Distance between separating circles and points

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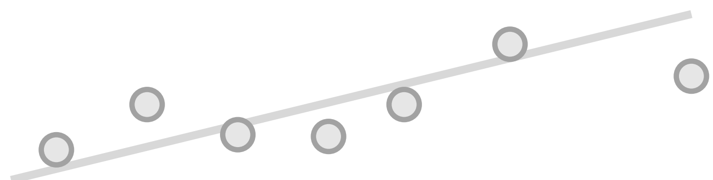
An important goal of digital geometry

To express a

a property that involves an infinite number of geometric representants

in terms of

a finite set of geometric conditions that are easy to verify



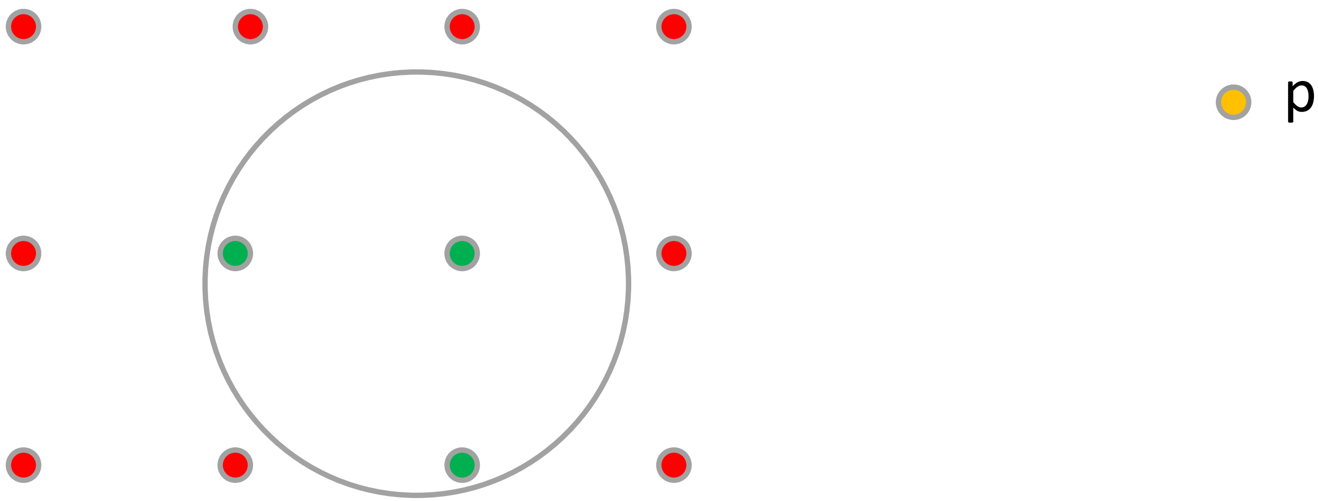
Classical example

the points of S lie at distance < 1 from a straight line

if

each of its 3-point subsets lie at distance < 1 from a straight line

Goal of this talk

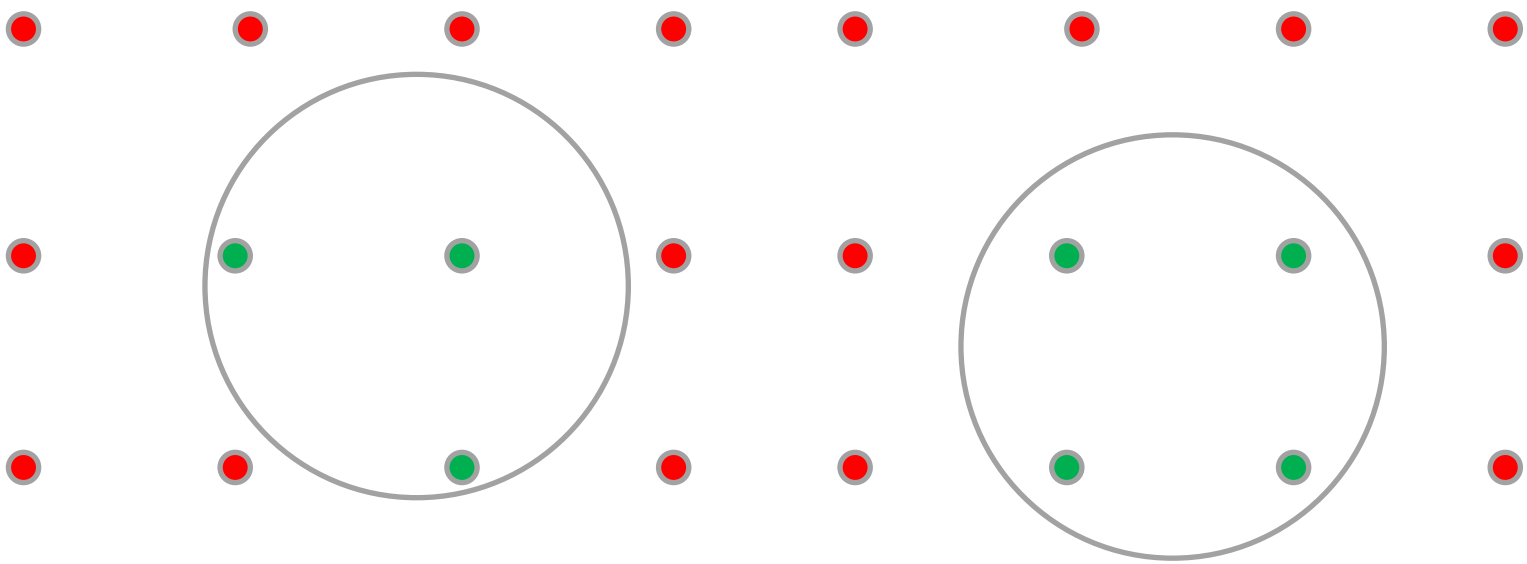


Given: an infinite family of separating circles

Problem: Find the smallest (largest) distance between p and this infinite family

Solution: Try to express this smallest distance by a finite set of circles

