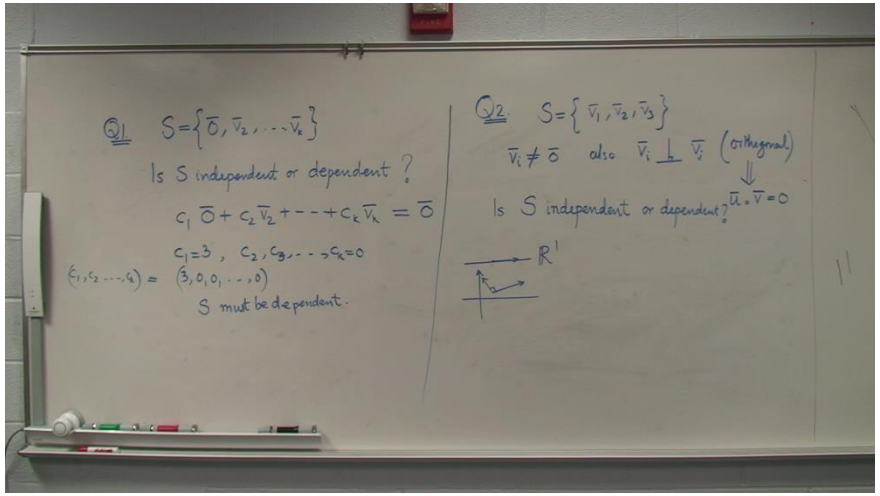


Potential Search Modes

LaTeX → Whiteboard

LaTeX → LaTeX

Lecture Video



Linear Independence

Question: Consider the set of vectors $S = \{\vec{0}, \vec{v}_2, ..., \vec{v}_k\}$. Is S independent or dependent? To answer this, we must ask whether the following homogeneous equation has a non-zero solution:

$$c_1\vec{0} + c_2\vec{v}_2 + ... + c_k\vec{v}_k = \vec{0}$$

Consider for instance the solution $c_1 = 3, c_2, c_3, ..., c_k = 0$. This is a non-zero or non-trivial solution, hence S must be dependent.

Question: Let $S = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$. Suppose none of the vectors in S are zero vectors, and each is perpendicular to one another. That is,

$$\vec{v}_i \neq \vec{0} \qquad \vec{v}_i \perp \vec{v}_j \qquad \text{for all } i \neq j$$

Lecture Notes

Whiteboard → Whiteboard

Whiteboard → LaTeX

Tangent-V Visual Search Engine

Applied to **Indexing** and **Retrieval** of formulae from **Lecture** materials

Based on Matching **Symbol Pairs** from **Line of Sight Graphs (LOS)**

Domain knowledge is given by Recognition Module

- Currently: **Mathematical Symbol Recognition**

Source **code** released: https://cs.rit.edu/~dprl/Software.html

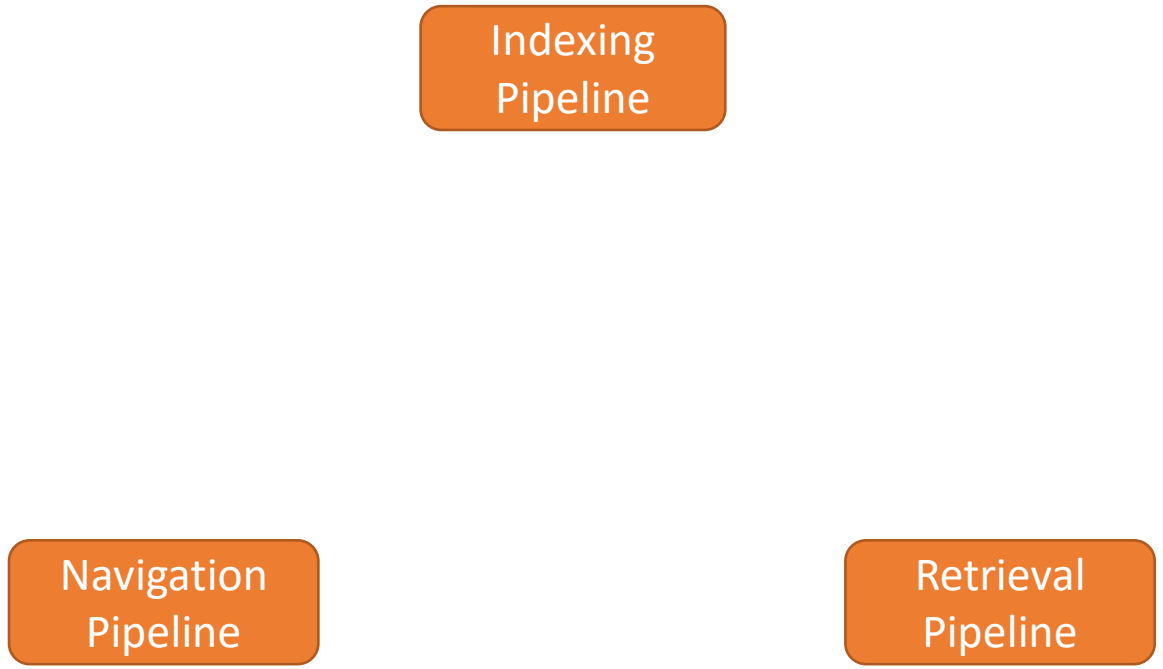
Related Work

Related fields:

- Content-Based Image Retrieval [1]
- Word Spotting [2]
- **Mathematical Information Retrieval** [3]
  - Formula Representation: Semantic vs **Appearance**
  - Retrieval Modality: Symbol vs **Image-based**
  - **Tangent-V** generalizes the **Tangent-S** formula retrieval model [4]

[1] J. Sivic & A. Zisserman, “Video Google: A text retrieval approach to object matching in videos,” in ICCV 2003
[2] S. Sudholt & G. A. Fink, “Phocnet: A deep convolutional neural network for word spotting in handwritten documents,” in ICFHR 2016
[3] R. Zanibbi & D. Blostein, “Recognition and retrieval of mathematical expressions,” IJDAR, vol. 15, no. 4, 2012.
[4] K. Davila & R. Zanibbi, “Layout and semantics: Combining representations for mathematical formula search,” SIGIR, 2017

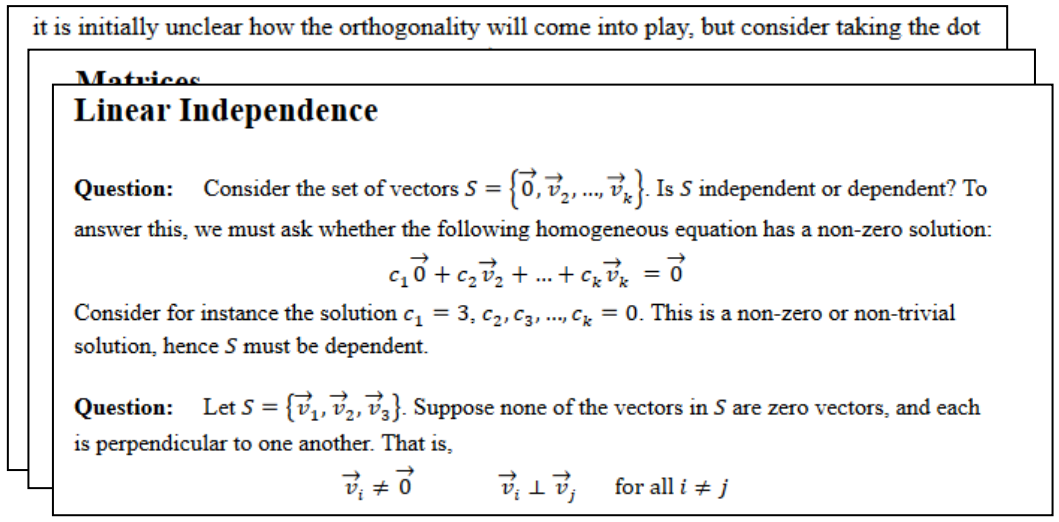
Tangent-V Overview



Supplementary Lecture Notes (LaTeX)

