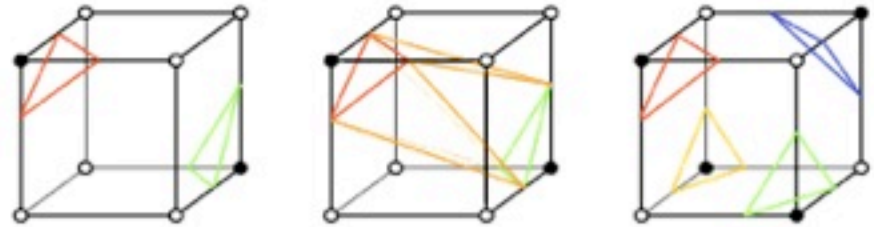
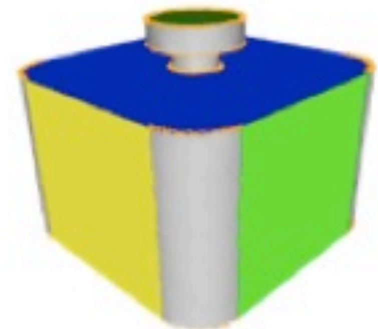
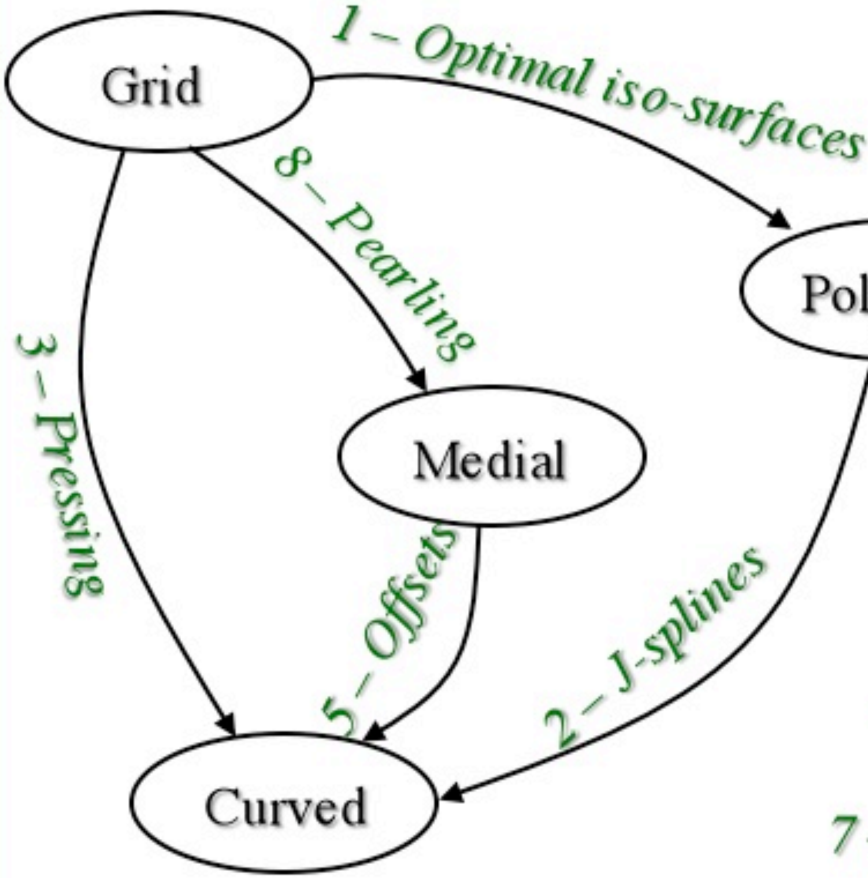
Ball-based measures, transformations, and animations

Jarek Rossignac





Outline

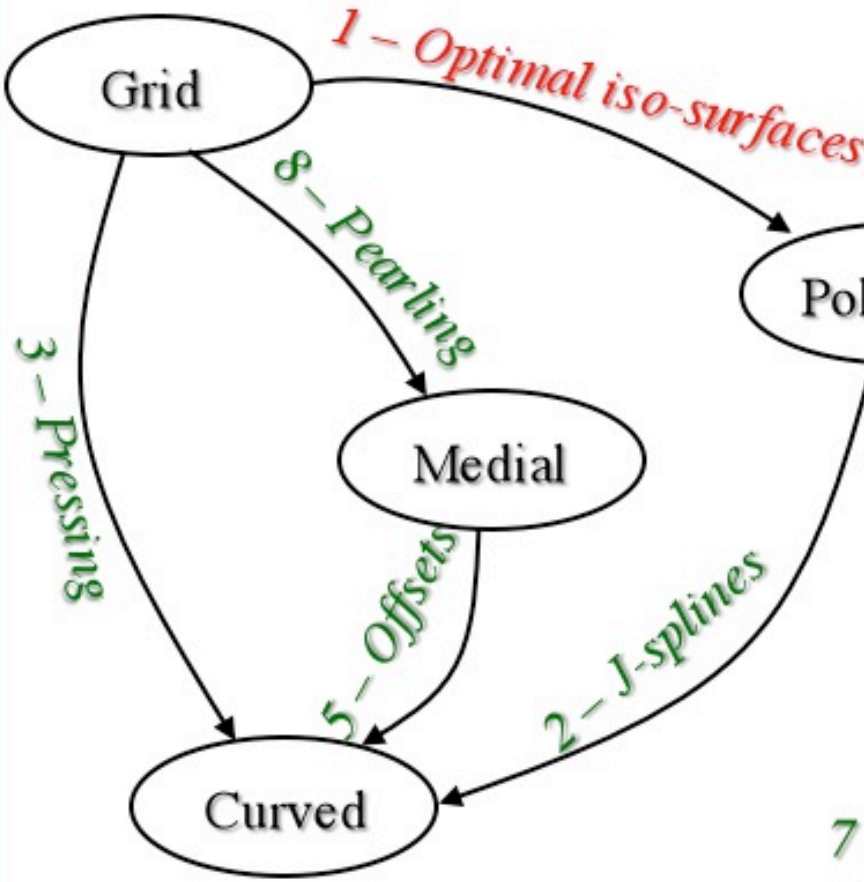


4 – Tightening

6 – Relative blending

7 – Ball-morphing

ISO-SURFACES



4 – Tightening

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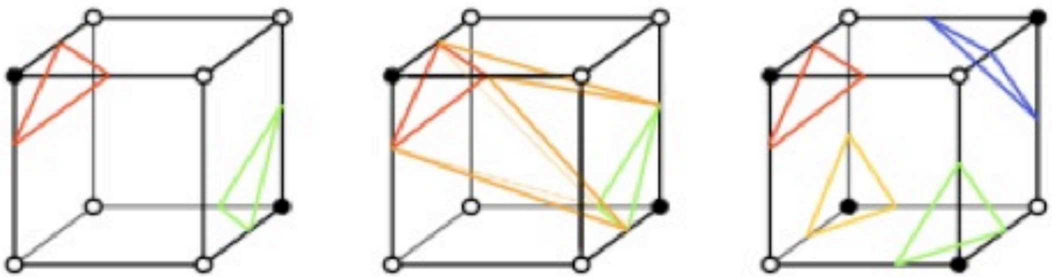
ISO-SURFACES

Goal: Minimize triangle count and optimize topology of a triangulated iso-surface

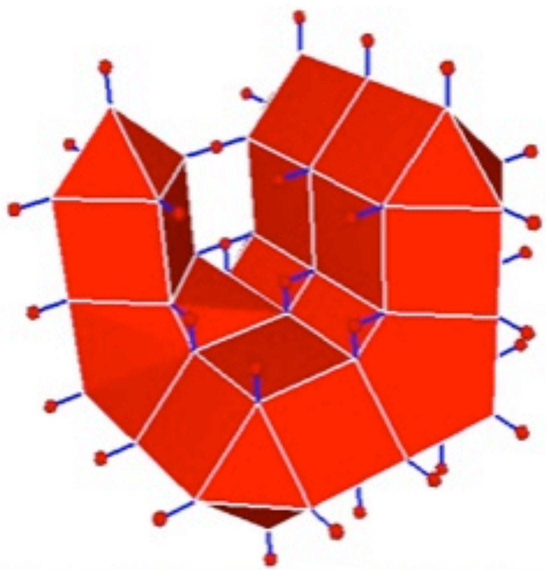


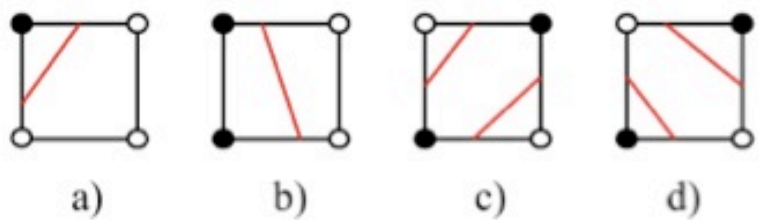
Optimizing the topological and combinatorial complexity of iso-Surfaces, C. Andujar, P. Brunet, A. Chica, I. Navazo, J. Rossignac, A. Vinacua. Journal of Computer-Aided Design & Applications, 37(8):847-857, 2005.

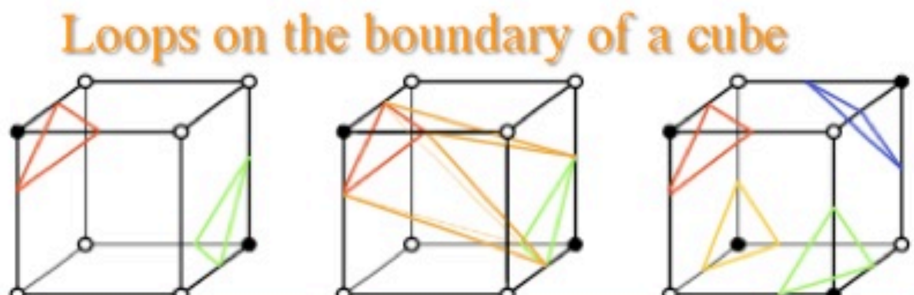
Optimal Iso-Surfaces, C. Andujar, P. Brunet, A. Chica, I. Navazo, J. Rossignac, A. Vinacua. Proc. CAD Conf. pp. 503-511, May 2004.

Iso-surface from discretized voxel model

- Iso-surface
  - Separates in and out samples
  - Has a vertex on each edge joining in and out samples
  - Is a manifold triangulation of these vertices
  - Does not cross axis-aligned edges anywhere else

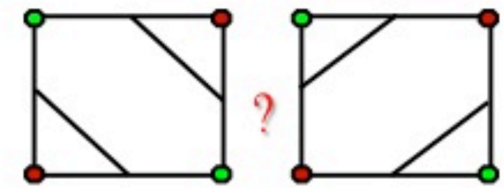




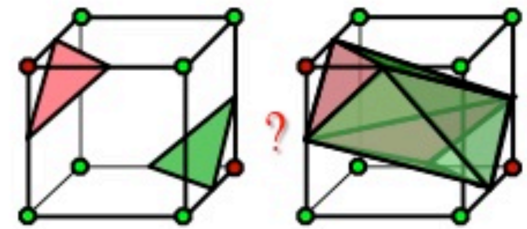


Options in Marching cubes

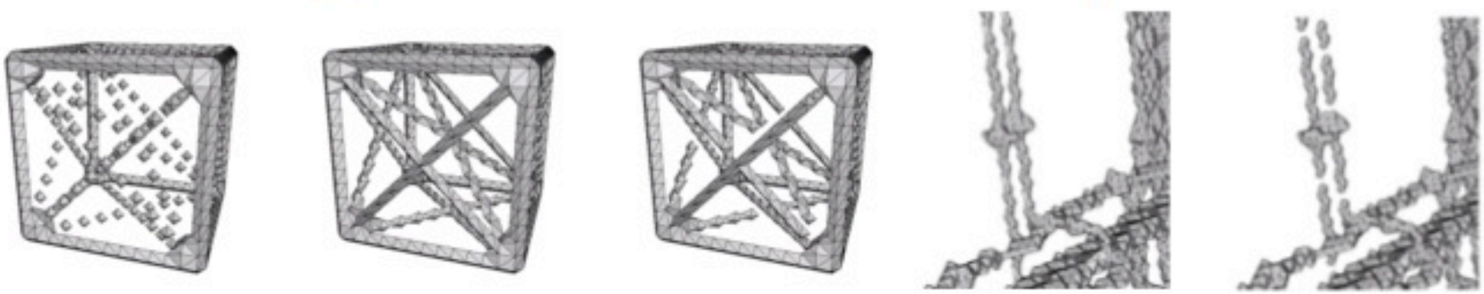
- How to cut an x-face?
  - Phase 1: Optimize number of loops