Broader context

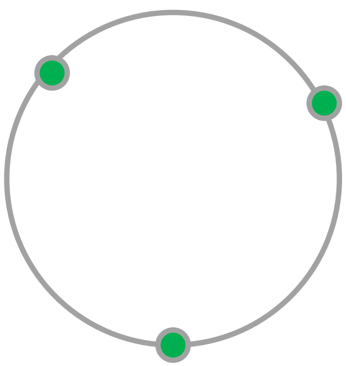


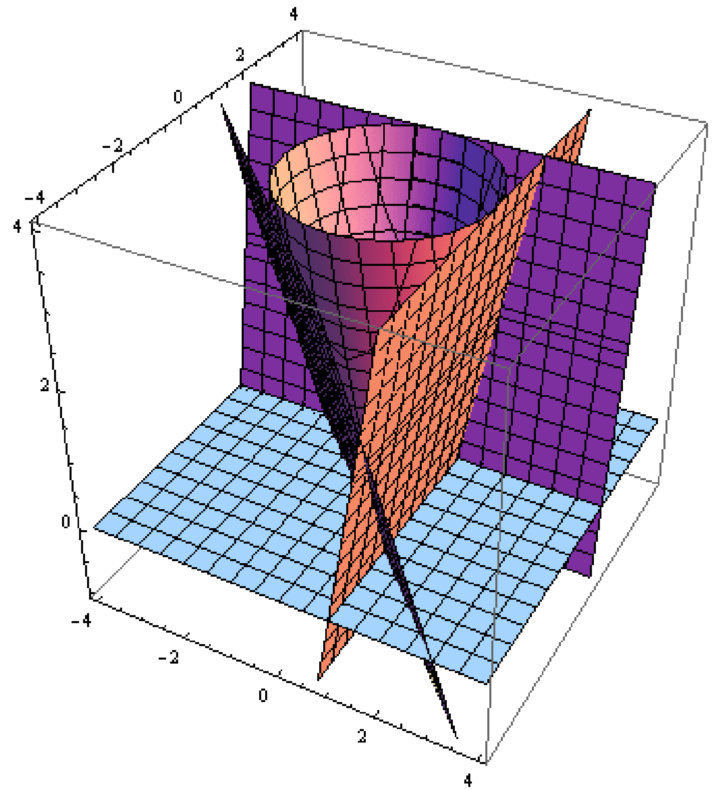
- Distance between two sets of separating circles
- Intersection problems
- Tangents …

Overview

- Infinite collections of circles
- Elementary circular separations
  - Circular and linear separability
- Properties that translate our infinite into a finite problem
  - Area covered by circles
  - Distance between point and circles

Circle passing through 3 points





Equation with linear parameters

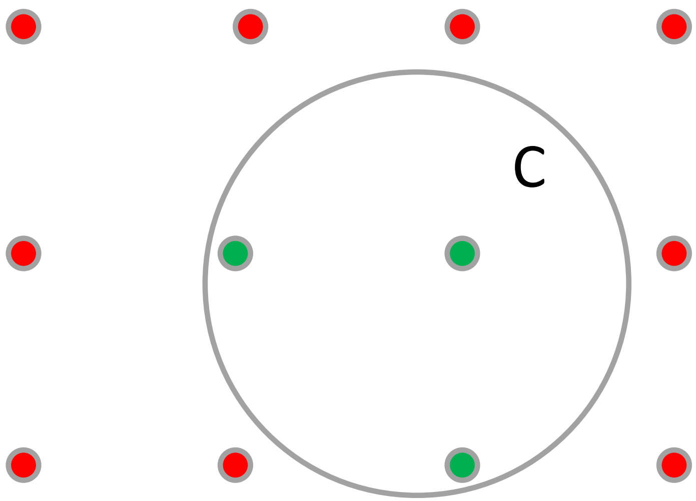
$$x^2 + y^2 - 2ax - 2by + c = 0$$

$$c \leq a^2 + b^2$$

Plane = parameters of all circles passing through 1 point.

Intersection 3 planes = parameters of 1 circle

Domain of separating circles



Dom(C)

