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Outline

Sparse Object Representations by Digital Distance Functions

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  - Thin
  - Centered
  - Topology preserving
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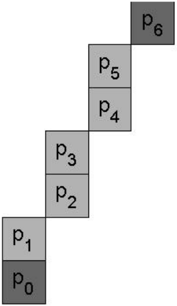
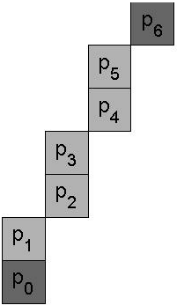
Weighted distances

The path below is a minimal cost path between $\mathbf{p}_0$ and $\mathbf{p}_6$ of cost $3\alpha + 3\beta$.



Distances based on neighborhood sequence

The path is a B-path for $B = (2, 2, 2, \dots) = (2)$ and $B = (1, 2)$, but not for $B = (1)$.



Weighted neighborhood sequences

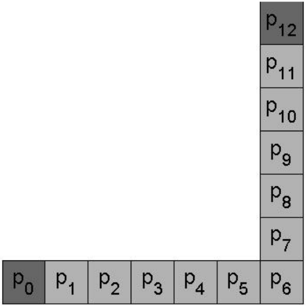
Definition

Given a neighborhood sequence (ns) B and two weights α and β, a path $\mathcal{P}$ is a *minimal cost* (α, β)*-weighted* B*-path* between two points $\mathbf{p}$ and $\mathbf{q}$ if

- $\mathcal{P}$ is a B-path and
- there is no other B-path between $\mathbf{p}$ and $\mathbf{q}$ with lower cost.

Weighted neighborhood sequences

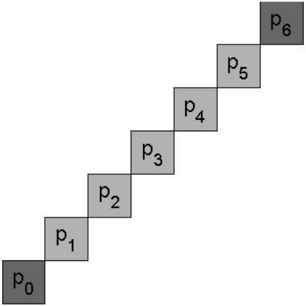
Let $B = (1, 2)$ and $\alpha = 2$, $\beta = 3$.



This path is a B-path of cost $12\alpha = 24$.

Weighted neighborhood sequences

Let $B = (1, 2)$ and $\alpha = 2$, $\beta = 3$.



This path is of cost $6\beta = 18$. It is *not* a B-path.

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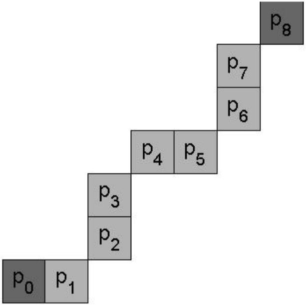
Weighted neighborhood sequences

Let $B = (1, 2)$ and $\alpha = 2$, $\beta = 3$.



This path is a *minimal cost* B-path. Its cost is $4\alpha + 4\beta = 20$.

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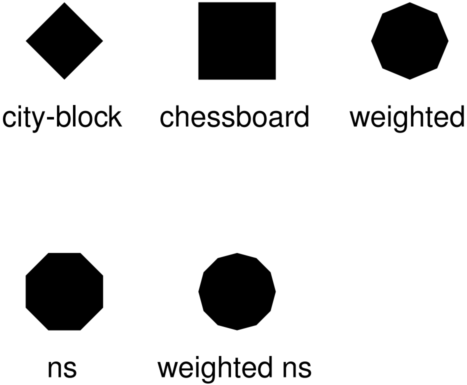
Path-based distances



Maximal balls

- A ball in an object X is a *maximal ball* if there is no other ball in X containing.
- An object X is equal to the union of its set of *maximal balls*.
- For the weighted distance with $\alpha = 3$ and $\beta = 4$, the set of centers of maximal balls can be obtained by a local check (finding the local maxima) in the distance transform. Relabeling of distance values that can not be attained is needed.
- Is this true also for (weighted) distances based on neighborhood sequences? To answer this question, we introduce the notion of maximal path-points, local B-maxima and local B^*-maxima.