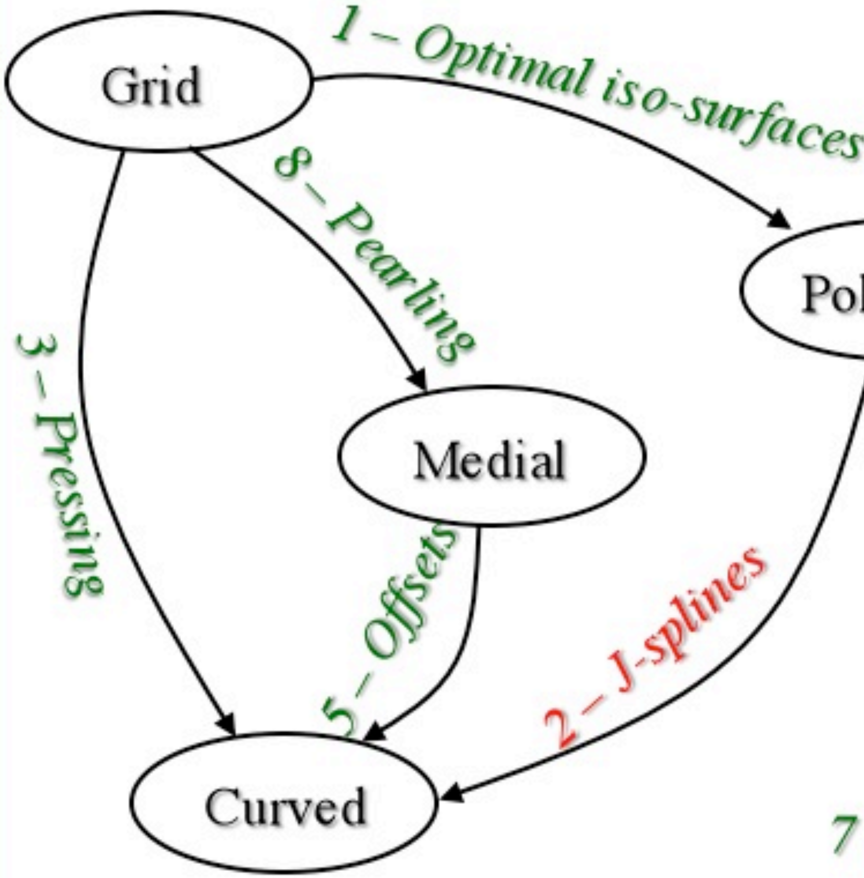
- How to triangulate the interior
  - Phase 2: Do not connect loops



- Make choices that “minimize topology”
  - Minimize triangle count T
  - Minimize genus or number of connected components



J-SPLINES



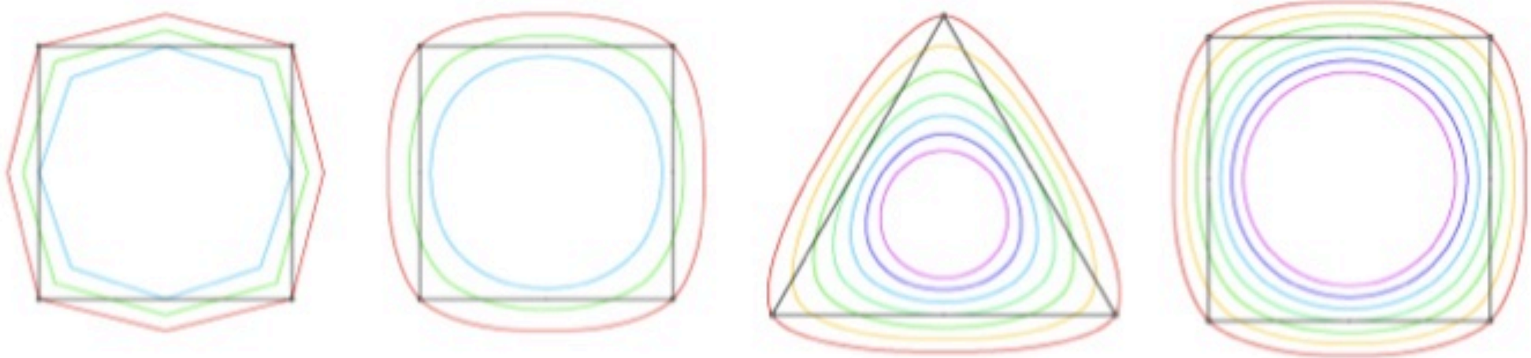
4 – Tightening

6 – Relative blending

7 – Ball-morphing

J-SPLINES

Goal: Increase smoothness of polygon and mesh subdivision schemes and reduce amount of temporary storage needed



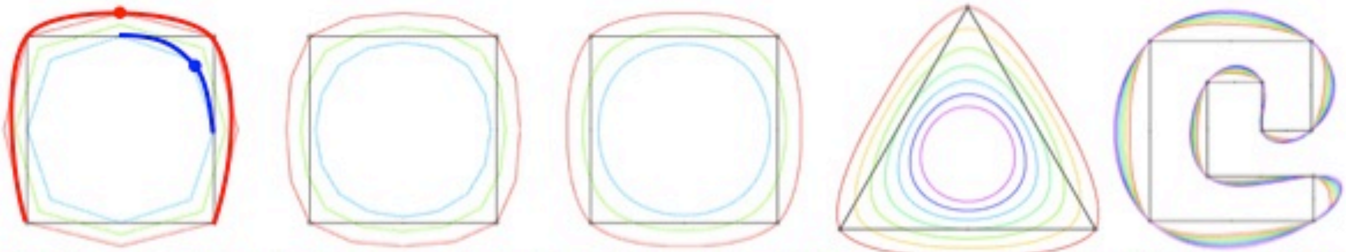
J-splines, J. Rossignac, S. Schaefer, Journal of Computer Aided-Design (JCAD), 40(10-11):1024-1032, Oct-Nov 2008

Ringing subdivision of curves and surfaces, J. Rossignac, A. Venkatesh, IEEE Computer Graphics & Applications, 2010

J-splines: Results

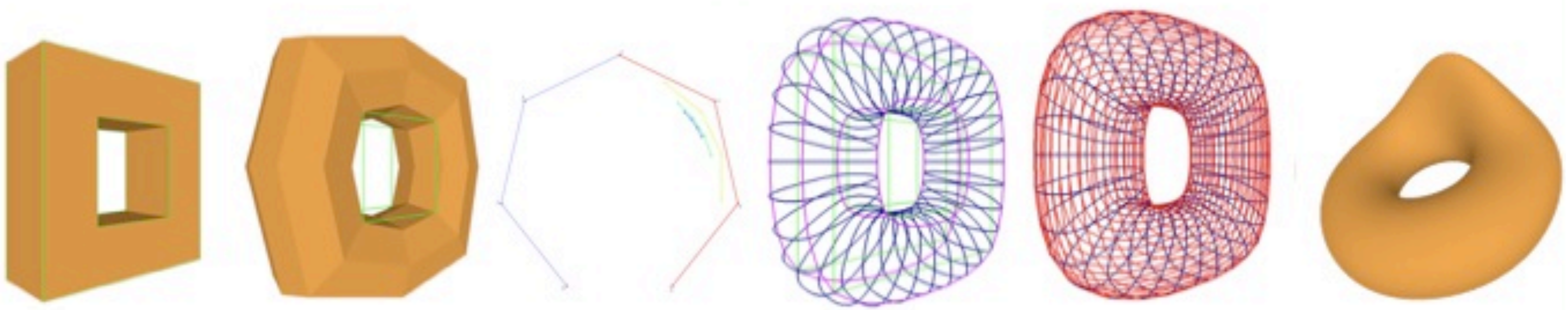
- 4-point: f_j, cubic B-spline: b_j, *J-spline:* $(1–s)f_j+sb_j$

  J_s is C^2 when $0<s<4$, C^3 when $1<s\leq 2.8$, and C^4 when $s=1.5$



"J-splines", J. Rossignac, S. Schaefer, Journal of Computer Aided-Design (JCAD). 40(10-11):1024-1032, October-November 2008.

- ***Ringing***: Reduces GPU footprint from $(n–5)2^r+5$ to **4r**

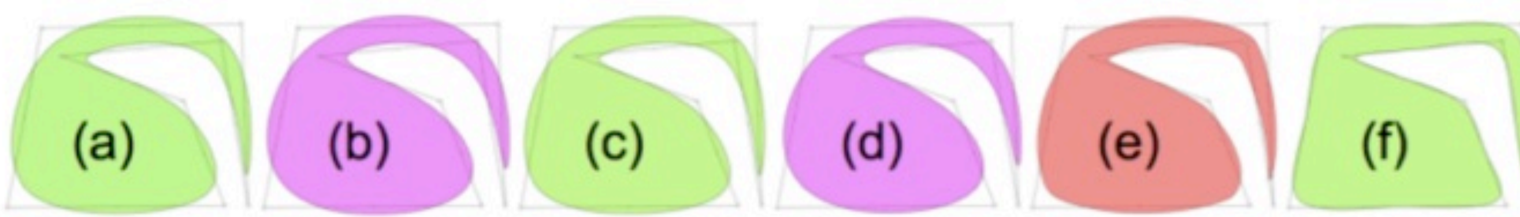


"Ringing: Frugal subdivision of curves and surfaces", J. Rossignac, A. Venkatesh, IEEE Computer Graphics and Applications (CG&A), 2010.

J-spline options

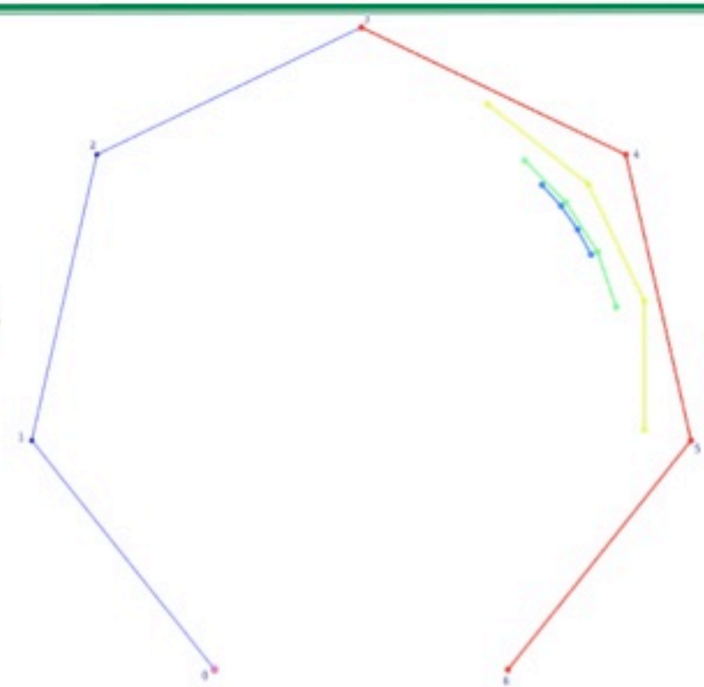
- Interpolate (4-points, adjust s, retrofit C^4)

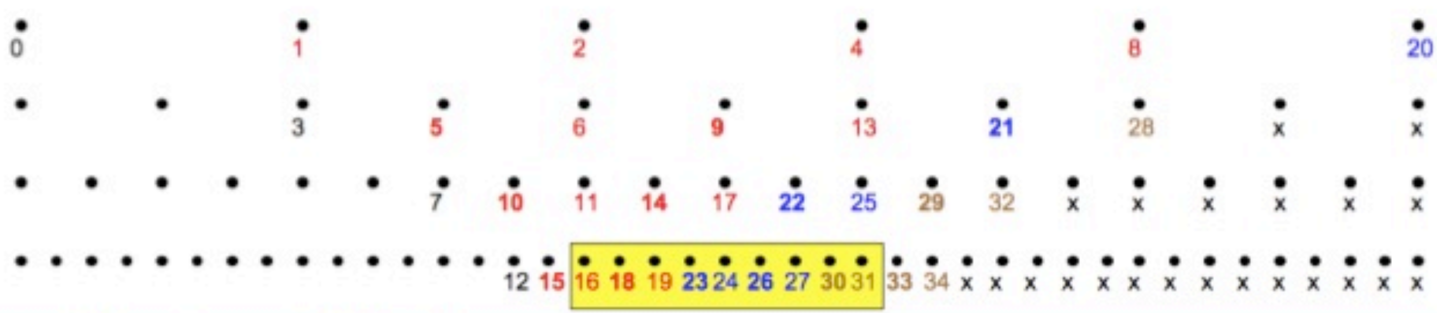
- Preserve area or perimeter length



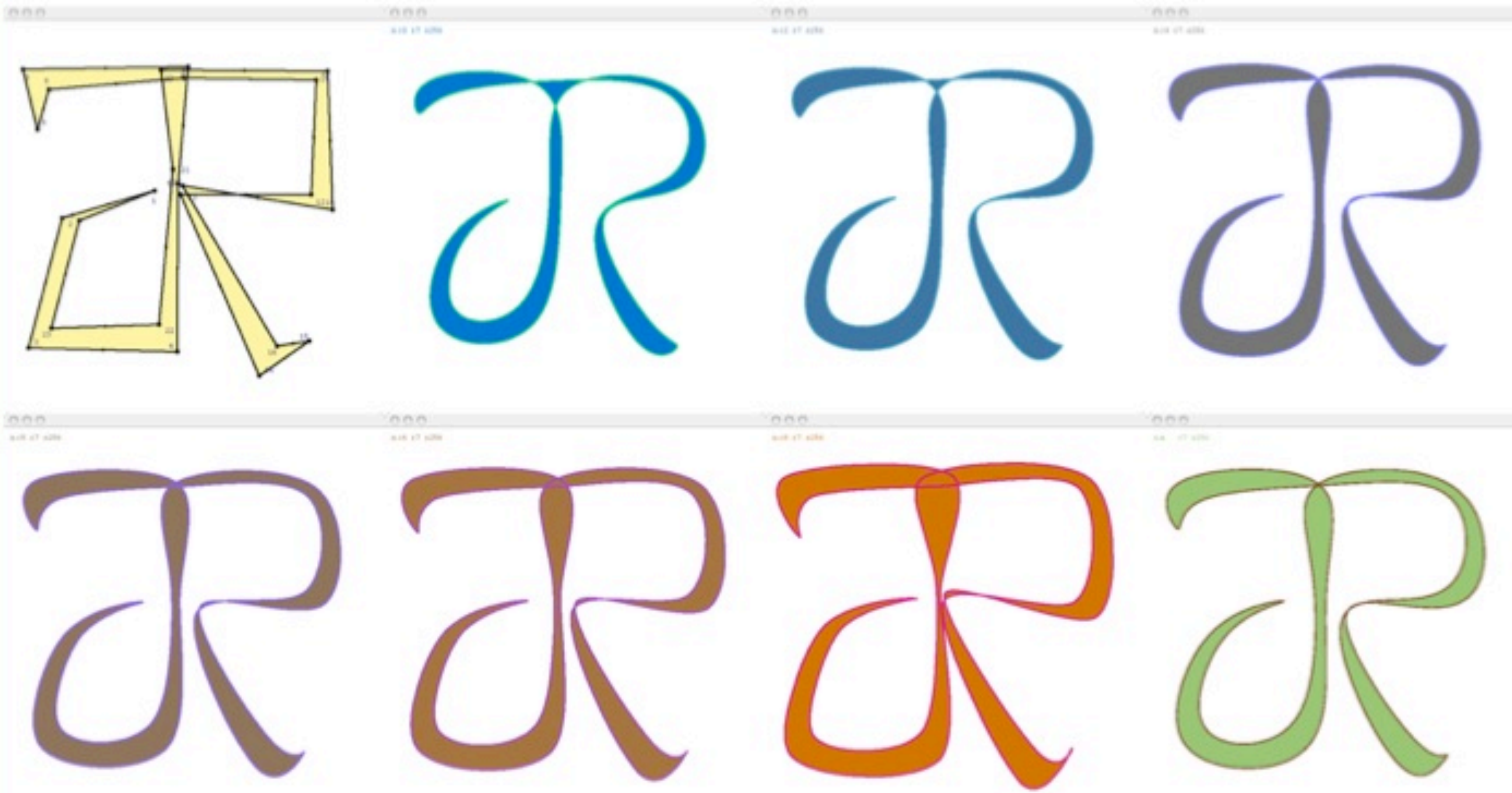
Ringing

- We evaluate points, one at a time, on the r^{th} level subdivision using only 4r point-registers to store results of intermediate calculations
  - Standard approach requires storing $(n–5)2^r+5$ points