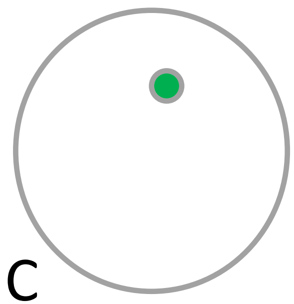
The separating circles define a domain bounded by

$$x_i^2 + y_i^2 - 2ax_i - 2by_i + c \leq 0, \quad (x_i, y_i) \in S^-$$

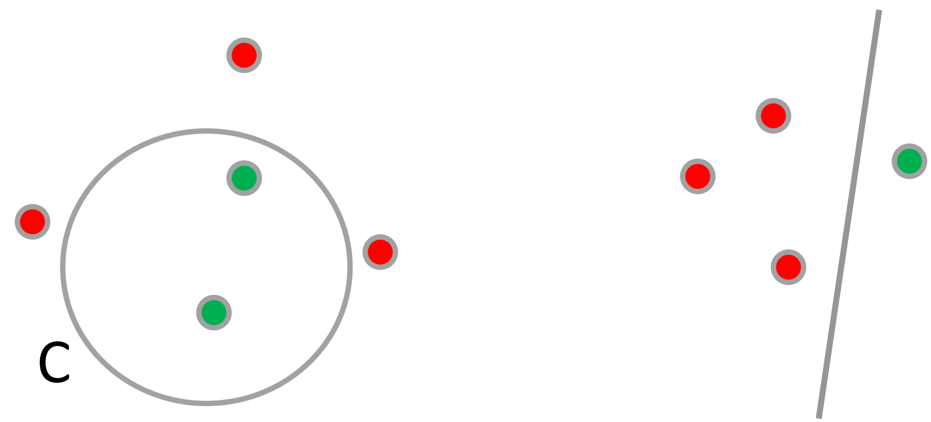
$$x_j^2 + y_j^2 - 2ax_j - 2by_j + c \geq 0, \quad (x_j, y_j) \in S^+$$

$$c \leq a^2 + b^2$$

Is a domain always a polytope?



Domain is a H-polyhedron
if S^- is non-empty



Domain is a (bounded) polytope
if S^- and S^+ cannot be
separated by a straight line

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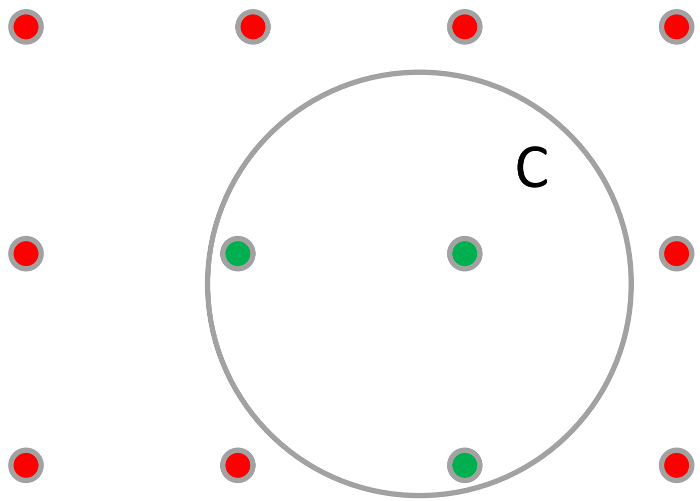
Elementary circular separations



We want to characterize a domain by a finite set of circles.
This leads to **elementary circular separations**.

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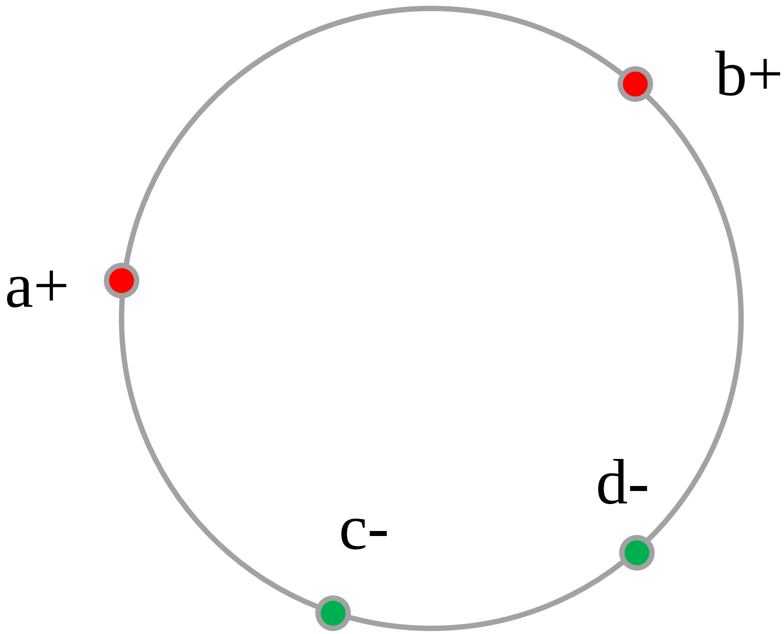
Elementary circular separations

Definition

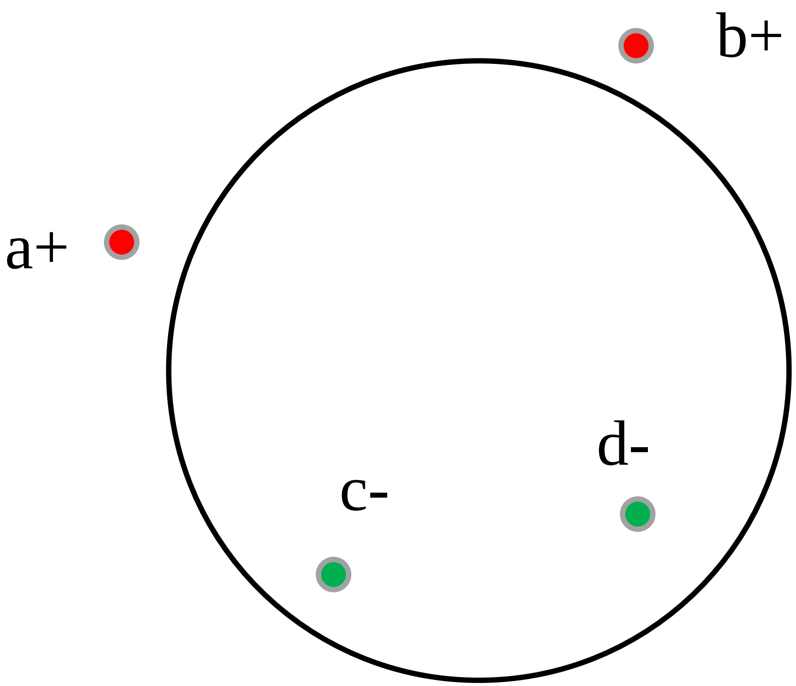
Let a, b, c, ... be N > 2 points on common circle