Input

Lecture Notes



it is initially unclear how the orthogonality will come into play, but consider taking the dot

Matrices

Linear Independence

Question: Consider the set of vectors $S = \{\vec{0}, \vec{v}_2, ..., \vec{v}_k\}$. Is S independent or dependent? To answer this, we must ask whether the following homogeneous equation has a non-zero solution:

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Consider for instance the solution $c_1 = 3, c_2, c_3, ..., c_k = 0$. This is a non-zero or non-trivial solution, hence S must be dependent.

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$$\vec{v}_i \neq \vec{0} \qquad \vec{v}_i \perp \vec{v}_j \quad \text{for all } i \neq j$$

LaTeX

Output

Math Expressions

$$S = \{\vec{0}, \vec{v}_2, \ldots, \vec{v}_k\} \quad S$$

$$c_1\vec{0} + c_2\vec{v}_2 + \cdots + c_k\vec{v}_k \quad S$$

$$c_1 = 3 \qquad c_2, c_3, \ldots, c_k = 0$$

$$S = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$$

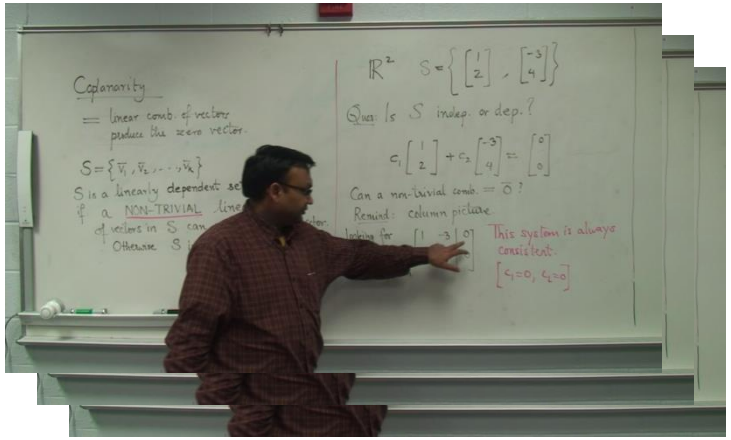
$$\vec{v}_i \neq \vec{0} \qquad \vec{v}_i \perp \vec{v}_j \quad \text{for all } i \neq j$$

Binary Images

Preprocessing Lecture Video Summarization [1]