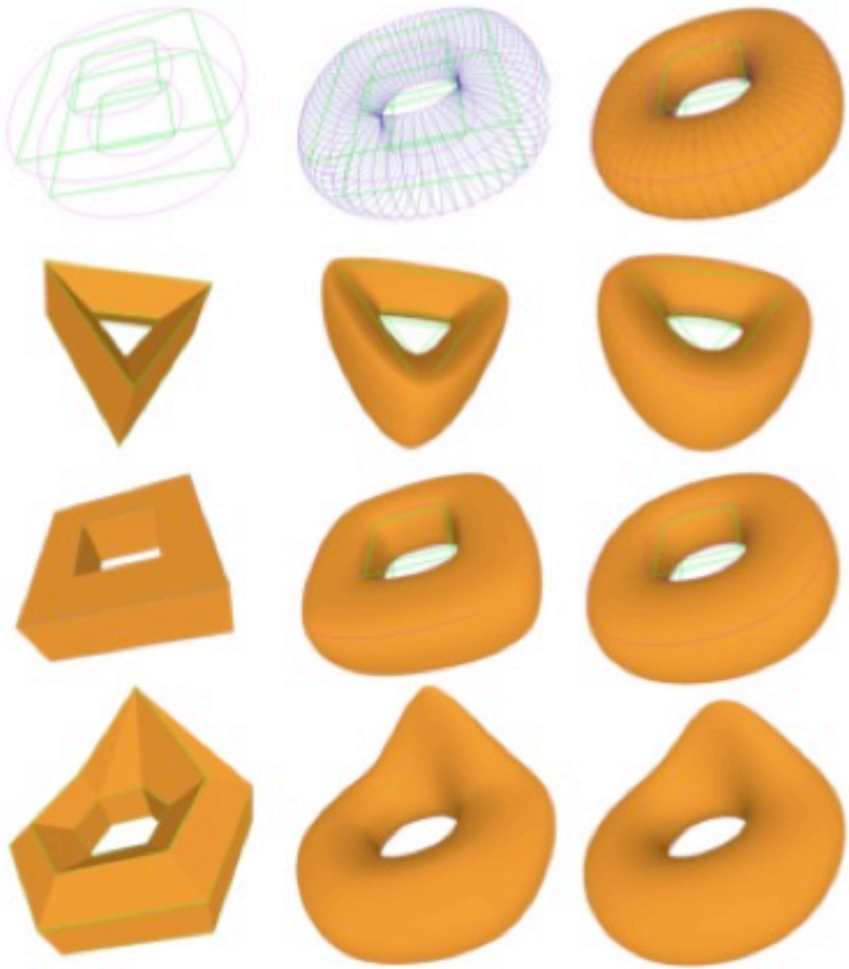




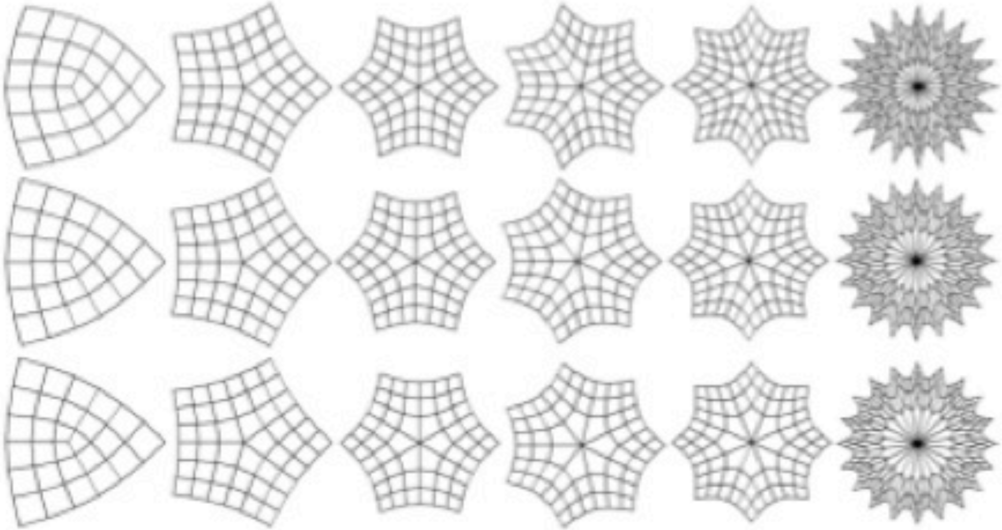
J-spline examples



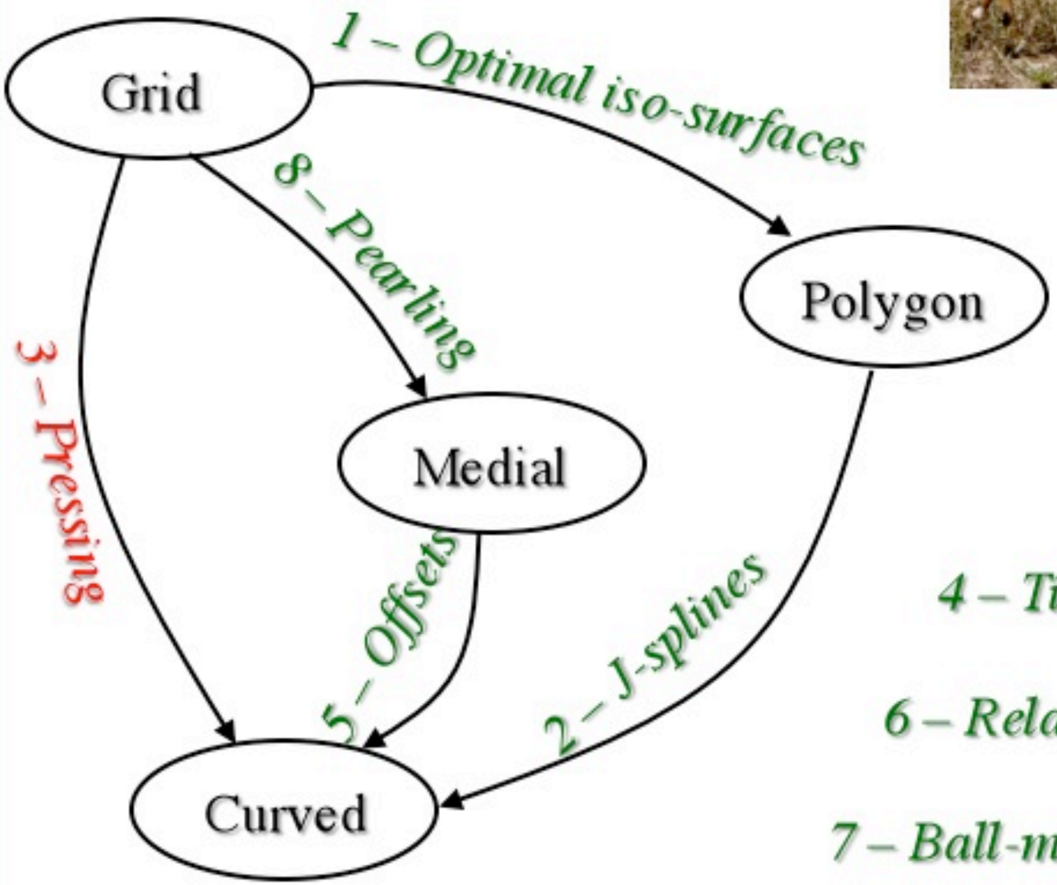
Tensor product J-spline surfaces



J-spline for general quad-meshes



PRESSING



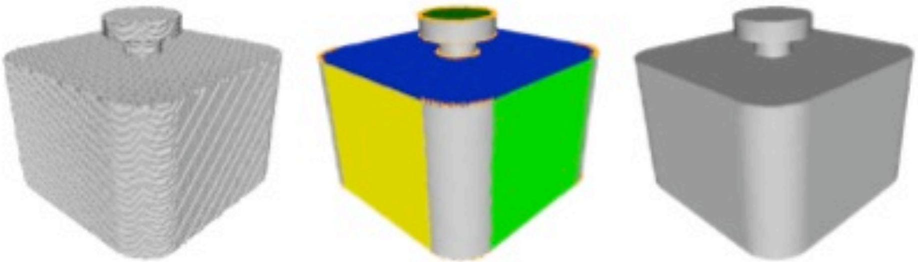
4 – Tightening

6 – Relative blending

7 – Ball-morphing

PRESSNG

Goal: Reverse engineer a 3D binary scan to recover large planar faces and smooth blends between them



Computing Maximal Tiles and Applications to Impostor-Based Simplification, C. Andujar, P. Brunet, A. Chica, J. Rossignac, I. Navazo, A. Vinacua. Computer Graphics Forum 23, pp. 401-410. Proc. Eurographics, September 2004.

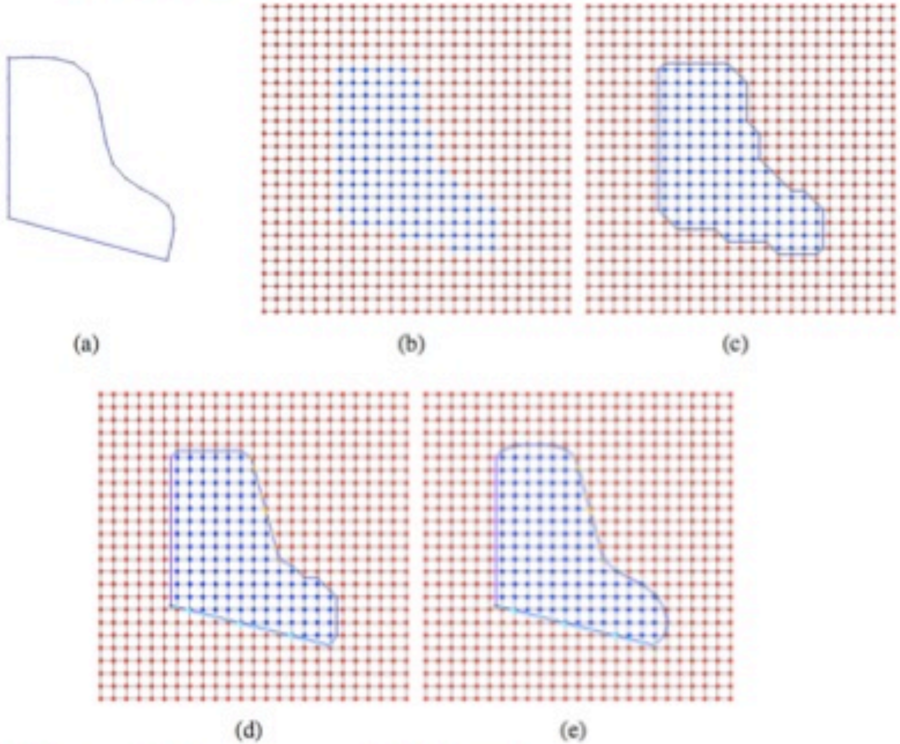
Pressing: Smooth Isosurfaces with Flats from Binary Grids, A. Chica, J. Williams, C. Andujar, P. Brunet, I. Navazo, J. Rossignac, A. Vinacua. Proc Eurographic's, Computer Gaphics Forum (CGF).

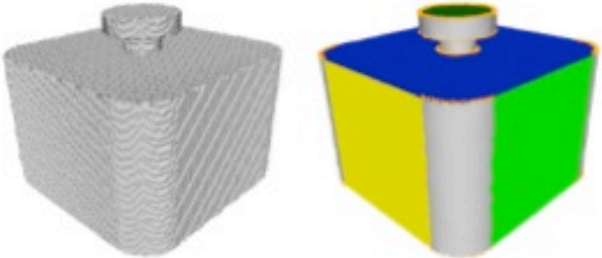
Sharpen&Bend: Recovering curved edges in triangle meshes produced by feature-insensitive sampling, M. Attene, B. Falcidino, M. Spagnuolo, J. Rossignac. **IEEE Transactions on Visualization and Computer Graphic**s (TVCG), vol 11, no 2, pp 181-192, March/April 2005.