Introduce signs, e.g., $a^+b^+c^-d^-$



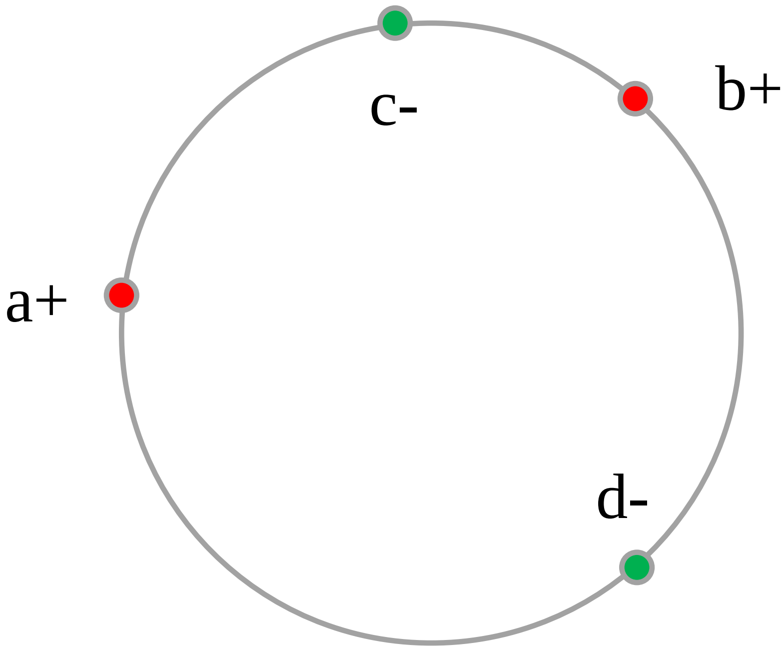
Then $a^+b^+c^-d^-$ is an elementary circular separation if …

Elementary circular separations



… if there is a second circle that separates a+ b+ from c- d-

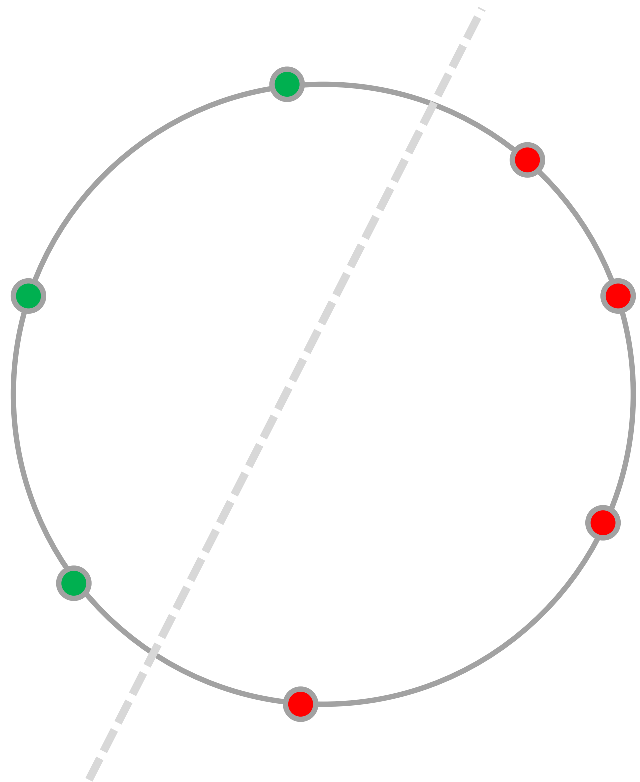
Elementary circular separations



Here a+ b+ c- d- is NOT an elementary circular separation …

Circular and linear separability

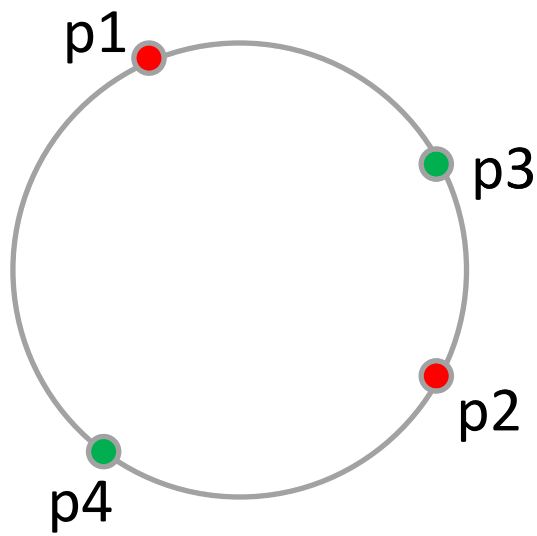
Property. We have an elementary circular separation if and only if the + points can be linearly separated from the - points.