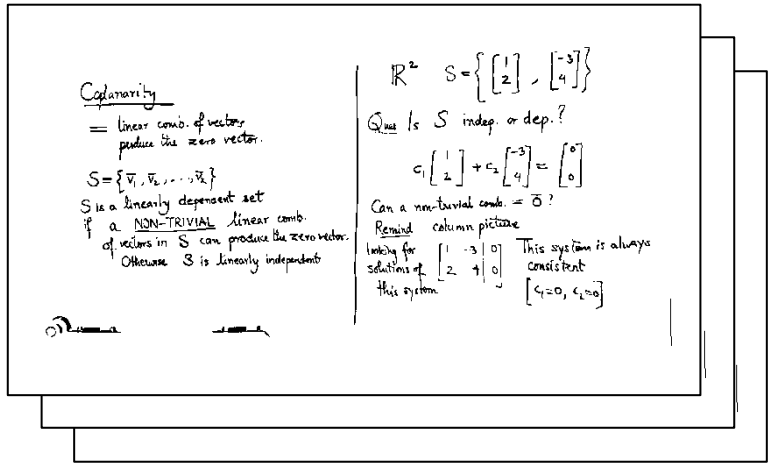
Input

Lecture Video



Output

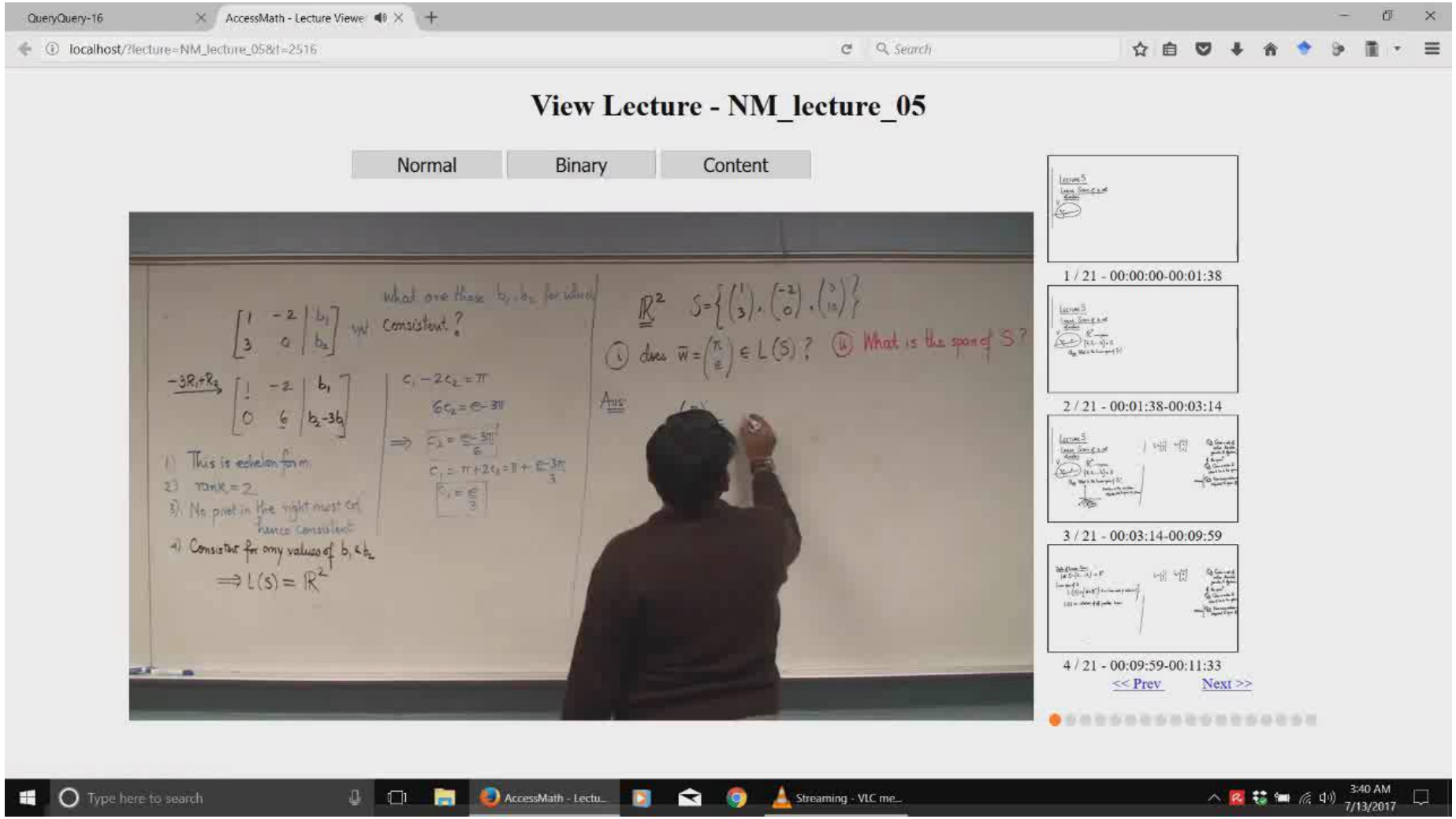
Whiteboard Contents Keyframes



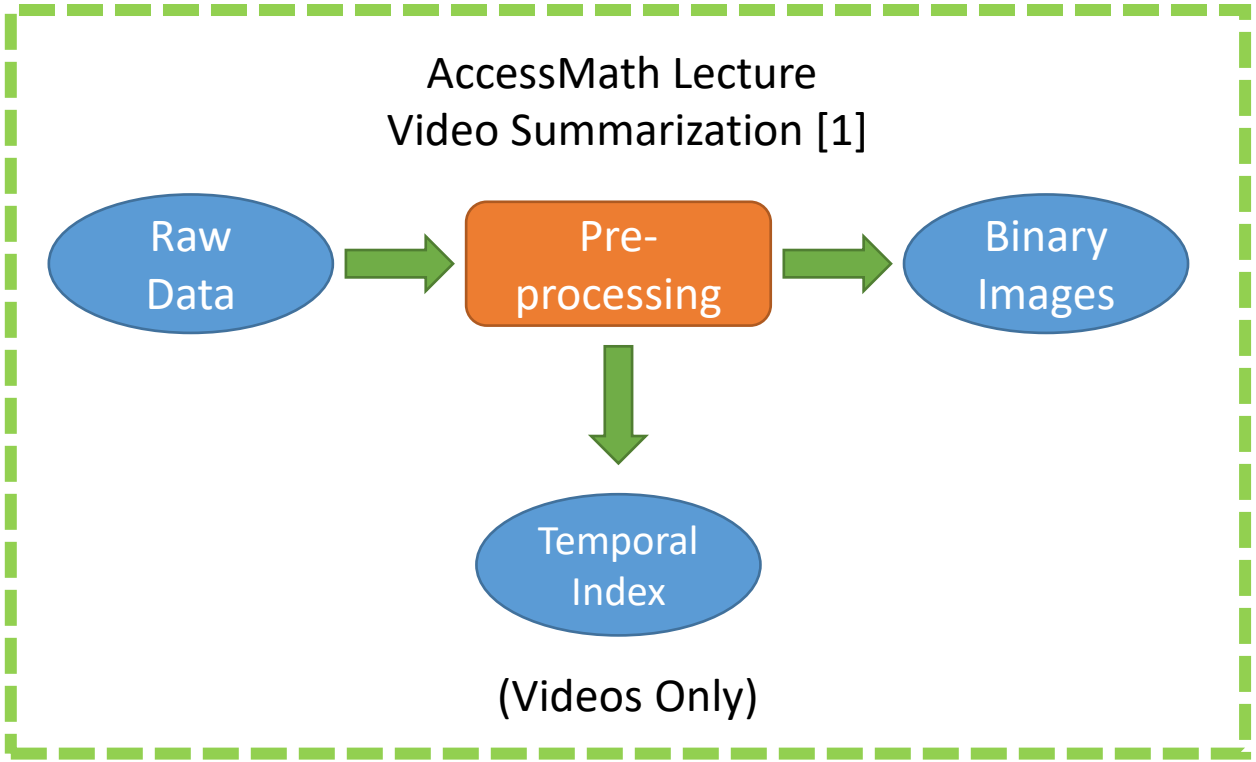
MTS/ MP4 → Content Extraction → Spatio-temporal Analysis → Temporal Index → Temporal Segmentation → Binary Images

[1] Davila, K., Zanibbi, R. Whiteboard Content Summarization via Spatio-Temporal Conflict Minimization in Lecture Videos. ICDAR 2017

Lecture Video Navigation from Keyframes



Indexing Pipeline (Overview)



[1] Davila, K., Zanibbi, R. Whiteboard Content Summarization via Spatio-Temporal Conflict Minimization in Lecture Videos. ICDAR 2017

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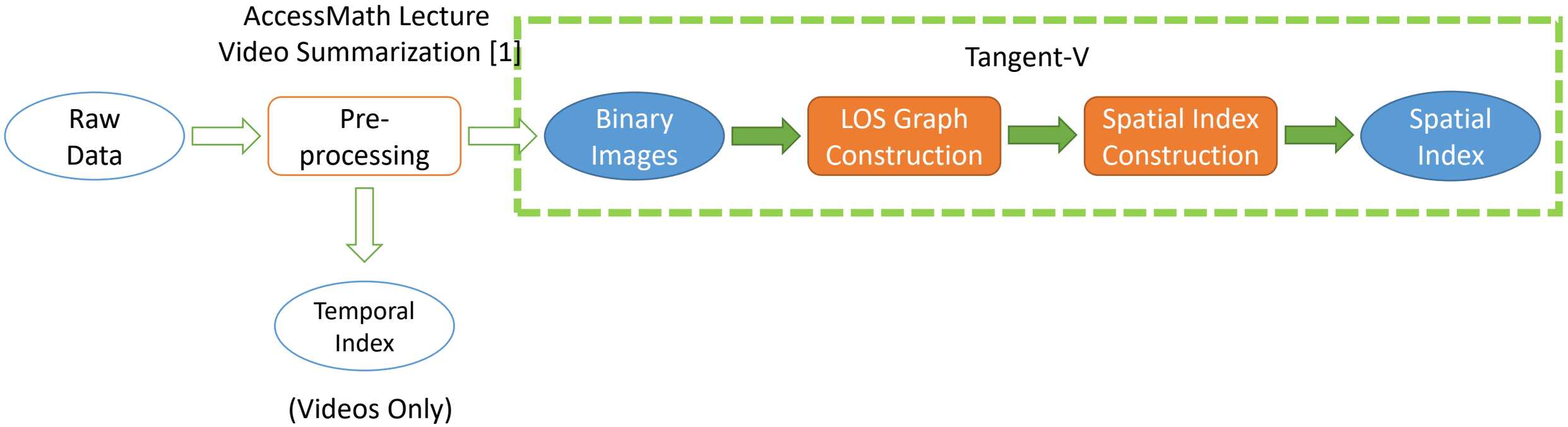