Surface reconstruction from grid

Identify **flats**, **blends**, and **sharp edges** in binary volumes

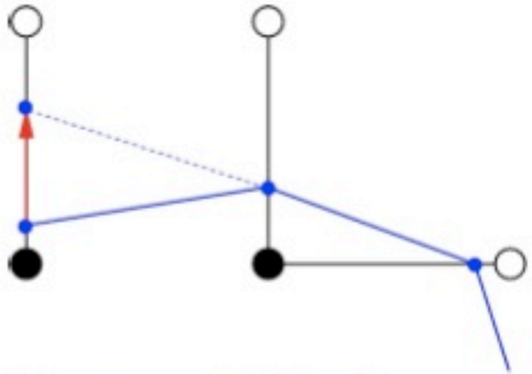
- grow maximal hyperplanes through red-green edges
- perform constrained smoothing of blends





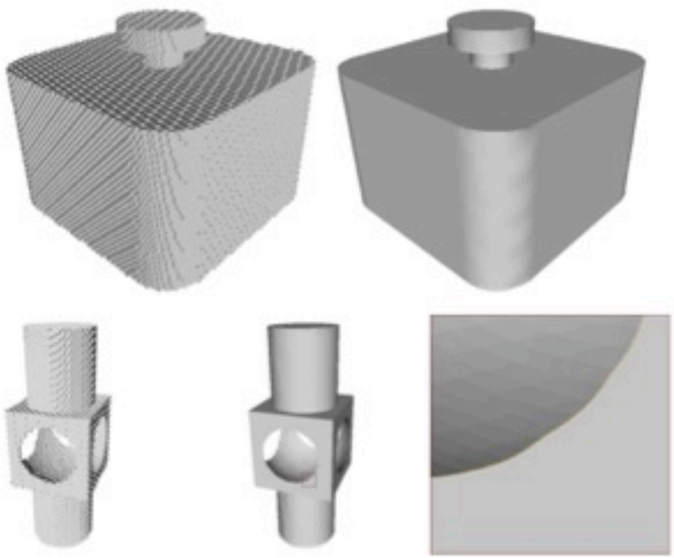
Growing planes

- Stick = grid edge between in and out samples
- We define set of all planes that stab a stick
- Planes that stab several sticks = intersections of these sets
- We grow groups of adjacent sticks that are stabbed by a plane
- We trim the stabbing planes at their intersections
- We slide vertices of stabbed sticks to their plane



Sharpening plane/plane intersections

- Subdivide edges with vertices on 2 stabbing planes



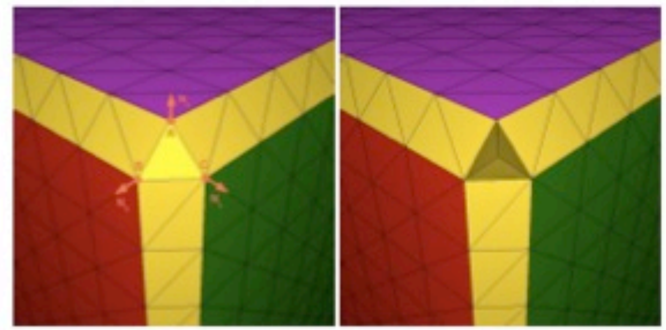


Figure 9: *Inserting a new vertex on a triangle with its vertices on three different faces.*

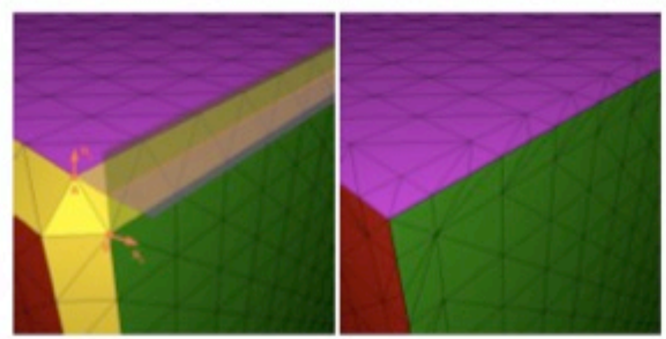
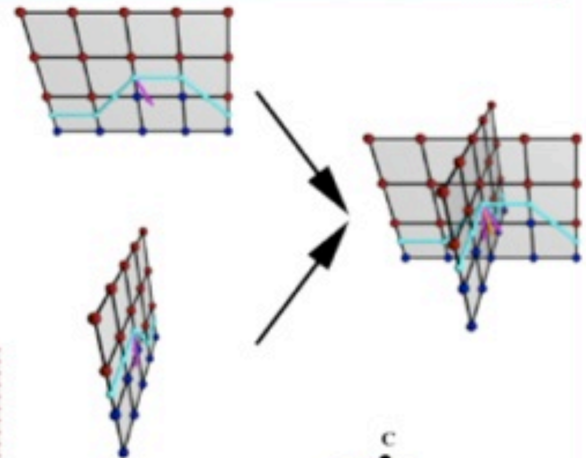
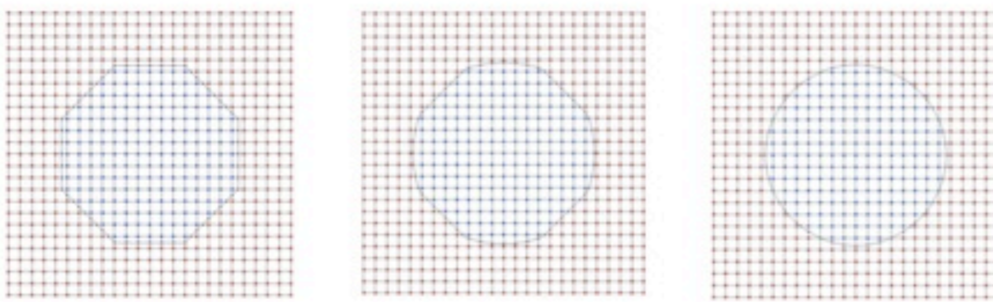


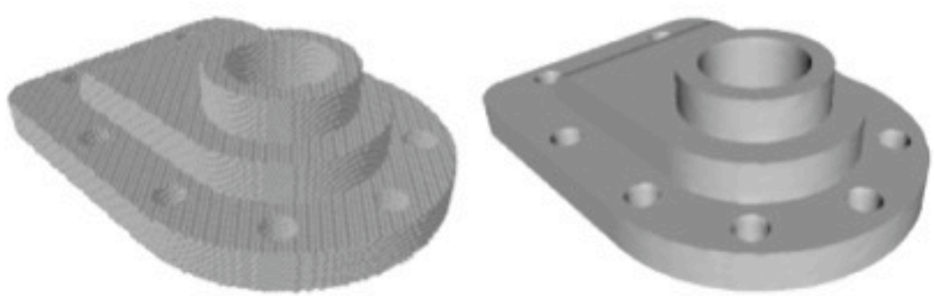
Figure 10: *Edges with vertices on two different faces are subdivided to recover a feature.*

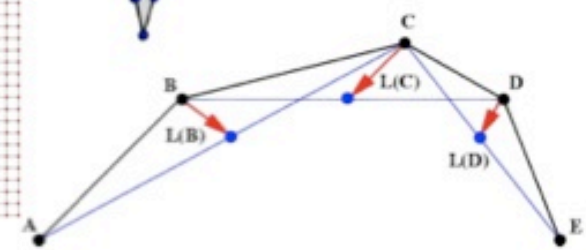
Smoothing the non-flat parts

- Slide the other vertices along their sticks using a variant of bi-Laplace smoothing on each plane that contains the edge

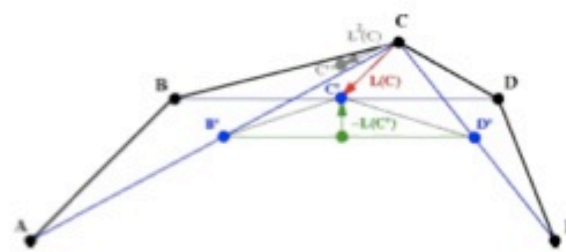






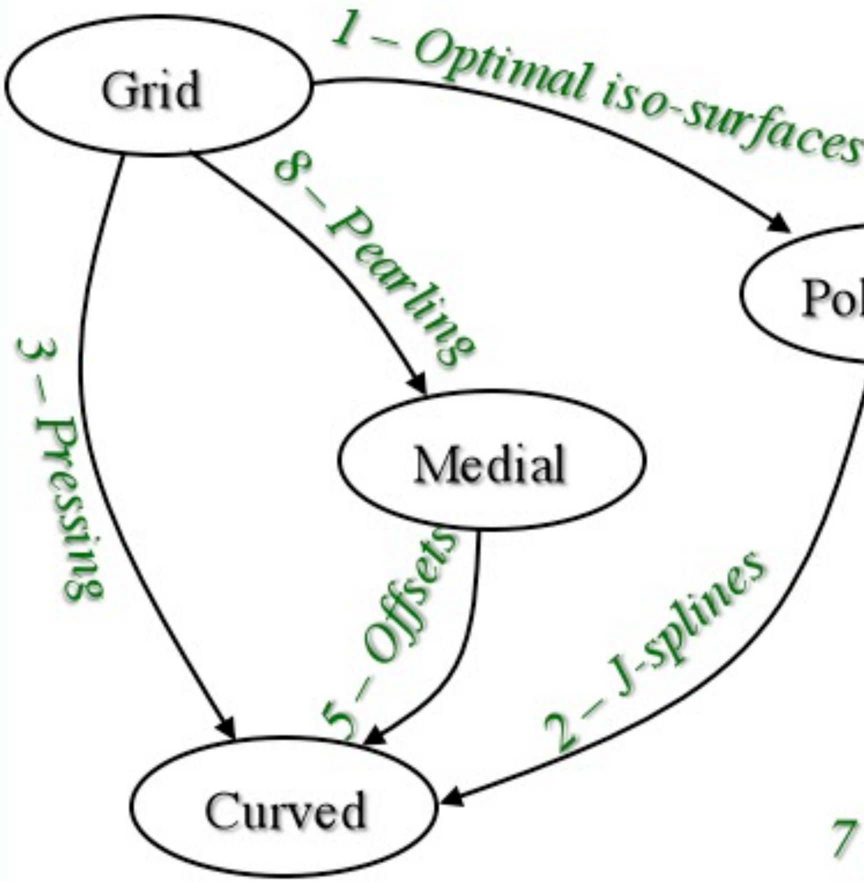


(a)



(b)

TIGHTENING



4 – Tightening

6 – Relative blending

7 – Ball-morphing

TIGHTENING

Goal: A symmetric morphological filter that produces a smooth shape and modifies a shape only in the r-closing of its boundary

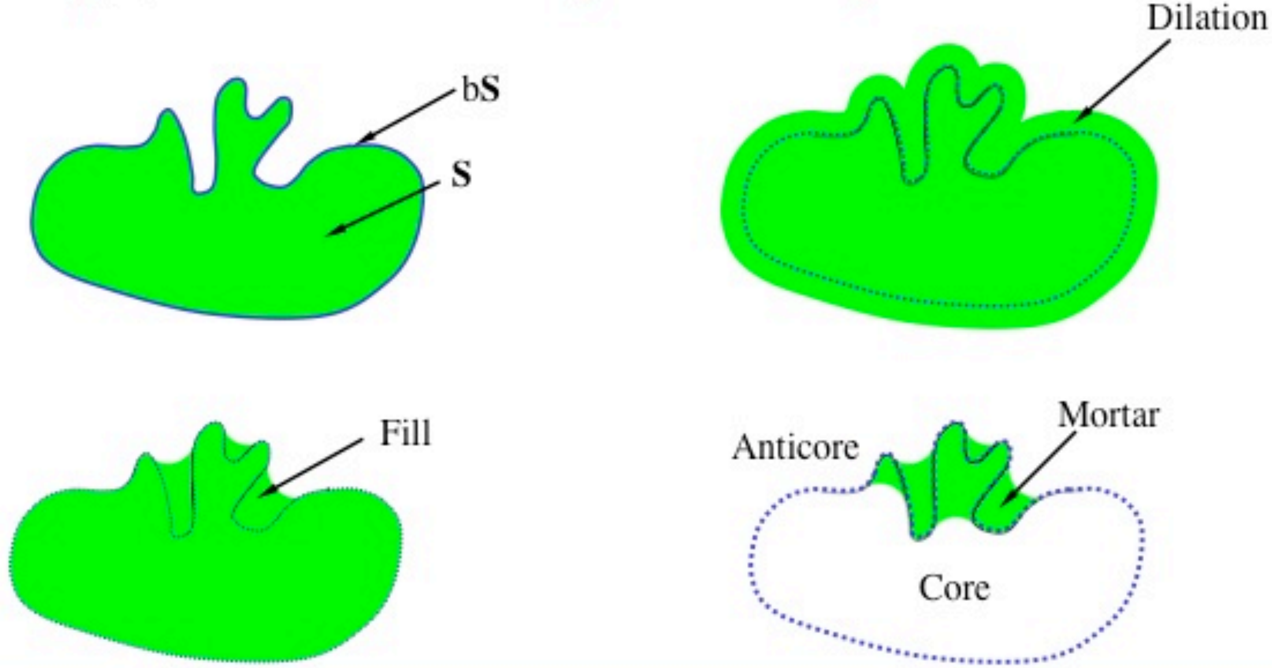
Mason: Morphological Simplification, Jason Williams and Jarek Rossignac**.** **Graphical Models**, 67(4)285:303, 2005.

Tightening: Curvature-Limiting Morphological Simplification., Jason Williams and Jarek Rossignac. ***ACM Symposium on Solid and Physical Modeling*** (SPM). pp. 107-112, June 2005.