Circular and linear separability

Proof that separating circle is impossible

Given: 4 points on a circle that cannot be separated linearly



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Circular and linear separability

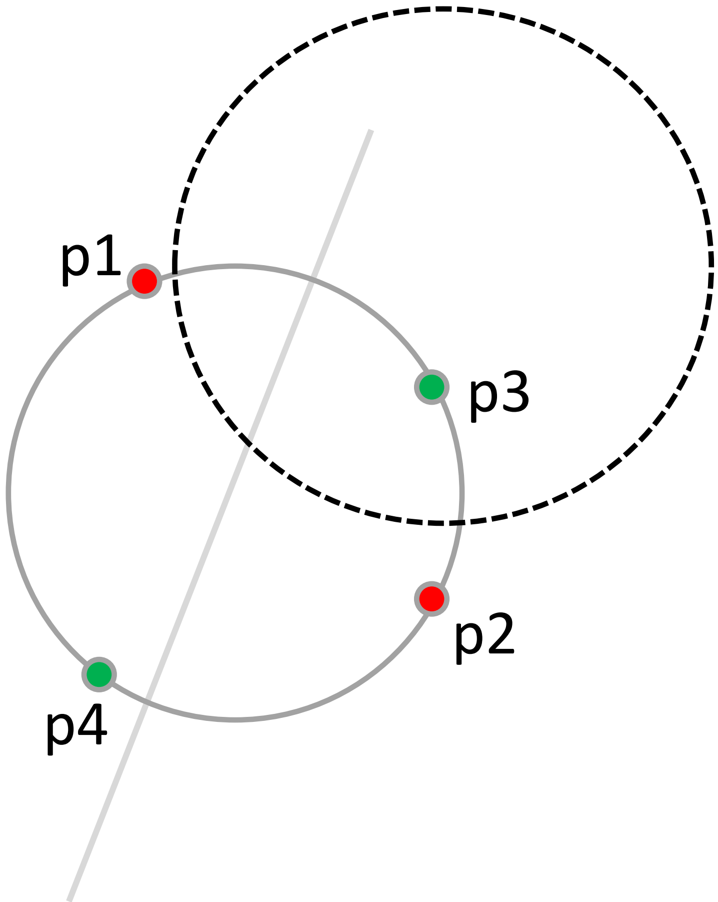
Proof that separating circle is impossible

Construct bisector of p1 and p3

- p1 outside
- p3 inside

⟹

center of separating circle must lie in halfplane containing p3



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Circular and linear separability

