Local turn-boundedness : a curvature control for a good digitization

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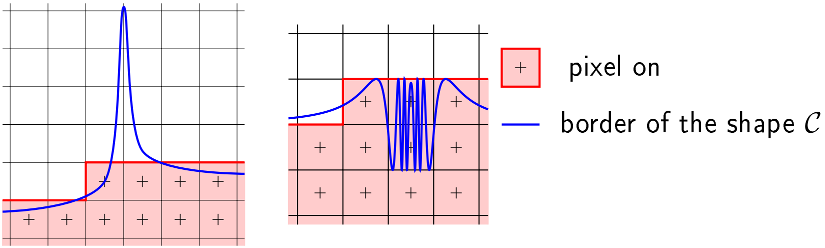
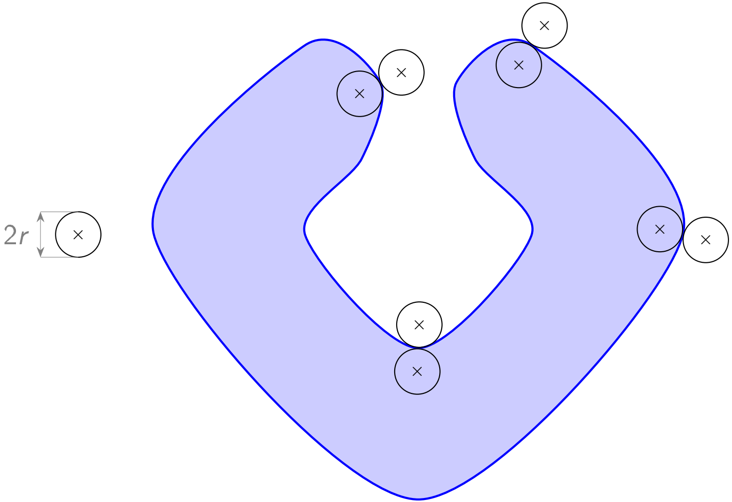
27 March 2019

Loss of information

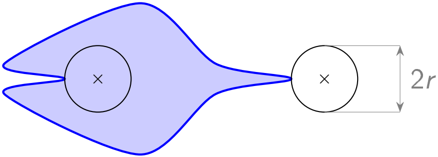


Add hypothesis to the border of the shape.

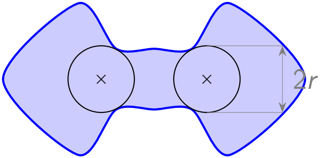
Par(r)-regularity [Pav82]



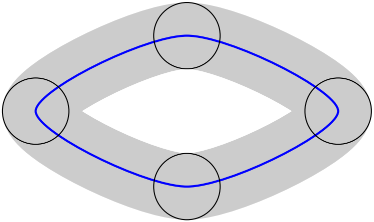
Generalizations of par(r)-regularity



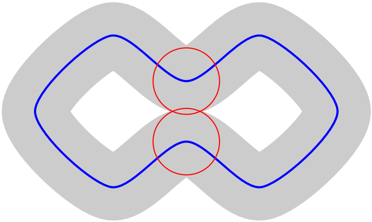
half(r)-regular [ST07]



not half(r)-regular

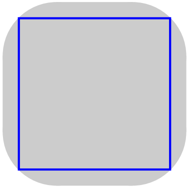


r-stable [MKS09]

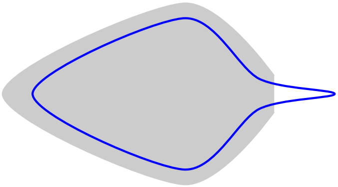


not r-stable

Generalizations of par(r)-regularity



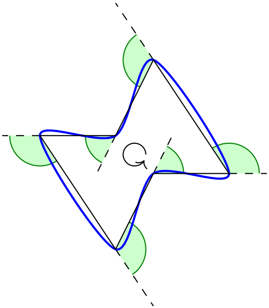
quasi(r)-regular [NKDRP17]



not quasi(r)-regular

	far from the digitization	oscillations
half(r)-regularity	Yes	No
r-stability	Yes	Yes
quasi(r)-regularity	No	Yes

Turn [Mil50], [AR89]



The turn of the polygon is the sum of the green angles.