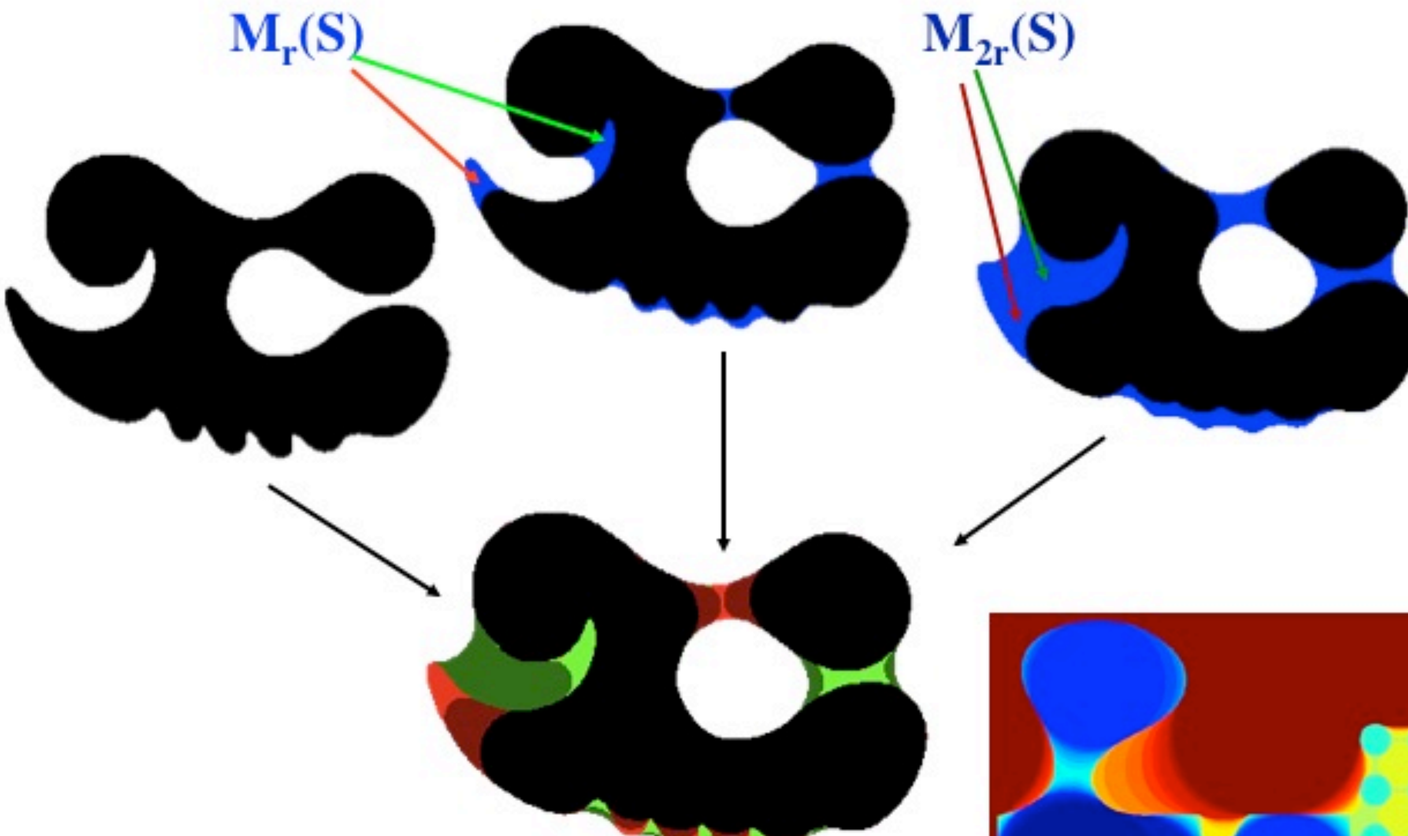
Tightening: Morphological Simplification, J. Williams, J. Rossignac. Int. J. of Computational Geometry and Applications (IJCGA), 17(5)487-503, Oct. 2007

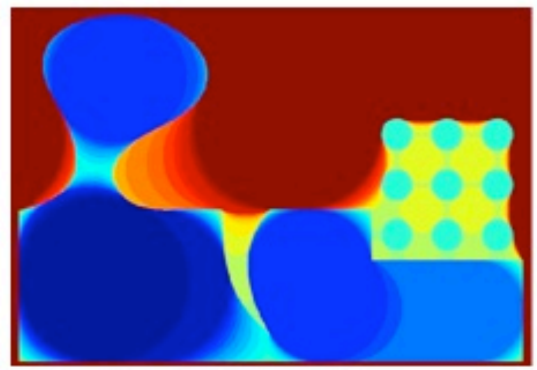
Morphological operators (summary)

- **Dilation S^r** : union of r-balls with centers in **S**
- **Round R_r(S)** : reachable by r-balls in **S** (opening)
- **Fill F_r(S)** : not reachable by r-balls disjoint from **S** (closing)
- **Mortar M_r(S)** : not reachable by r-balls disjoint from b**S**



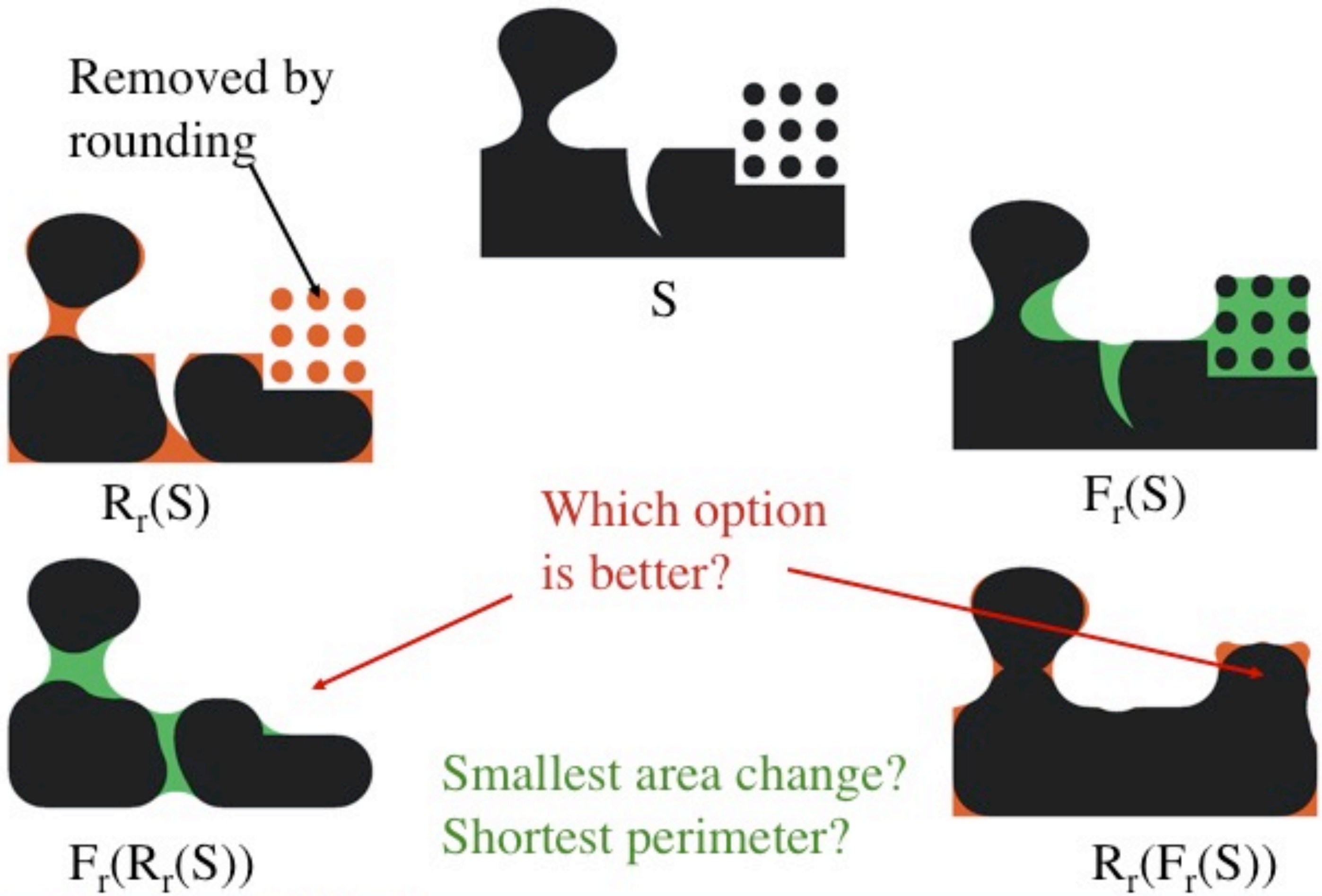
Mortar for multi-resolution analysis of space





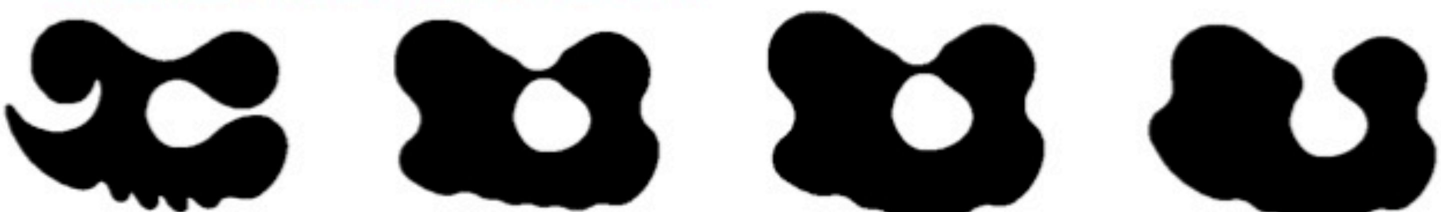
Regularity of P: min r for which P is in $M_r(S)$

Which is better: FR or RF?



The Mason filter

For each connected component M of $M_r(S)$ replace $M \cap S$ by $M \cap F_r(R_r(S))$ or by $M \cap R_r(F_r(S))$, whichever best **preserves the shape** (minimize area change in M)

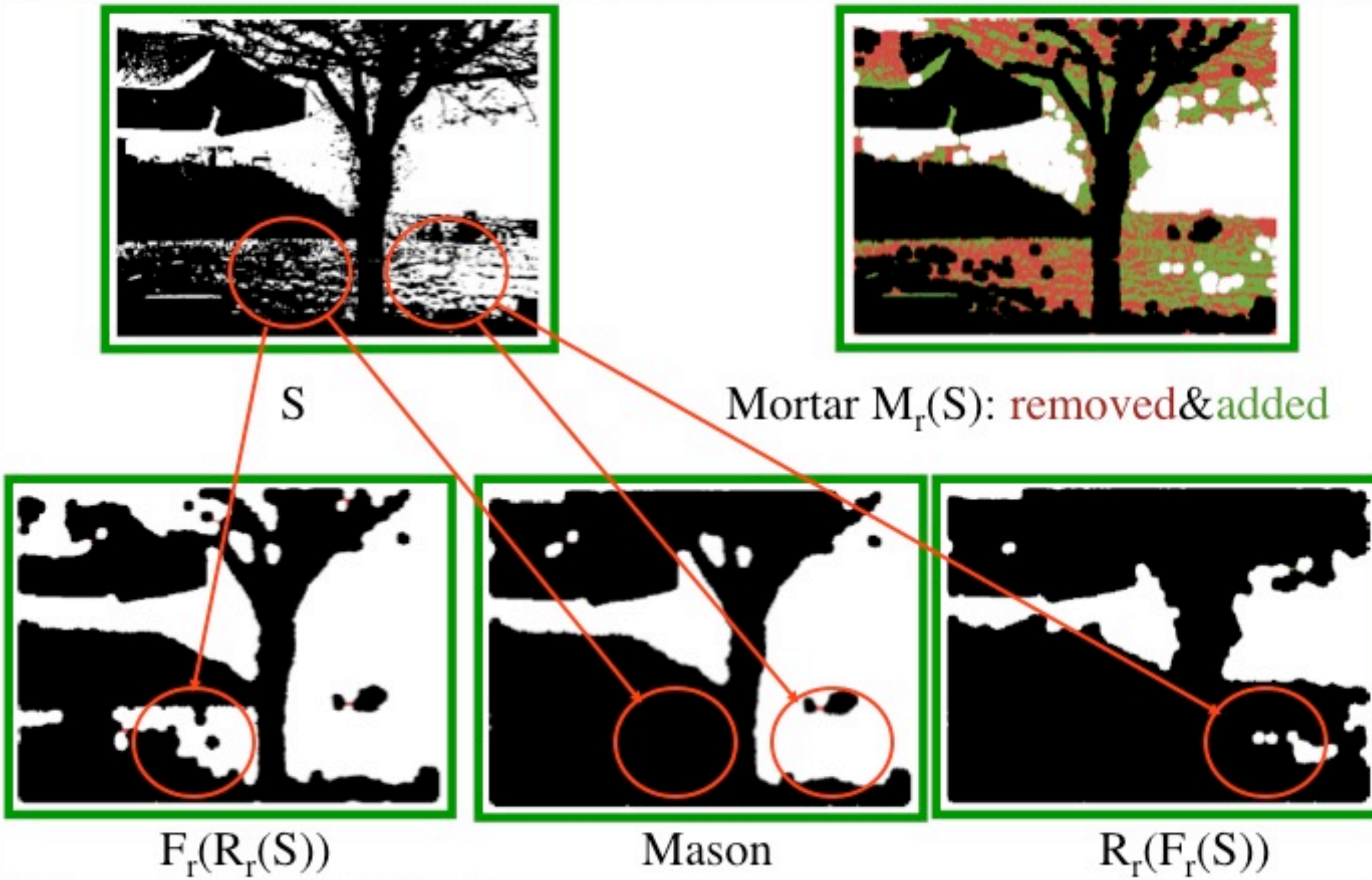


S $F_r(R_r(S))$ Mason $R_r(F_r(S))$

Preserves density (average area) better than a global $F_r(R_r(S))$ or $R_r(F_r(S))$, but does not guarantee smoothness nor minimality of perimeter

"Mason: Morphological Simplification", J. Williams, J. Rossignac. Graphical Models, 67(4)285:303, 2005.