- p1 outside
- p3 inside
- p2 outside

⇒

center must lie in S3

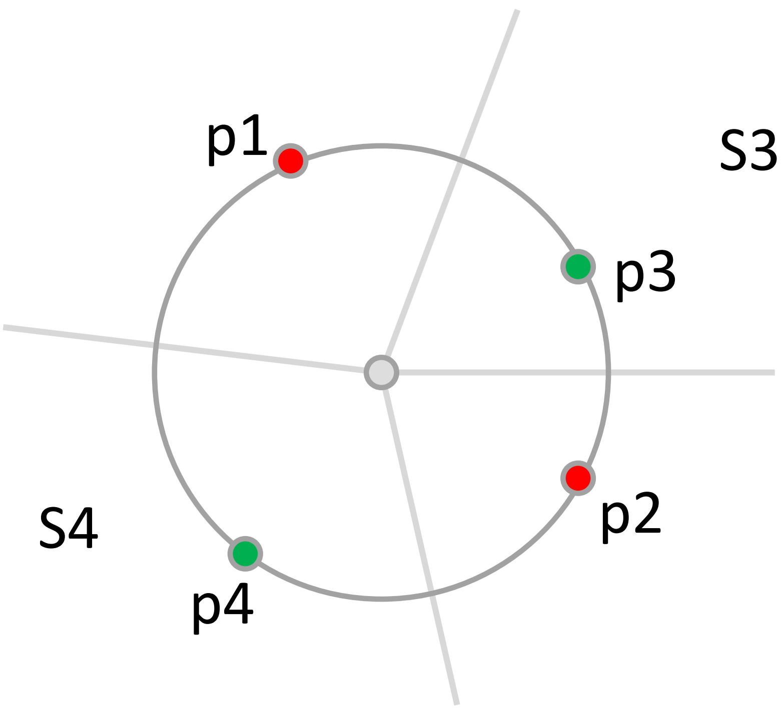
- p1 outside
- p4 inside
- p2 outside

⇒

center must lie in S4 and S3

⇒

impossible

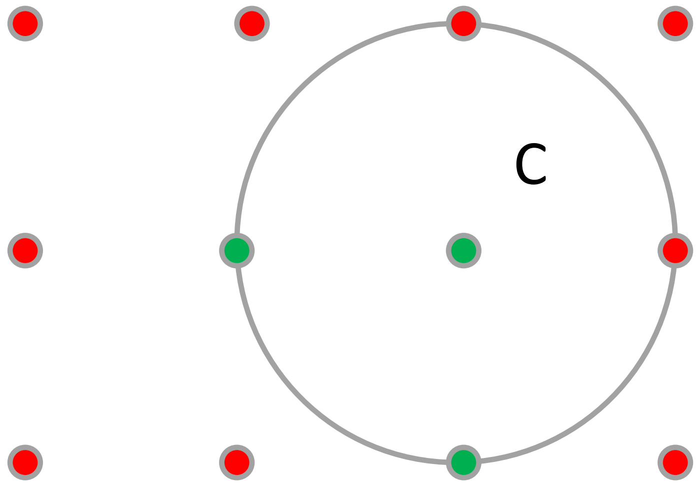


Elementary circular separations

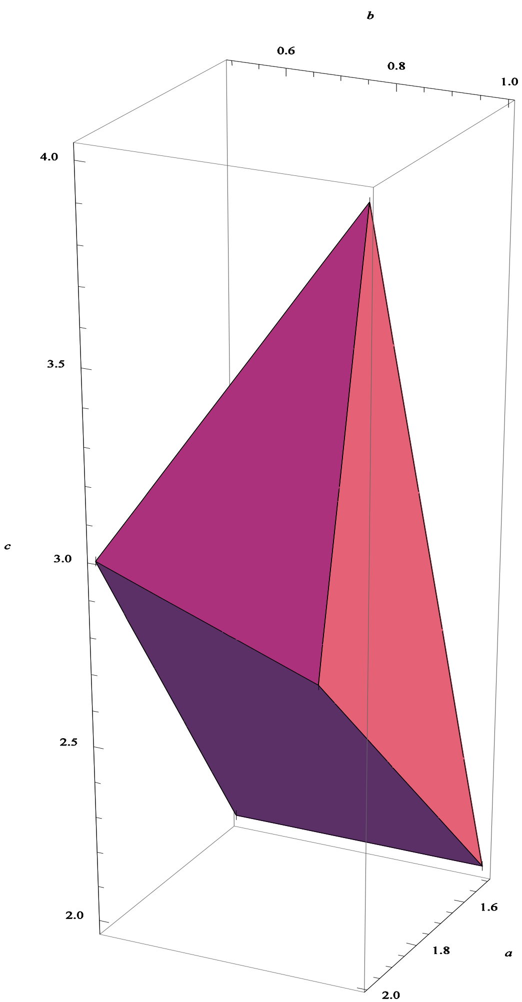
An elementary circular separation characterizes a domain unambigously when we attribute signs to the remaining points of S

It is a **minimal** characterizing subset of a signed set S.

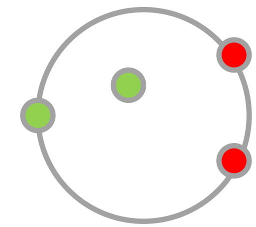
However, it is **not unique** (unless we impose additional constraints, such as an order on the points of S).



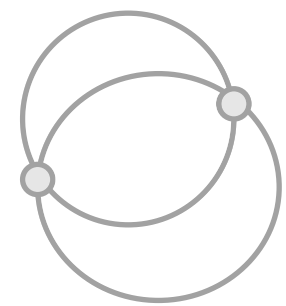
Elementary circular separations



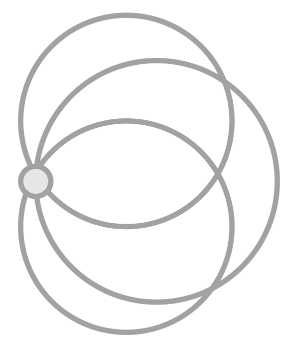
Each **vertex** of a domain corresponds to an elementary circular separation



Each **edge** corresponds to a pencil of circles passing through two common points.



Each **face** corresponds to a pencil of circles passing through one common point.