Definition

The *turn of a curve* $\kappa(\mathcal{C})$ is the supremum of turn of polygons inscribed in it.

Basic properties of turn

The turn independent of the orientation of $\mathcal{C}$!

Proposition

For a curve parametrized by arc length γ of class C^2,

$$\kappa(\gamma) = \int_0^{L(\gamma)} |k(s)| ds,$$

$k(s)$ the curvature of γ at point $\gamma(s)$.

Theorem (Fenchel's Theorem)

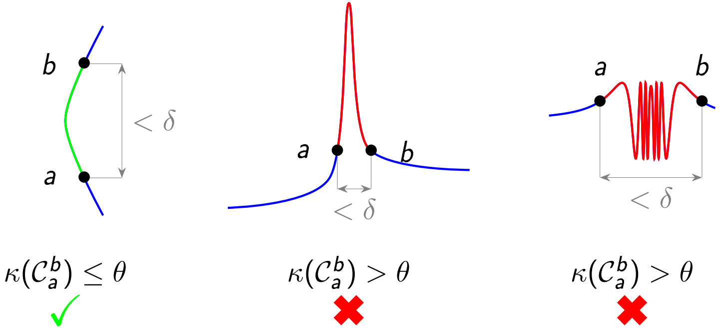
For any Jordan curve $\mathcal{C}$, $\kappa(\mathcal{C}) \geq 2\pi$.
Equality only on the convex case.

Definition of Local turn-boundedness

Definition

On a locally turn-bounded curve $\mathcal{C}$ with parameters (θ, δ):

$$a, b \in \mathcal{C}, ||b - a|| < \delta \Rightarrow \exists \mathcal{C}_a^b, \kappa(\mathcal{C}_a^b) \leq \theta.$$

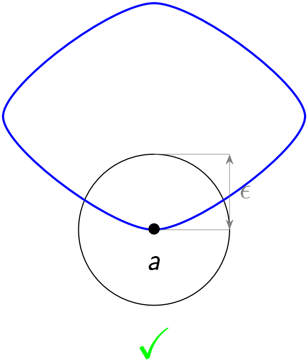


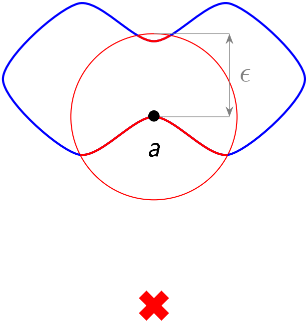
Local connectedness

Proposition

- *$\mathcal{C}$ Jordan curve locally turn-bounded with parameters $(\theta \in (0, \pi/2], \delta)$,*
- *$a \in \mathcal{C}$, $\epsilon \leq \delta$.*

Then $\mathcal{C} \cap B(a, \epsilon)$ is path-connected.

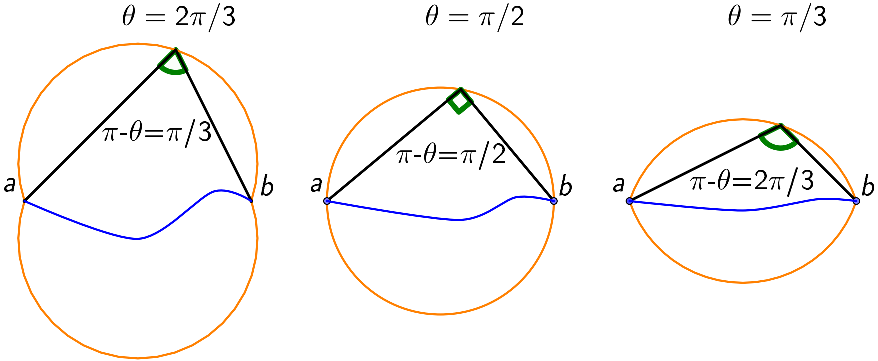




Proposition

- $\mathcal{C}$ *a simple curve locally turn-bounded with parameters* $(\theta \in (0, \pi), \delta)$,
- $\|a - b\|_2 < \delta$.