Mason in Granada



3D Mason

Tightening

An r-**tightening**, $\mathbf{T}_r(\mathbf{S})$, of a set S may be obtained by tightening b**S** in the r-mortar $\mathbf{M}_r(\mathbf{S})$

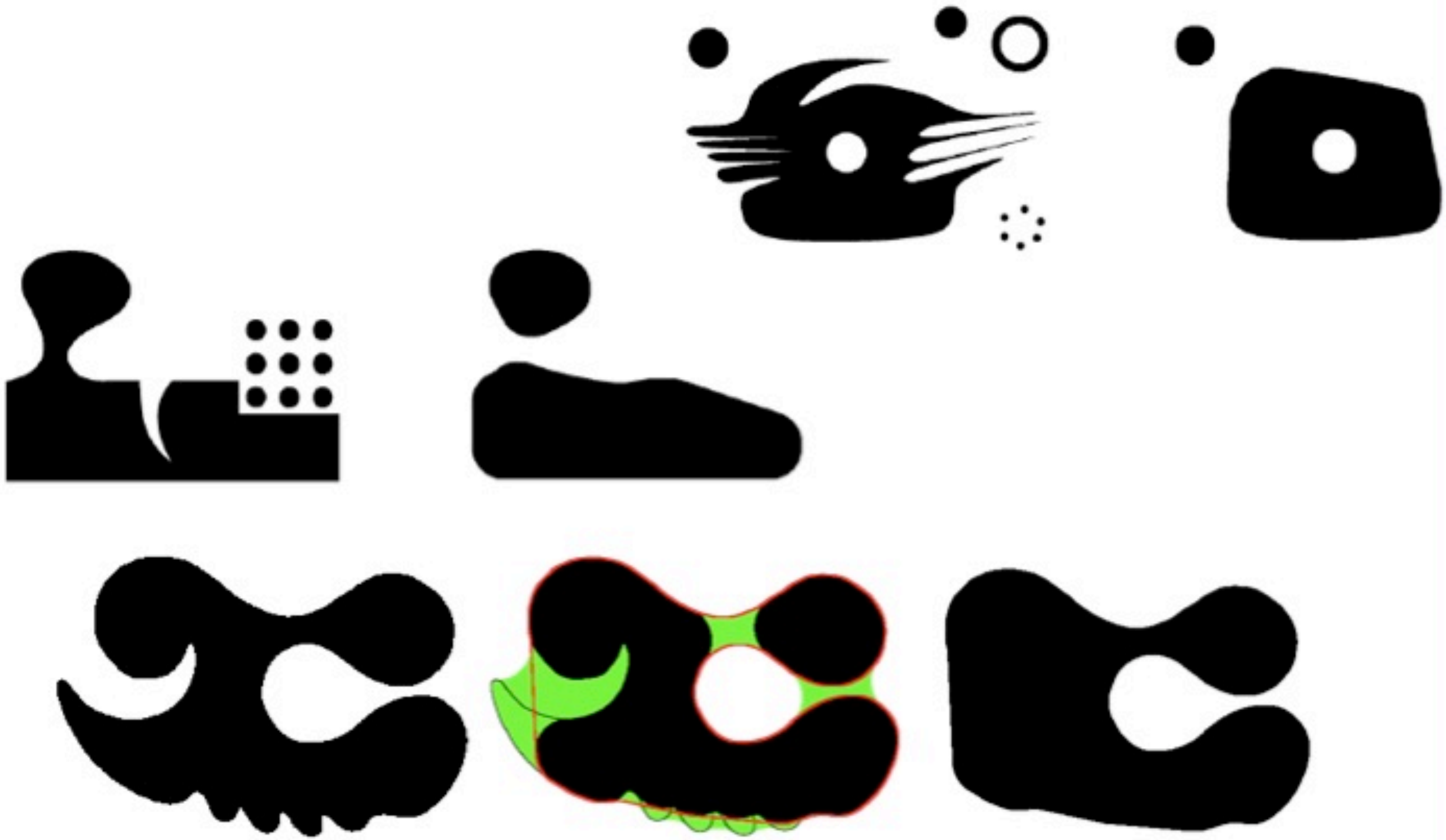
2D: The shortest of all *homotopic* curves in $M_r(A)$

3D: Minimal area surface in Mortar

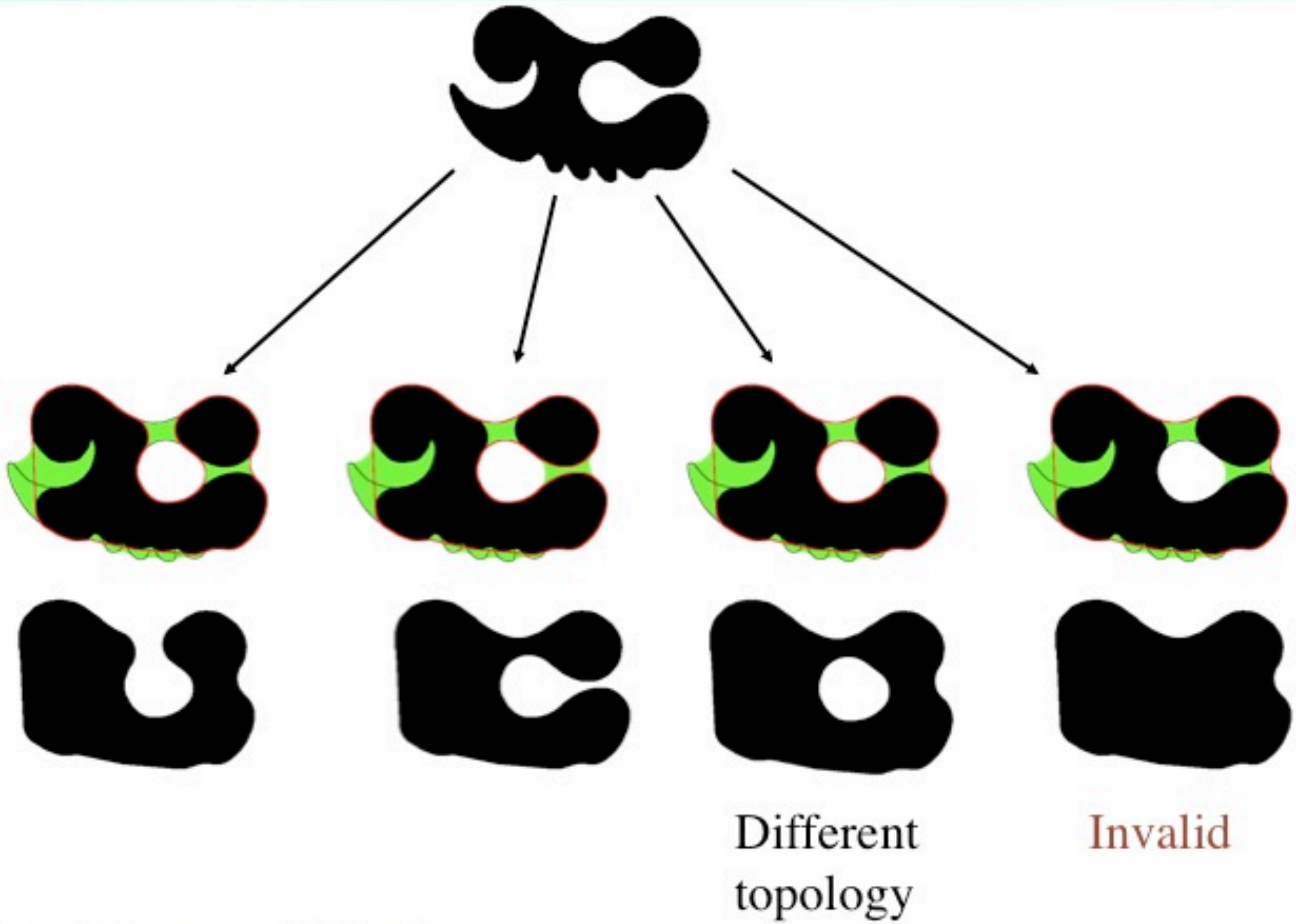
Related to the Tight Hull [Sklansk&Kibler76 , Robinson97, Mitchell04], but the hull is the mortar!

Tightening: Curvature-Limiting Morphological Simplification. J. Williams & J. Rossignac. Sketch in the *ACM Symposium on Solid and Physical Modeling* (SPM). pp. 107-112, June 2005.