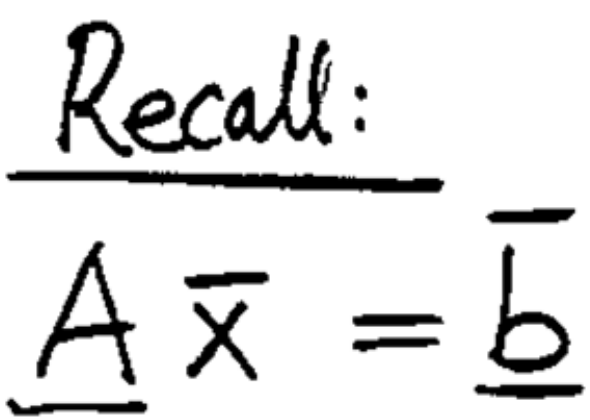
[1] Davila, K., Zanibbi, R. Whiteboard Content Summarization via Spatio-Temporal Conflict Minimization in Lecture Videos. ICDAR 2017

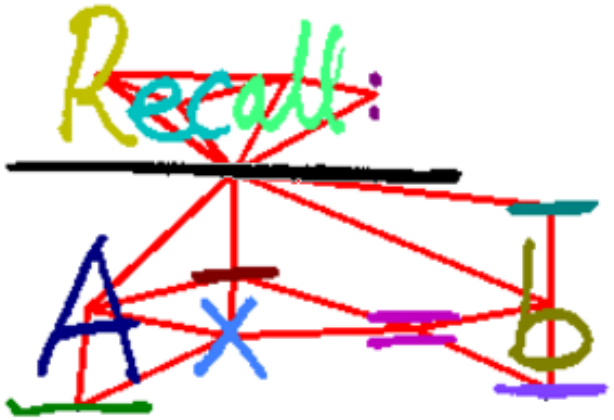
Line of Sight (LOS) Graphs

Uses **Connected Components (CC)** as **Nodes**

Two **nodes** are **connected** if

- One can **see** the other
- Max. distance factor considered for whiteboard content (2 times median size)





Line of Sight (LOS) Graphs

True **Node Labels/Relationships** are **unknown**

- After Symbol Recognition, each **Node** has top **k labels** with **probabilities**
  - $\sum_{\omega \in \Omega} p(\omega | s_x) \geq 80\% \quad k \leq 10$
- **Edges** have **3D unit vectors** indicating direction

2^x

(0.707, 0.707, 0.000)

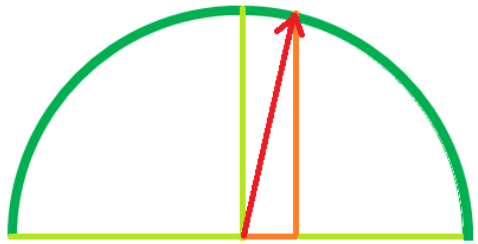
$2x$

(1.000, 0.000, 0.000)

x^2

(-0.707, -0.707, 0.000)

$\sqrt{x}$



(0.146, -0.146, 0.978)

Spatial Indexing using Symbol Pairs

Inverted Index for Symbol Pairs

Entries: Pairs of symbol labels $(\boldsymbol{\omega_1}, \boldsymbol{\omega_2})$

Posting lists: Pair locations in images with $\left(\boldsymbol{ID}, \boldsymbol{p_1}, \boldsymbol{p_2}, \vec{\boldsymbol{c}}, \boldsymbol{s_p}(\boldsymbol{c_1}, \boldsymbol{c_2})\right)$

Top k-labels per node $(\boldsymbol{\Omega})$

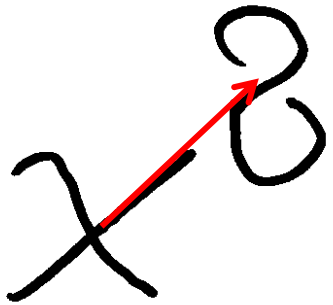
Tuples Generated $(\boldsymbol{\Omega_1 \times \Omega_2})$

$$\left(\boldsymbol{\omega_1}, \boldsymbol{\omega_2}, \boldsymbol{p_1}, \boldsymbol{p_2}, \vec{\boldsymbol{c}}, \boldsymbol{s_p}\right)$$

$\boldsymbol{p_x}$ - $\boldsymbol{p}(\boldsymbol{\omega_x}|\boldsymbol{s_x})$

$\vec{\boldsymbol{c}}$ - 3D Unit Vector from $\boldsymbol{s_1}$ to $\boldsymbol{s_2}$

$\boldsymbol{s_p}$ - Size Ratio between $\boldsymbol{s_1}$ and $\boldsymbol{s_2}$