Then $\mathcal{C}_a^b$ *bound by the orange curve.*

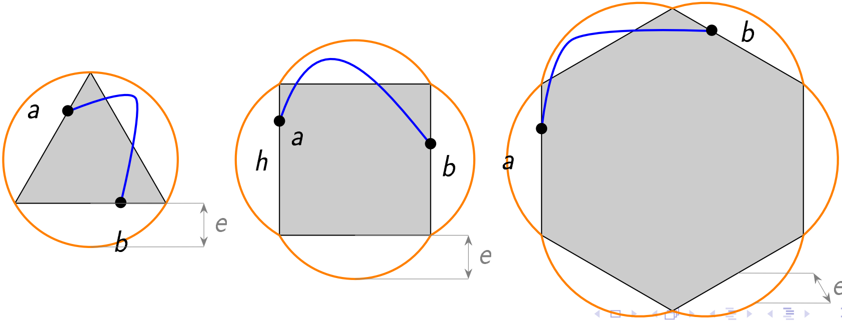


Control of the curve thank the turn-step

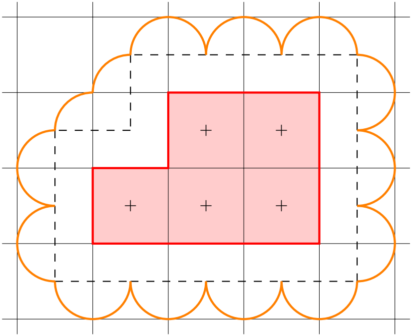
Proposition (Hausdorff distance between the curve and a pixel)

- *T a n-regular polygon with $n = 3, 4, 6$,*
- *$\mathcal{C}$ a locally turn-bounded Jordan curve with parameters $(\theta < 2\pi/n, \delta > h\sqrt{n-2})$.*

Then the arc delimited by the first and the last intersection of $\mathcal{C}$ and T is bound by the orange curve.



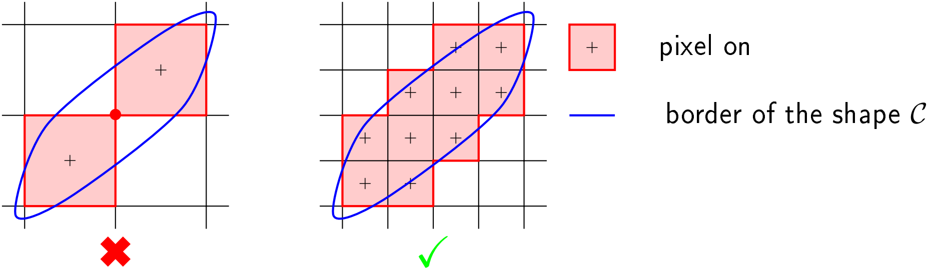
Control of the curve thank the turn-step




pixel on

domain where $\mathcal{C}$ lies

Well-composedness



Proposition

- *$\mathcal{C}$ locally turn-bounded Jordan curve with parameters* $(\theta \in (0, \pi/2], \delta)$,
- $\delta \leq$ *diam*$(\mathcal{C})$,
- *grid step* $h < \delta/\sqrt{2}$.

Then, the Gauss digitization of $\mathcal{C}$ is almost surely well-composed.

Future work

- Well-composedness without “almost surely”
- 4-connectedness of the digitized shape.
- Link between θ-turn step and par(r)-regularity.
- Estimation of geometric features.

Thanks for your attention.

References I

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References II

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