Example of tightening in 2D

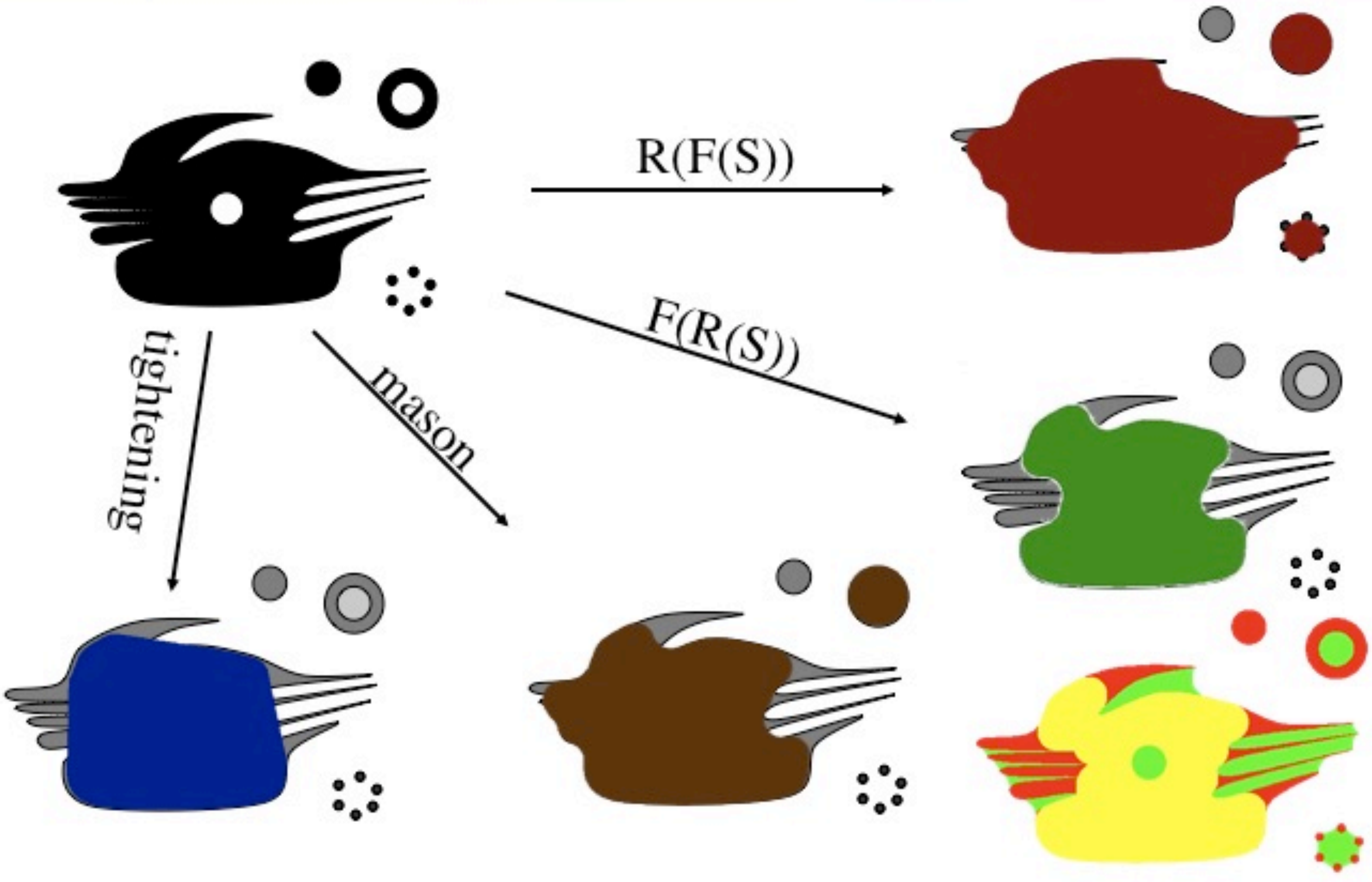


Topological choices of tightening

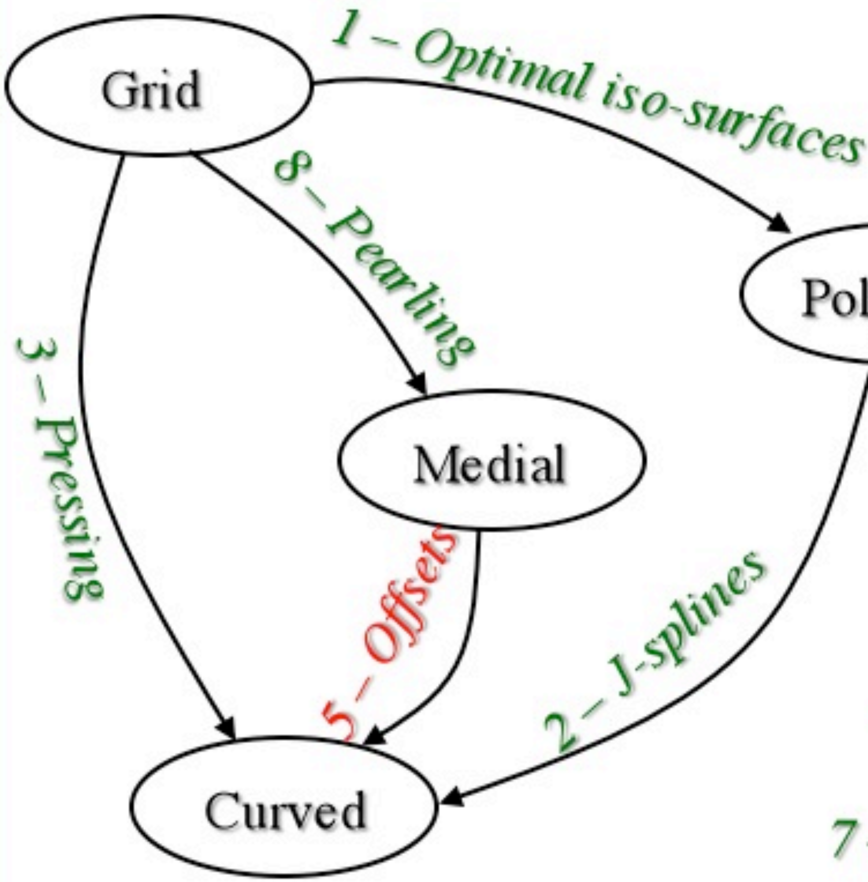


Examples of 3D tightening

Comparing morphological simplifications



OFFSETS



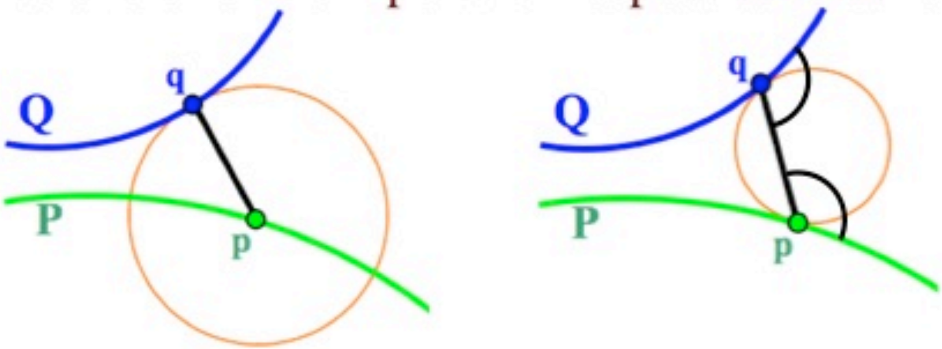
4 – Tightening

6 – Relative blending

7 – Ball-morphing

OFFSETTING

Goal: Compare mappings between shapes and variable distance offset formulations of one shape with respect to another.



OrthoMap: Homeomorphism-guaranteeing normal-projection map between surfaces, F. Chazal, A. Lieutier, and J. Rossignac. ACM Symposium on Solid and Physical Modeling (SPM). pp. 9-14, June 2005.

Normal map between normal-compatible manifolds, F. Chazal, A. Lieutier, and J. Rossignac., Int. J. of Computational Geometry & Applications (IJCGA), 17(5)403-421 Oct. 2007.

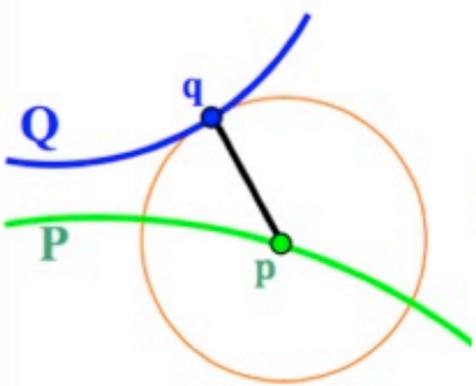
Ball-map: Homeomorphism between compatible surfaces, F. Chazal, A. Lieutier, J. Rossignac, B. Whited, International Journal of Computational Geometry and Applications.

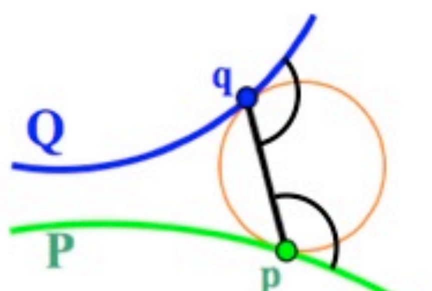
Three variable distance offset formulations

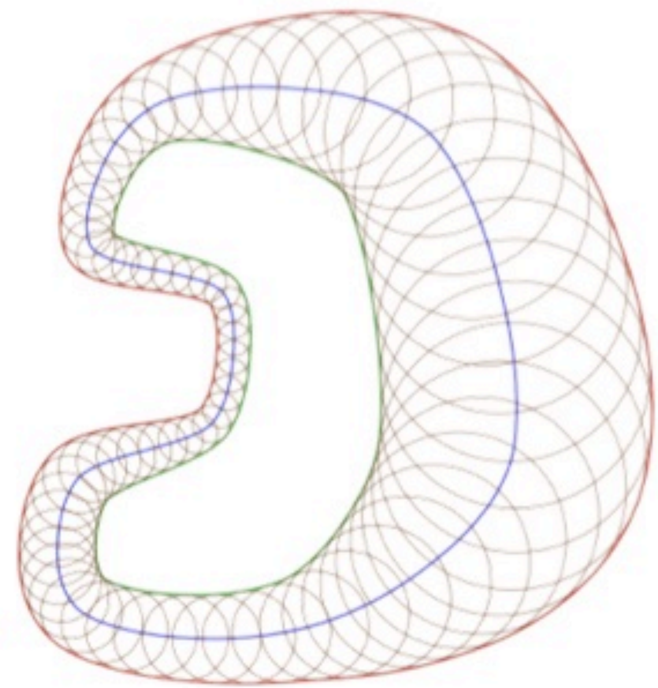
- Orthogonal $O_d(P)$: move P along normal to P+dN
- Radial $R_d(P)$: move P(s) to closest projection on envelop
  - Move P to P+dD , along direction $D = -d'T + \sqrt{(1 - d'^2)}\,N$
- Ball $B_d(P) = R_r(O_r(P))$ with r=d/2.
  - Envelop swept by ball of varying diameter d(s) sliding on curve

Relation between offset formulations

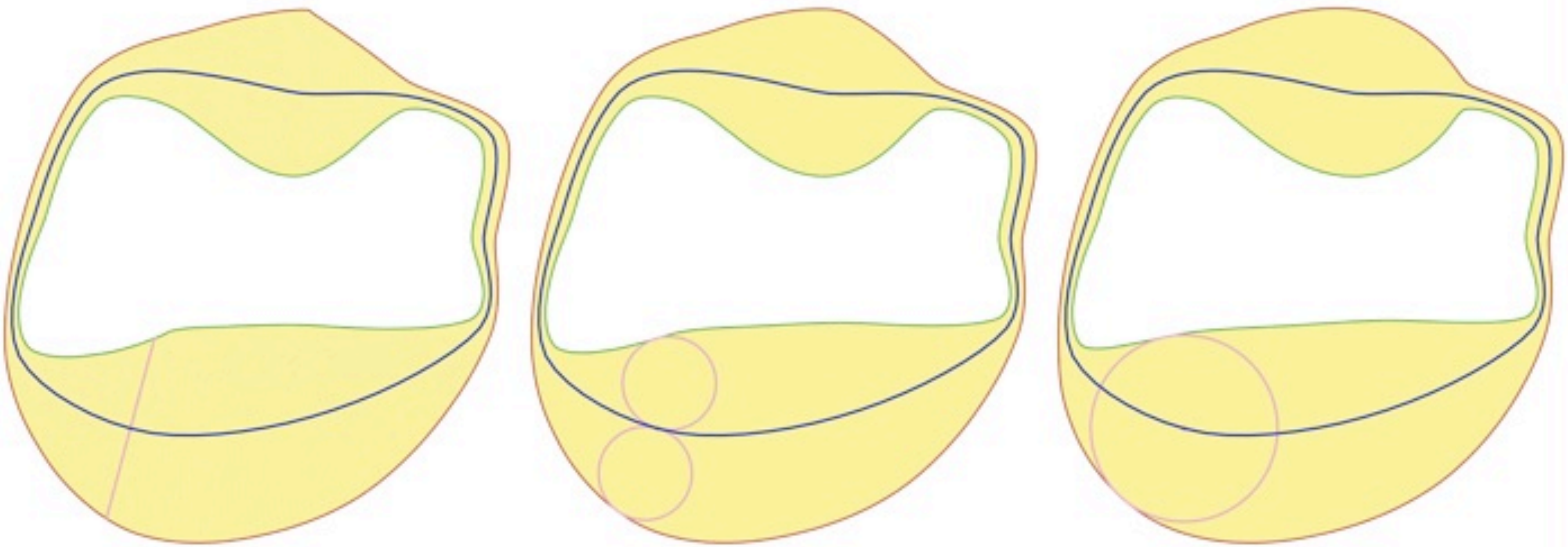
- Radial is inverse of orthogonal (different d)
  - Red and green are radial offsets of blue
  - Blue is orthogonal offset of red and of green
- Ball is inverse of itself (same d)
  - Red is the ball offset of green
  - Green is the ball offset of red





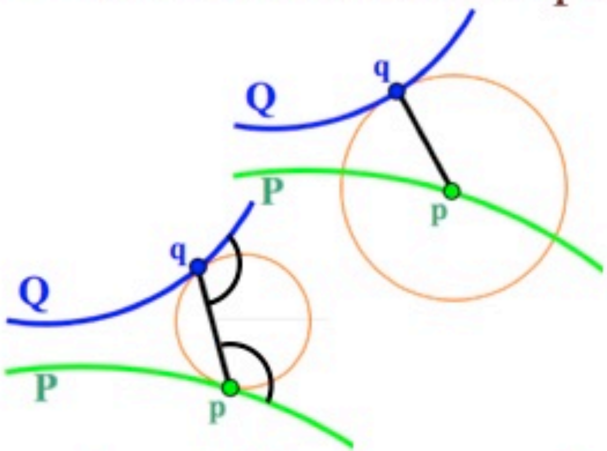


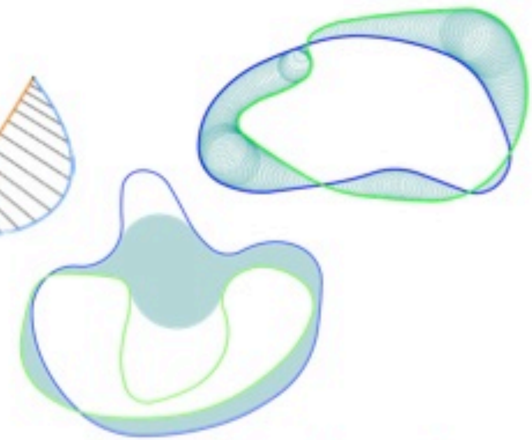
Comparing the 3 offsets



Compatibility

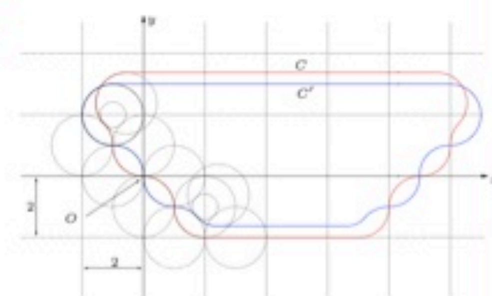
- Two sets are x-compatible if each is the x-offset of the other





- A and B are ***normal-compatible*** (orthogonal offset of each other), and hence also radial-compatible, **if**

  H(A,B) < (2–√2)min(mfs(A),mfs(b)), tight



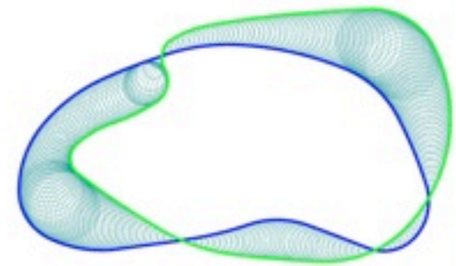
- A and B are ***ball-compatible*** if

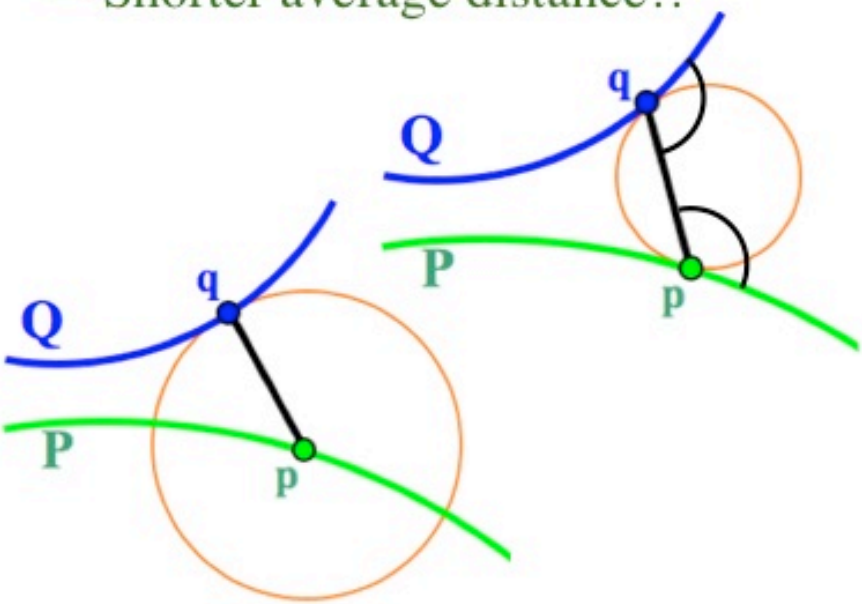
  H(A,B) < min(mfs(A),mfs(b)), tight bound

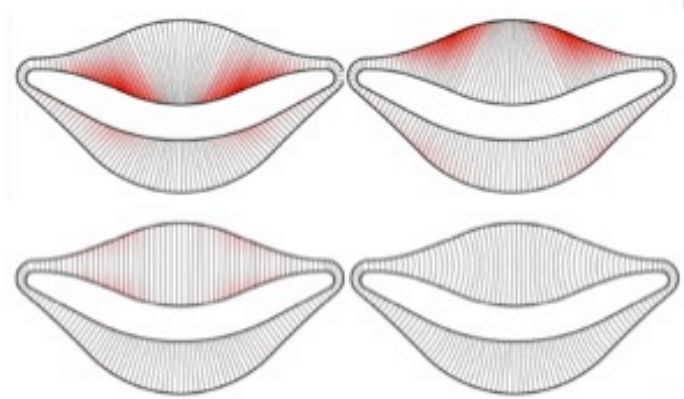
- A and B ball-compatible => H(A,B) = F(A,B) Frechet=Hausdorff

Ball-Map

- *Ball-map*: Maps contact points of maximal balls in P xor Q
- Advantages over *Closest Point map*
  - Symmetric (same incidence angle)
  - No distortion between symmetric features
  - Shorter average distance!!