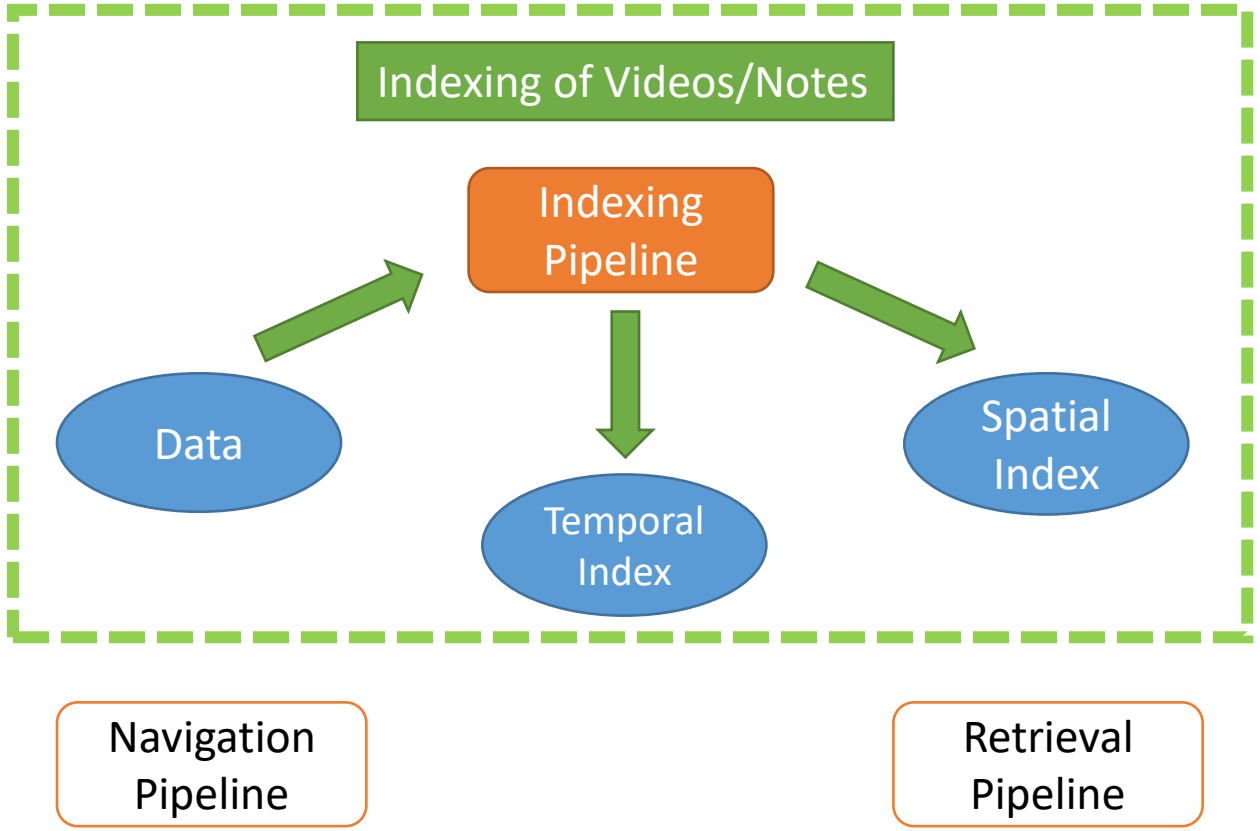
$$S_1 = x \qquad S_2 = 8$$

$$\Omega_1 = \{(x, 0.8), (X, 0.2)\} \qquad \Omega_2 = \{(8, 0.6), (\&, 0.3)\}$$

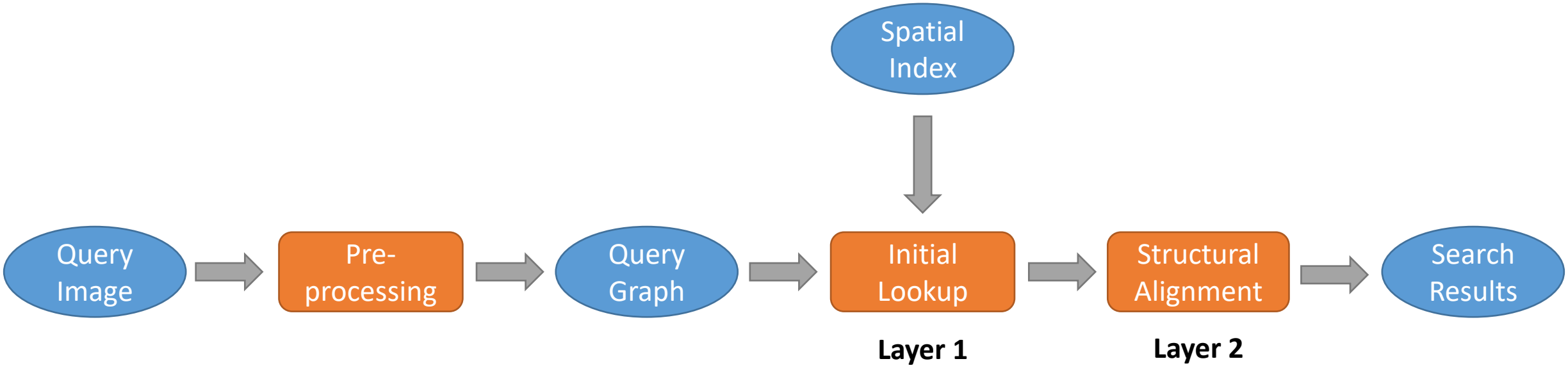
$$\vec{\boldsymbol{c}} = \langle \boldsymbol{0.71}, \boldsymbol{-0.71}, \boldsymbol{0.00} \rangle$$

$$\boldsymbol{s_p} = 1.26$$

Tangent-V Overview



Tangent-V Retrieval Model



Layer 1: Initial Lookup

Query **symbol pairs** are used to find matches on their corresponding **entries** on the **inverted index** structure

A **match** between index symbol pair $P^c = (c_1, c_2)$ and query pair $P^q = (q_1, q_2)$ will be accepted as **valid** if and only if:

1 - They are **spatially consistent**:

$$\vec{\boldsymbol{c}} \cdot \vec{\boldsymbol{q}} \geq \cos(45^\circ)$$

2 - Optionally, if they have **consistent size ratios** (not too small/large)

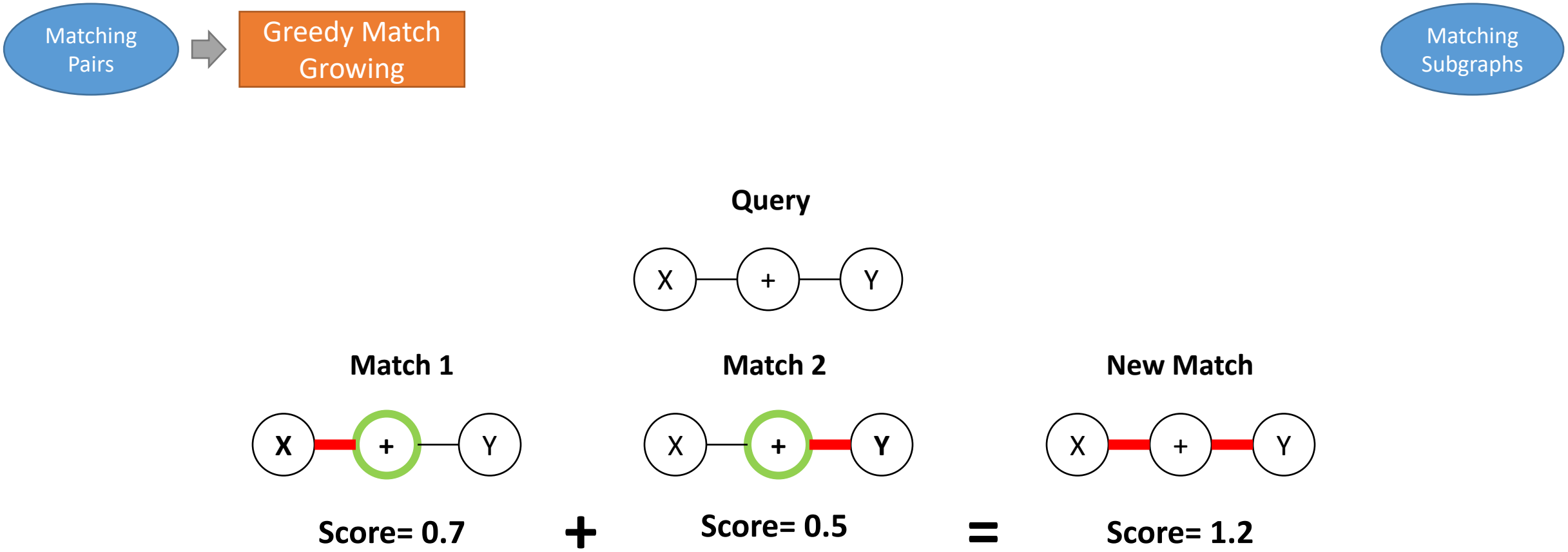
Matching Pairs Scores are then **aggregated** by unique **Graph Pair IDs**