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Distance transform

X is the object, $\overline{X}$ is the background.
Given B and weights α, β, the value of $DT_{\mathcal{C}}(\mathbf{p})$ for any point $\mathbf{p} \in X$ is

$$\min_{\mathbf{q} \in \overline{X}} \{\text{cost of the minimal cost } B\text{-path between } \mathbf{p} \text{ and } \mathbf{q}\}.$$

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Local maxima

Definition

A point $\mathbf{p} \in X$ is a local maximum if $DT_{\mathcal{C}}(\mathbf{p}) > DT_{\mathcal{C}}(\mathbf{p} + \mathbf{n}_i) - w_i$, where the set $\{\mathbf{n}_i\}$ defines the neighborhood at $\mathbf{p}$ and w_i are the corresponding weights.

3	3	3	3
3	6	4	3
3	3	3	

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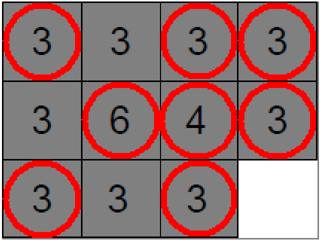

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Relabeled distance values

1	1	1	1
1	5	4	1
1	1	1	

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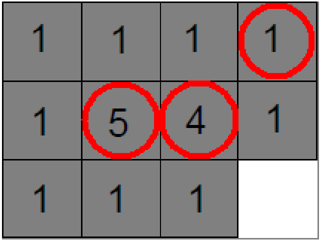

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Which neighborhood should be used?

- weighted distances – fixed neighborhood.
- ns-distances – the size of the neighborhood is determined by the length of the path $=$ the distance value.
- weighted ns-distances – the size of the neighborhood can *not* be determined by $DT_{\mathcal{C}}$.

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Maximal path-points

Definition

A point $\mathbf{p} \in X$ is a *maximal path-point* if there is a point $\mathbf{q} \in \overline{X}$ such that

- $\langle \mathbf{q} = \mathbf{p}_0, \mathbf{p}_1, \ldots, \mathbf{p}_n = \mathbf{p} \rangle$ is a minimal cost (α, β)-weighted B-path of length n that defines $DT_{\mathcal{C}}(\mathbf{p})$ *and*
- there is no $\mathbf{p}'$ such that $\langle \mathbf{q} = \mathbf{p}_0, \mathbf{p}_1, \ldots, \mathbf{p}_n, \mathbf{p}_{n+1} = \mathbf{p}' \rangle$ is a minimal cost (α, β)-weighted B-path of length $n+1$ that defines $DT_{\mathcal{C}}(\mathbf{p}')$.

We denote the set of maximal points by MP.

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Maximal path-points

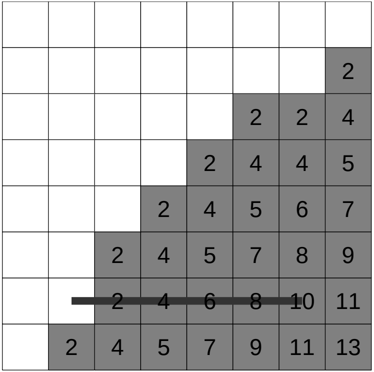

Figure: $DT_{\mathcal{C}}$, $B = (1, 2)$ and $(\alpha, \beta) = (2, 3)$

Path of length five.

Maximal path-points

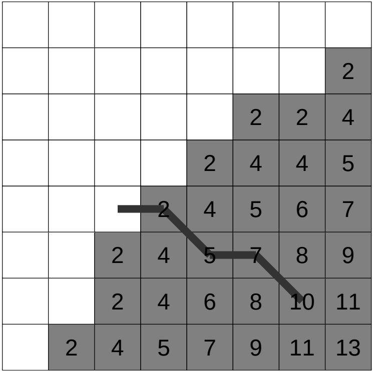

Figure: $DT_{\mathcal{C}}$, $B = (1, 2)$ and $(\alpha, \beta) = (2, 3)$

Path of length four. This path defines a maximal path point.

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Maximal path-points

							1
					1	1	2
				1	2	2	2
			1	2	2	3	3
		1	2	2	3	4	4
		1	2	3	4	4	5
	1	2	2	3	4	5	6

Figure: $DT_{\mathcal{L}}$, $B = (1, 2)$ and $(\alpha, \beta) = (2, 3)$

Representability, maximal path-points

Theorem

If either

$$\alpha < \beta \leq 2\alpha \text{ or} \tag{1}$$

$$\alpha = \beta \text{ and } (B = (1) \text{ or } B = (2)), \tag{2}$$

then X can be recovered from the set of maximal path-points.