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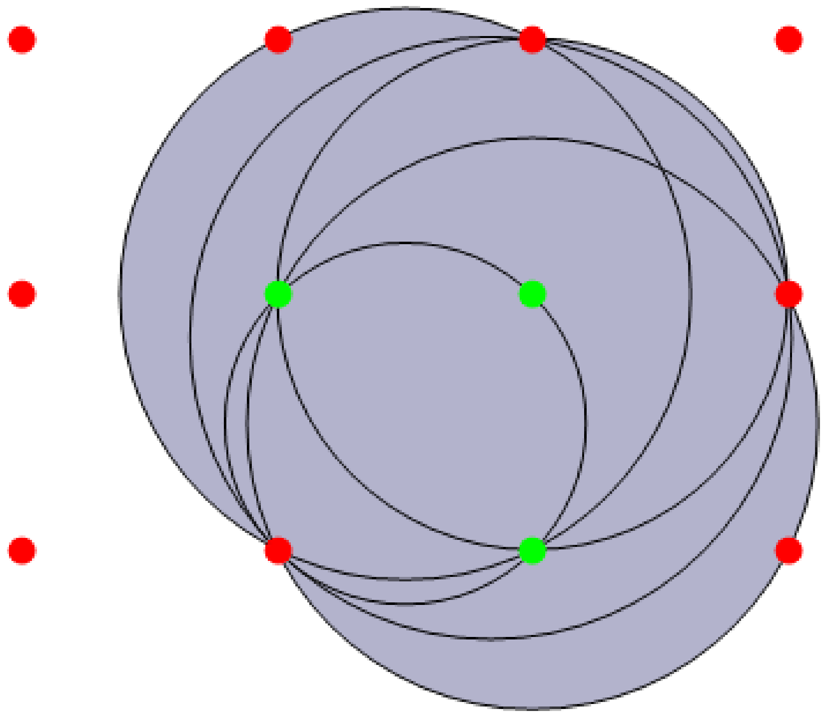
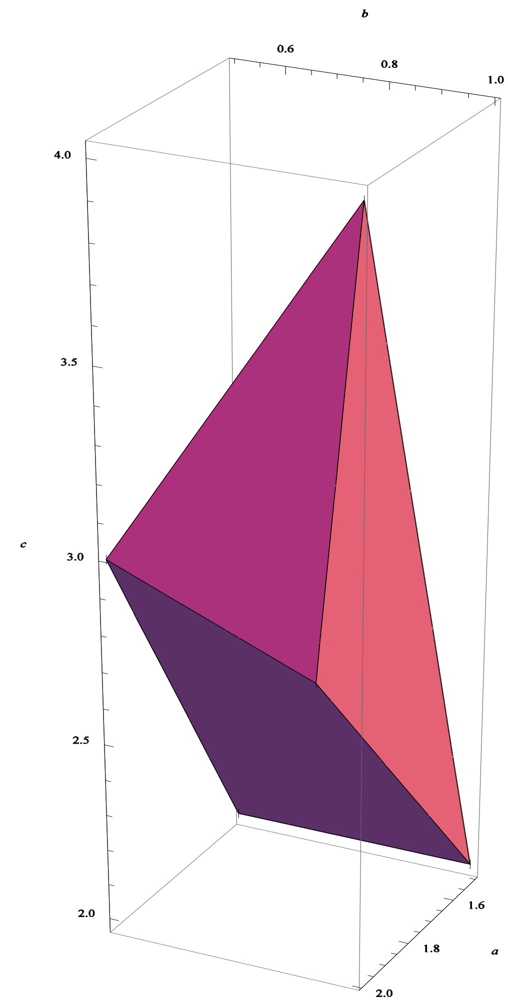
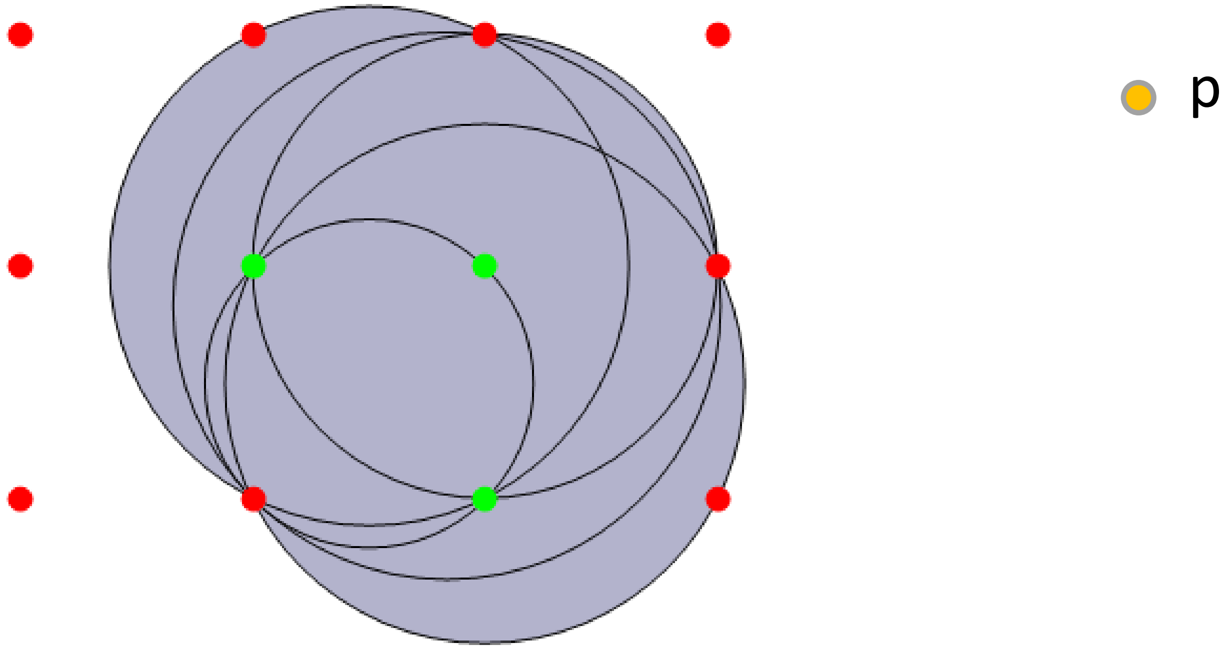
Area covered





Property. Area covered by all circles of domain is the same as area covered by circles of elementary circular separations.

Distance between point and circles



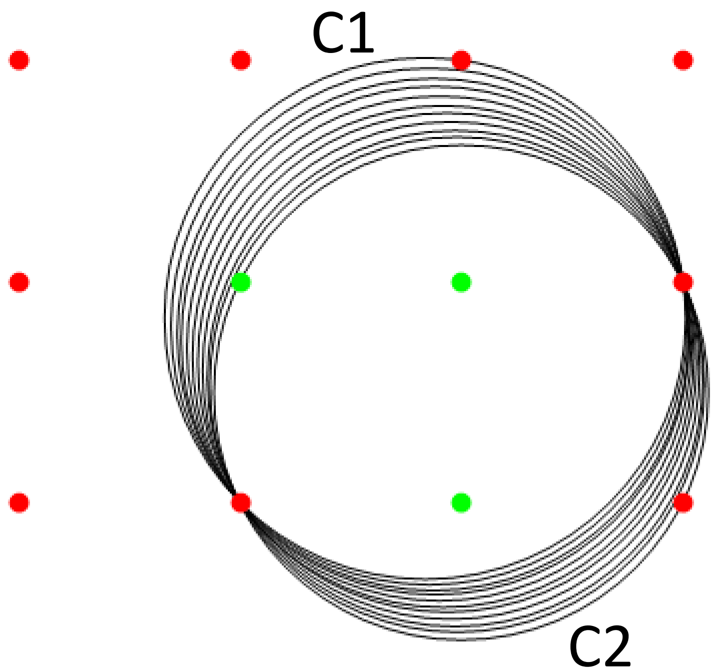
Theorem (for points outside covered area)