Layer 2: Structural Alignment

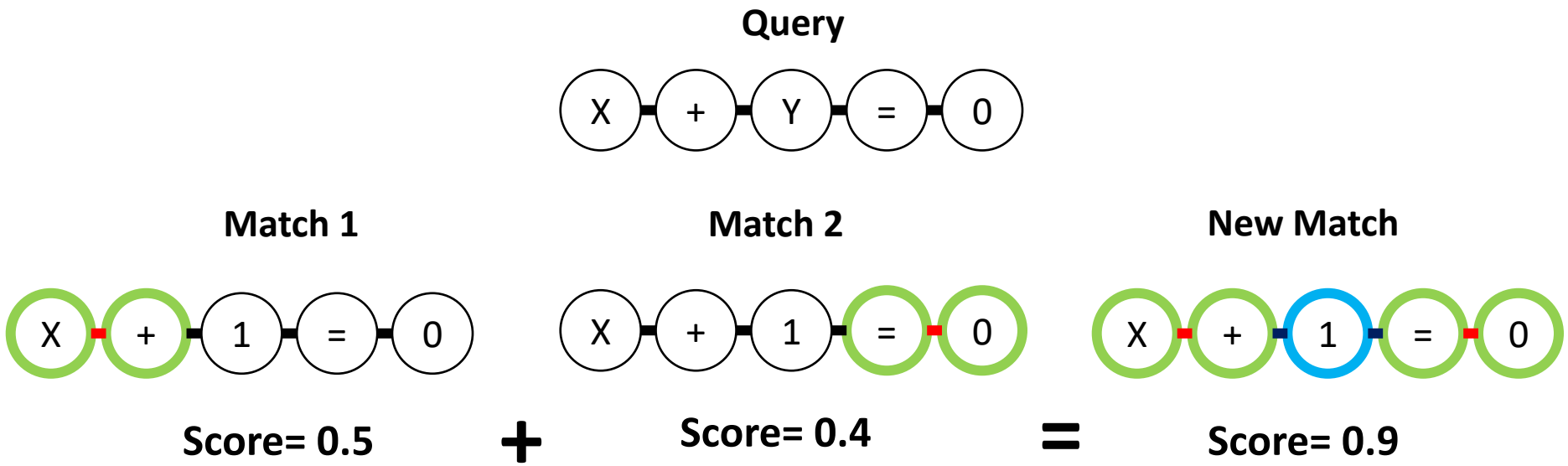
Matching Pairs

Matching Subgraphs

Layer 2: Structural Alignment



Layer 2: Structural Alignment



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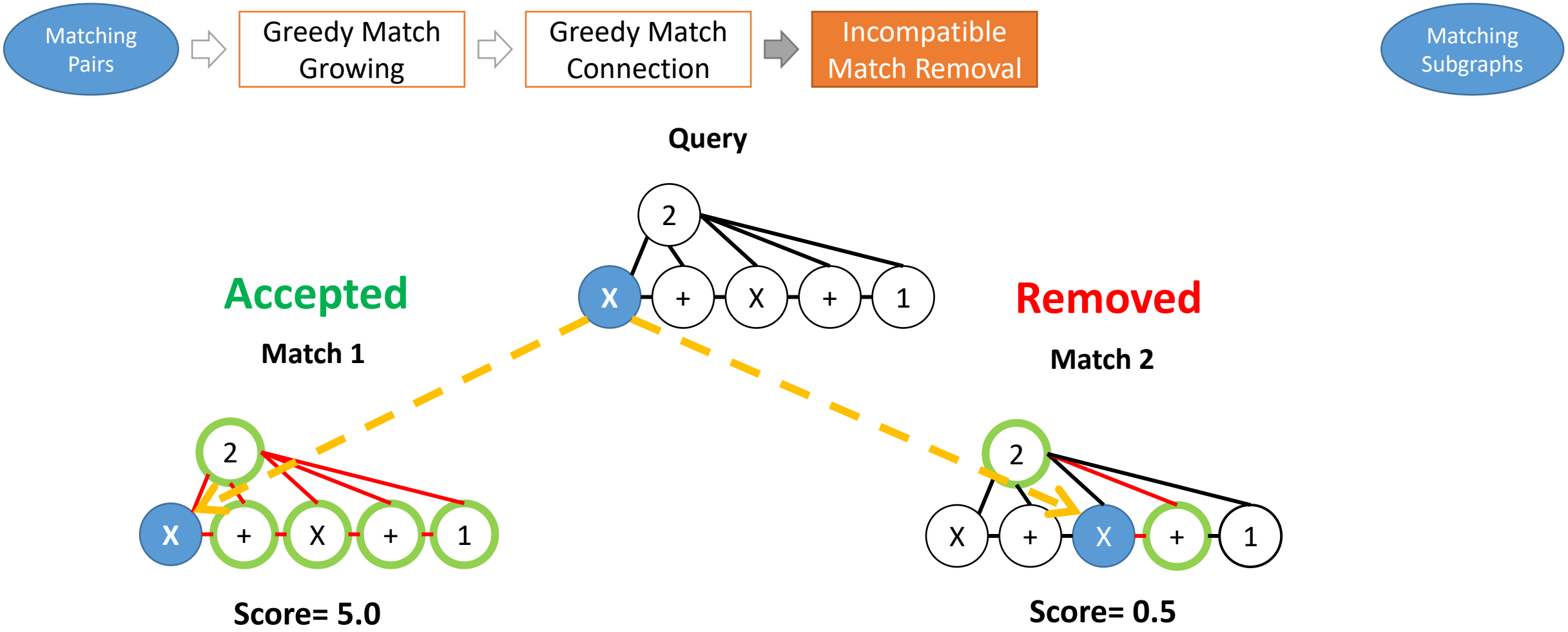
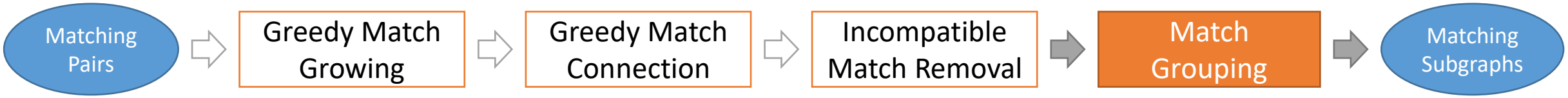
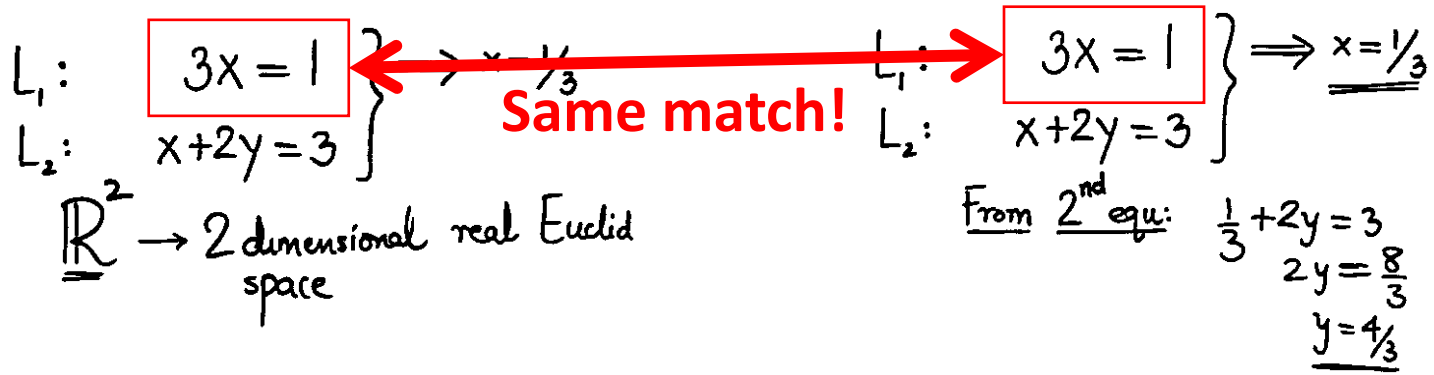