Note: not ns-distances in general!

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Smallest neighborhood

Definition

Given an object X, a ns B and weights α, β, we say that $DT_{\mathcal{L}}$ associated with $DT_{\mathcal{C}}$ *holds information about the smallest neighborhoods* if it holds information about the minimal cost-path with smallest size of the neighborhood at each point of X.

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Local B-maxima

A local B-maximum is a local maximum obtained by using the neighborhood defined by $b\left(DT_{\mathcal{L}}\left(\mathbf{p}\right)+1\right)$ at the point $\mathbf{p}$.

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Local B-maxima

Theorem

If $DT_{\mathcal{L}}$ holds information about the smallest neighborhoods, then the set of maximal path points equals the set of local B-maxima.

This theorem gives an efficient way to compute this set of points.

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Local B^*-maxima

A local B^*-maximum is a local maximum obtained by using the neighborhood defined by $b\left(DT_{\mathcal{L}}\left(\mathbf{p}\right)\right)$ at the point $\mathbf{p}$.

Representability, local B^*-maxima

Theorem

If $DT_{\mathcal{L}}$ holds information about the smallest neighborhoods, then the original object can be recovered from the set of local B^*-maxima.

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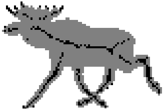
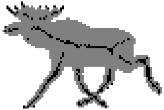
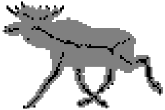
Examples

local B-maxima (the set MP)	local B^*-maxima	CMBs
$B = (1), (\alpha, \beta) = (1, 1)$		
 222 elements	 222 elements	 222 elements

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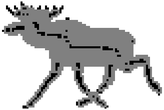
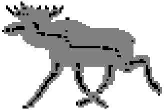
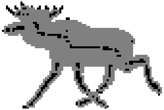
Examples

local B-maxima (the set MP)	local B^*-maxima	CMBs
$B = (2)$, $(\alpha, \beta) = (1, 1)$		
 250 elements	 250 elements	 250 elements

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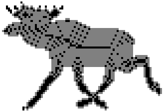
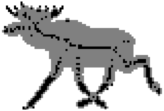
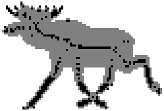
local B-maxima (the set MP)	local B^*-maxima	CMBs
$B = (1,2)$, $(\alpha, \beta) = (1,1)$		
 418 elements	 295 elements	 295 elements

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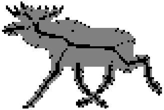
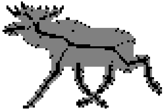
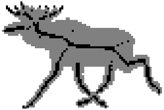
$B = (2)$, $(\alpha, \beta) = (3, 4)$		
 453 elements	 453 elements	 317 elements

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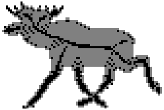
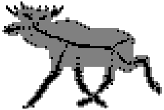
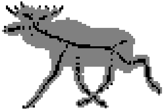
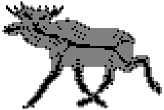
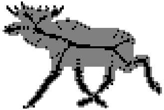
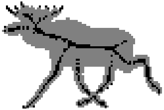
Examples

local B-maxima (the set MP)	local B^*-maxima	CMBs
$B = (1,2),\ (\alpha,\beta) = (2,3)$		
 451 elements	 406 elements	 278 elements

Examples

local B-maxima (the set MP)	local B^*-maxima	CMBs
$B = (1, 2, 1, 2, 2), (\alpha, \beta) = (4, 5)$		
 471 elements	 435 elements	 289 elements

Conclusions, Summary

- The maximal path-points are the points that do not propagate distance information when computing the DT.
- The set of maximal path-points MP can be computed efficiently by finding local B-maxima.
- Local B^*-maxima.
- If the weights α and β are equal, then the set of B-maxima is not reversible.
- When a constant neighborhood sequence is used, the two approaches are equal

Future work:

- Relabeling

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