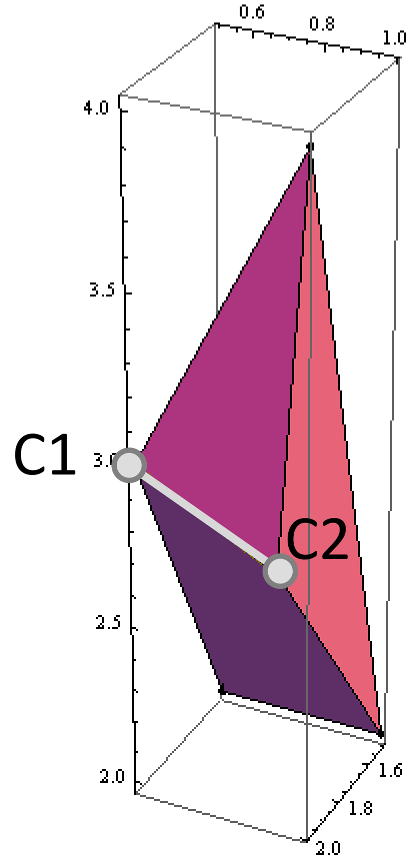
smallest (largest) distance between p and any member of separating family is equal to

smallest (largest) distance between p and circles that correspond to elementary circular separations of domain

Sketch of proof

Sketch of proof smallest distance (along an edge of the domain)





Sketch of proof

Sketch of part of proof (along an edge of the domain)

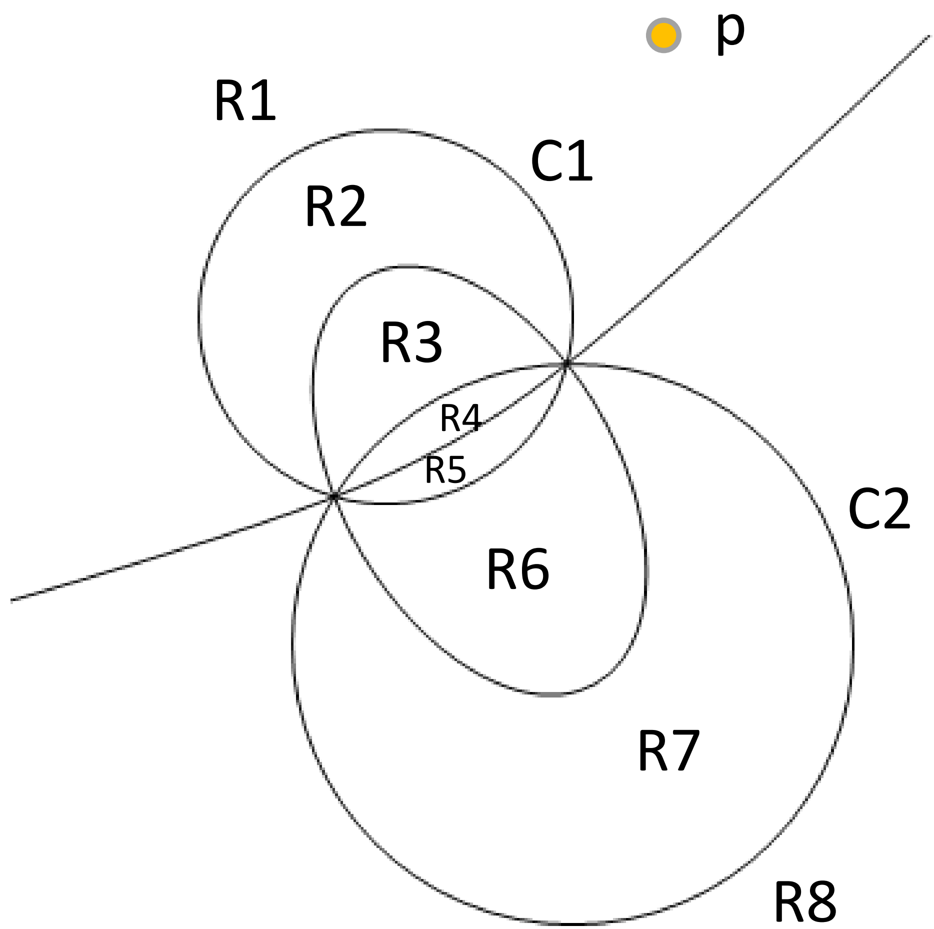
If p in R2, R3, R6 or R7 then there is a circle passing through p

If p in R1 or R5 then closest circle is C1

If p in R4 or R8 then closest circle is C2

⇒

Either distance is zero or closest circle is C1 or C2



Concluding remarks

- Elementary separation is a general concept (also possible for lines, planes, …)
- Proofs are not difficult but require some care
- The computation of the domain is the most time consuming part