Layer 2: Structural Alignment



Query: $3x = 1$



Lecture 01 – KF #5 **Lecture 01 – KF #6**

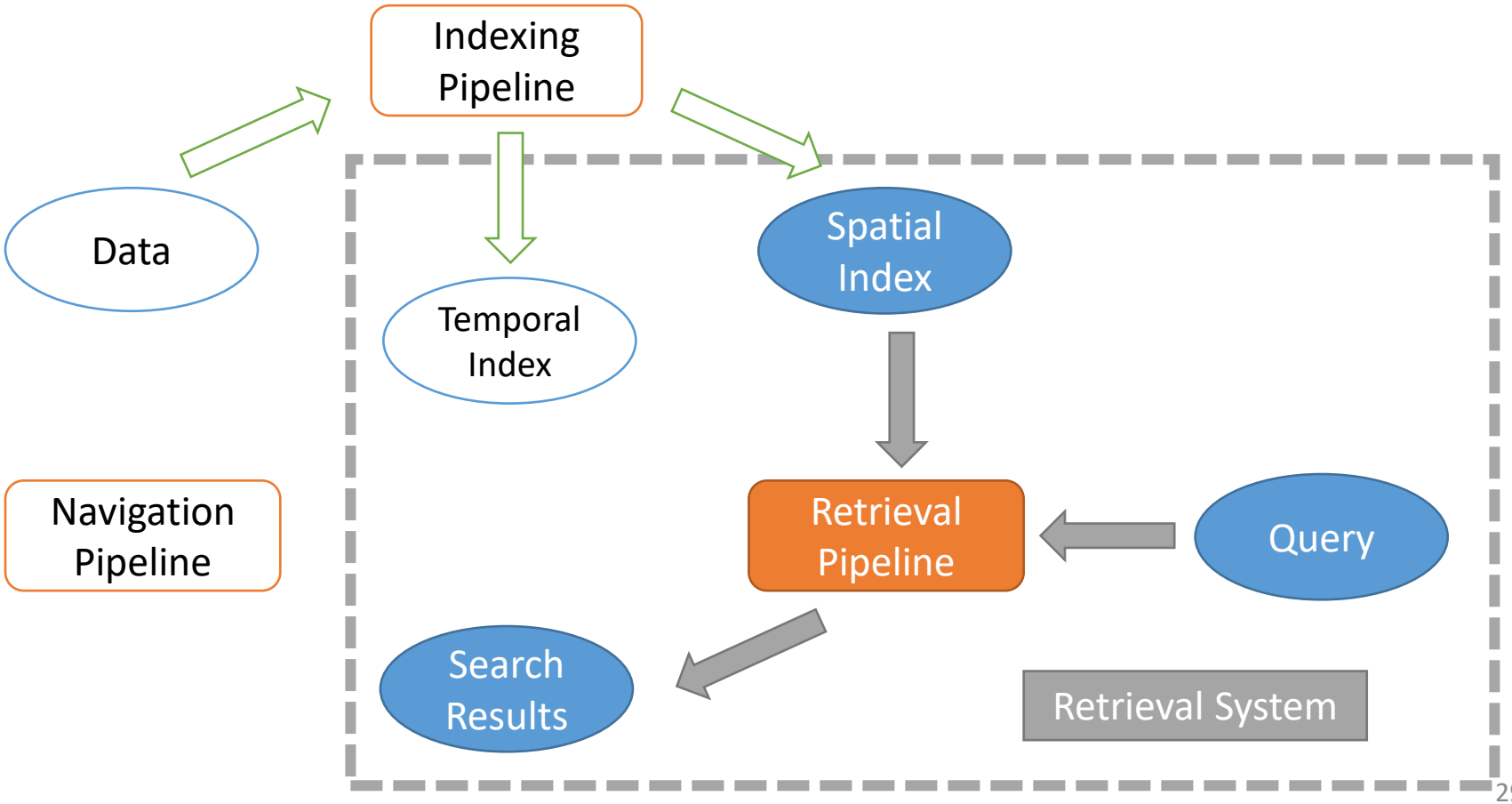
Match Scoring and Ranking

We introduce two **scoring schemes**: **α** and **h**

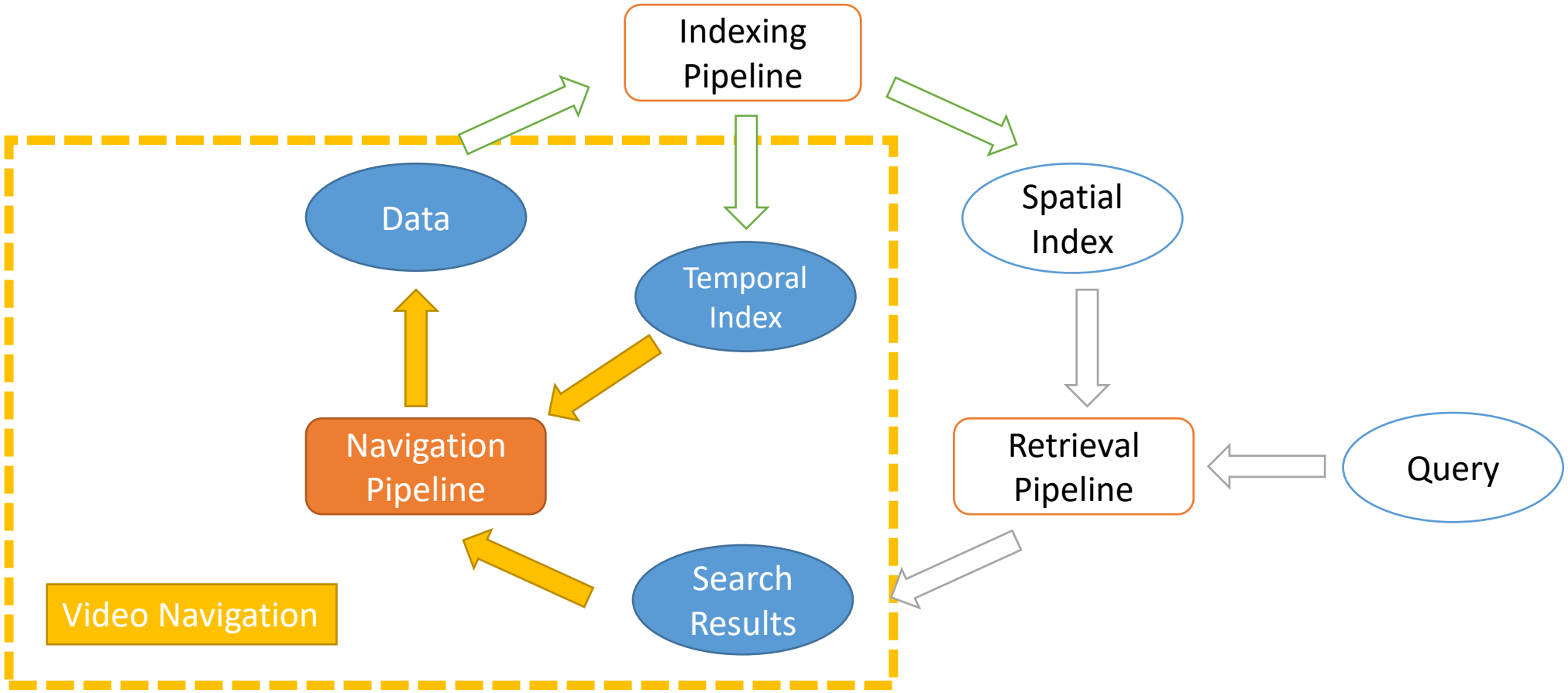
Item	$\alpha(M)$	$h(M)$
Description	A weighted **edge recall**	Harmonic mean of weighted **edge recall** and **node recall**
Edge weighting	**pair-wise** symbol alignments and **scaled cosine similarity**	**scaled cosine similarity**
Node weighting	-	**Individual** symbol alignments
Based on	-	**Maximum Subtree Similarity (MSS)** [1]
Execution Times	Faster	Slower

[1] R. Zanibbi, K. Davila, A. Kane, & F. Tompa, "Multi-stage math formula search: Using appearance-based similarity metrics at scale," SIGIR, 2016

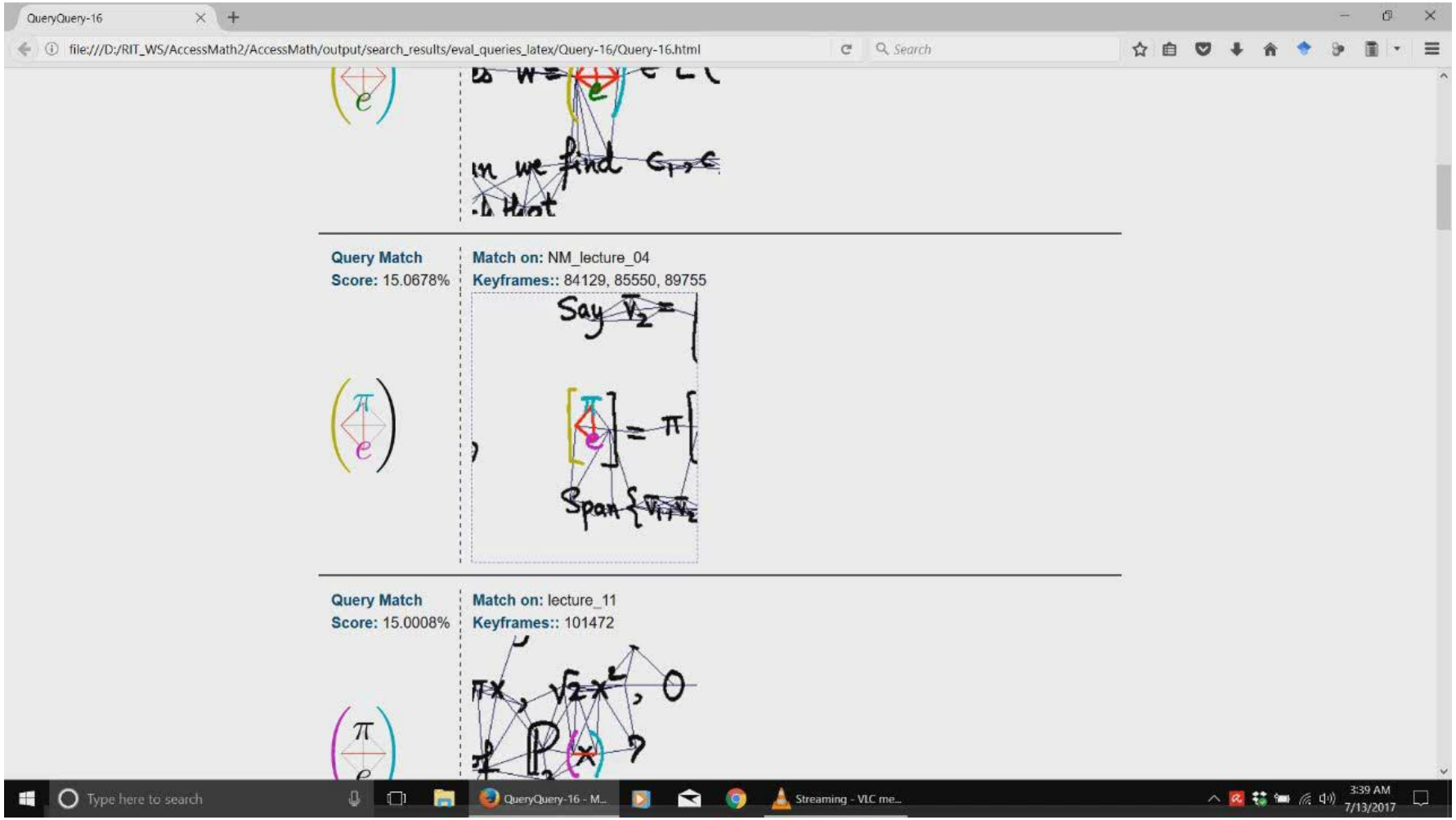
Tangent-V Overview



Tangent-V Overview



Lecture Video Navigation from Search Results



Check our demo at: https://youtu.be/gn24qo1MLN0

Experiments

AccessMath **Dataset**

- 13 Lecture videos with supplementary notes

A total of 20 evaluation **queries** were chosen with rejection sampling

A total of 4 combinations of Query-vs-Index **modalities**

- Handwritten expressions
- Typeset expressions

For a given query, the **target** is to find a math expression that **contains** the **whole query graph**

- query is **same expression**
- query is **sub-expression**

Evaluation Metrics

Two **metrics** are considered

- Recall @ 10: Target found @ rank ≤ 10
- MRR @ 10: Mean of Reciprocal Rank (RR), with

$$RR = \begin{cases} \frac{1}{r} & 1 \leq r \leq 10 \\ 0 & otherwise \end{cases}$$

Results: Recall @ 10

		Weighted Edge Recall α			Harmonic Mean h		
Query	Index	$\boldsymbol{\alpha}$	$\boldsymbol{\alpha_{\wedge}}$	$\boldsymbol{\alpha_{\wedge s}}$	$\boldsymbol{h}$	$\boldsymbol{h_{\wedge}}$	$\boldsymbol{h_{\wedge s}}$
LaTeX		**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
Whiteboard		0.95	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
LaTeX	Whiteboard	0.95	0.95	0.90	0.95	**1.00**	0.95
Whiteboard	LaTeX	0.80	0.85	0.85	**0.90**	**0.90**	**0.90**

Results: MRR @ 10

		Weighted Edge Recall α			Harmonic Mean h		
Query	Index	$\boldsymbol{\alpha}$	$\boldsymbol{\alpha_{\wedge}}$	$\boldsymbol{\alpha_{\wedge s}}$	$\boldsymbol{h}$	$\boldsymbol{h_{\wedge}}$	$\boldsymbol{h_{\wedge s}}$
LaTeX		0.98	**1.00**	**1.00**	0.98	**1.00**	**1.00**
Whiteboard		0.93	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
LaTeX	Whiteboard	0.66	0.69	0.71	**0.89**	0.84	0.86
Whiteboard	LaTeX	0.63	0.71	0.74	0.74	0.78	**0.84**

Conclusions

Tangent-V is effective for search between Typeset and Handwriting
- **Multiple labels help** finding targets when recognition accuracy is low

Tangent-V can also be used to create **navigational tools**

New symbol recognizers can be used for indexing of **new domains**
- **Code** is released for others to try on new domains (http://cs.rit.edu/~dprl/Software.html)

Future work:
- Test unsupervised symbol classification
- Explore Vector formats
- Speed-up search

Thank You!

Source code: www.cs.rit.edu/~dprl/Software.html

This material is based upon work supported by the **National Science Foundation** (USA) under Grants No. IIS-1016815 and HCC-1218801.

We also thank **Anurag Agarwal** for helping in the creation of the lecture videos used to evaluate our system.