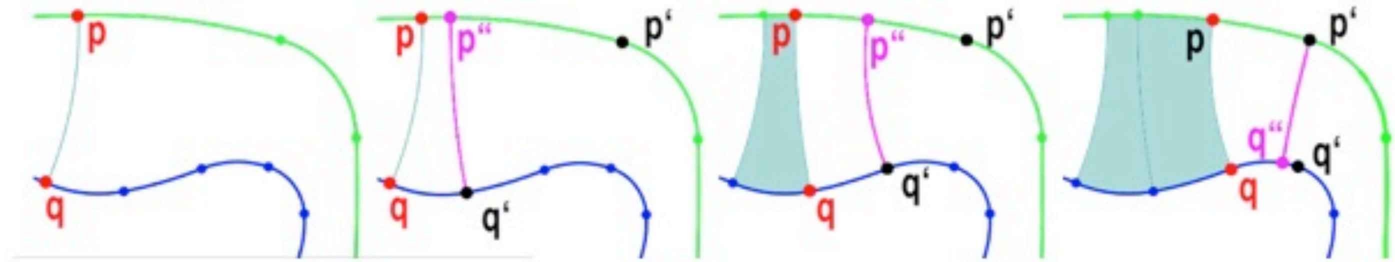




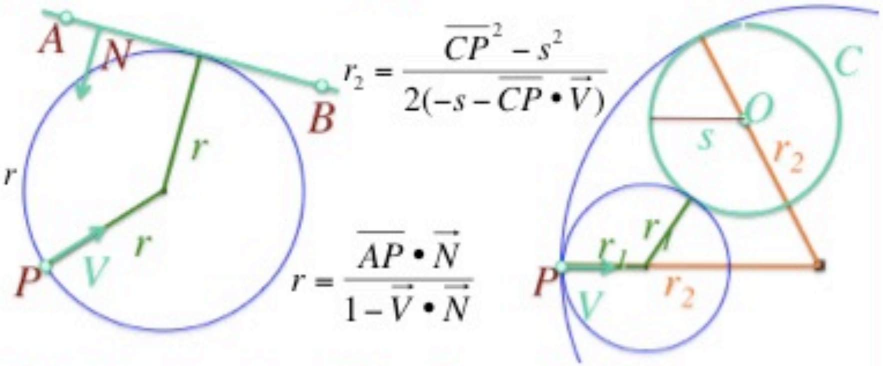


Ball map computation for ***PCCs*** in 2D

- Map the *first* end of current arc-pair and advance



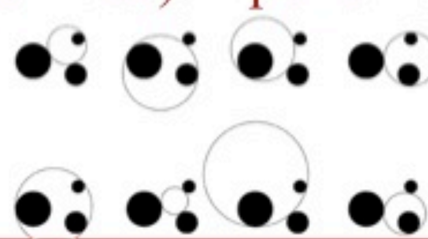
$$\left\|\left(P + \vec{V}r\right) - Q\right\| = r$$



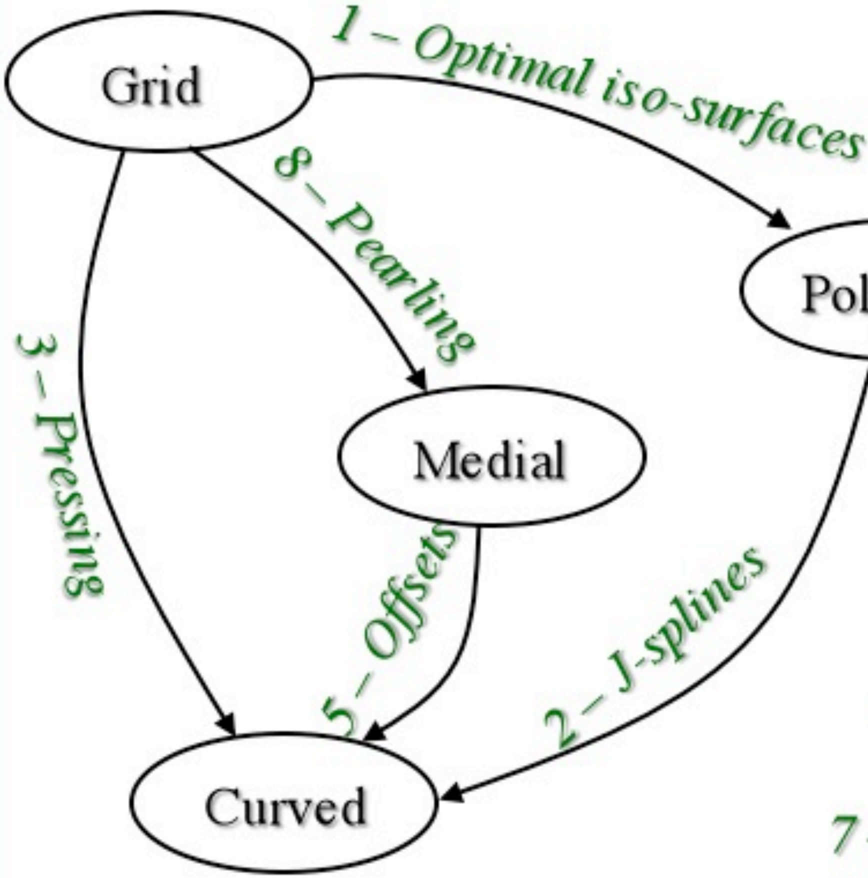
$$r_2 = \frac{\overline{CP}^2 - s^2}{2(-s - \overline{CP} \bullet \vec{V})}$$

$$r = \frac{\overline{AP} \bullet \vec{N}}{1 - \vec{V} \bullet \vec{N}}$$

- Detect branches (incompatibilities): Apollonius circle



BLENDING



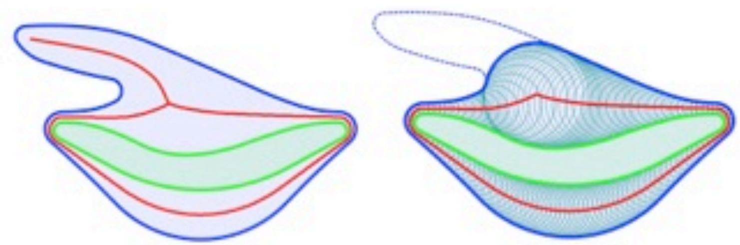
4 – Tightening

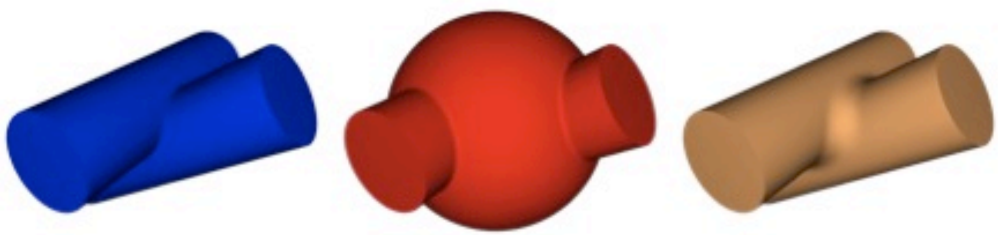
6 – Relative blending

7 – Ball-morphing

RELATIVE BLENDING

Goal: Specify the variable blending radius using the radius of the ball map to a reference surface

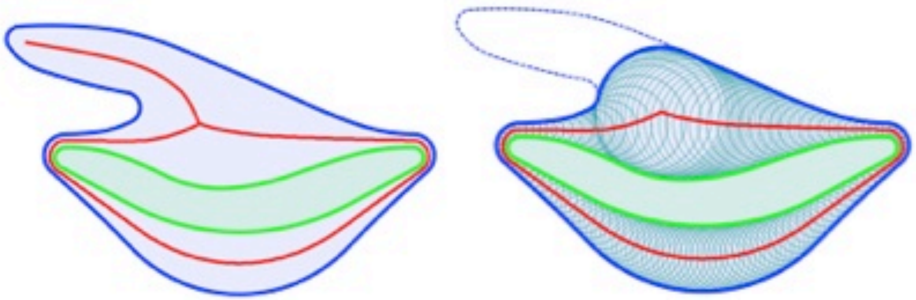




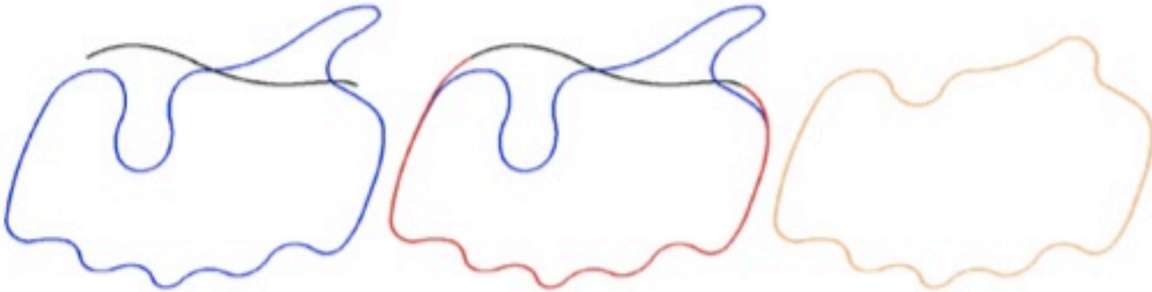
"Relative blending", B. Whited, J. Rossignac. Journal of Computer-Aided Design (JCAD). 2009

Motivations

- Make two shapes ball compatible



- Provide useful tool for defining variable radius blending

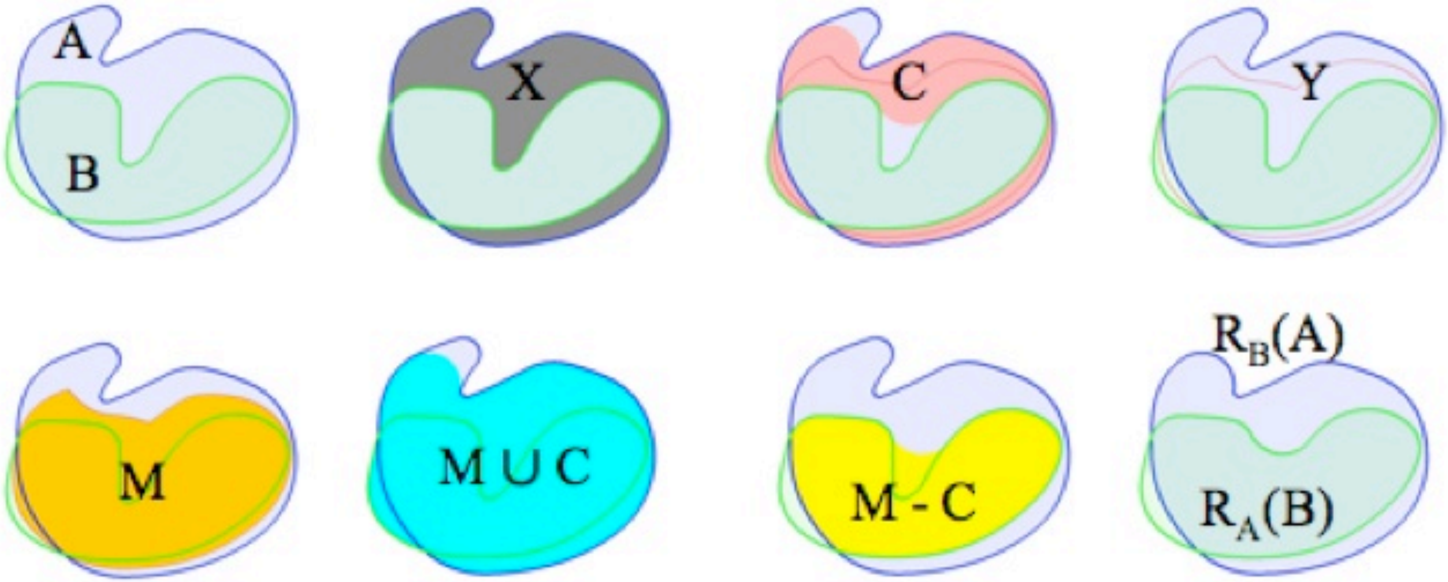


Relative Blending: Set theoretic formula

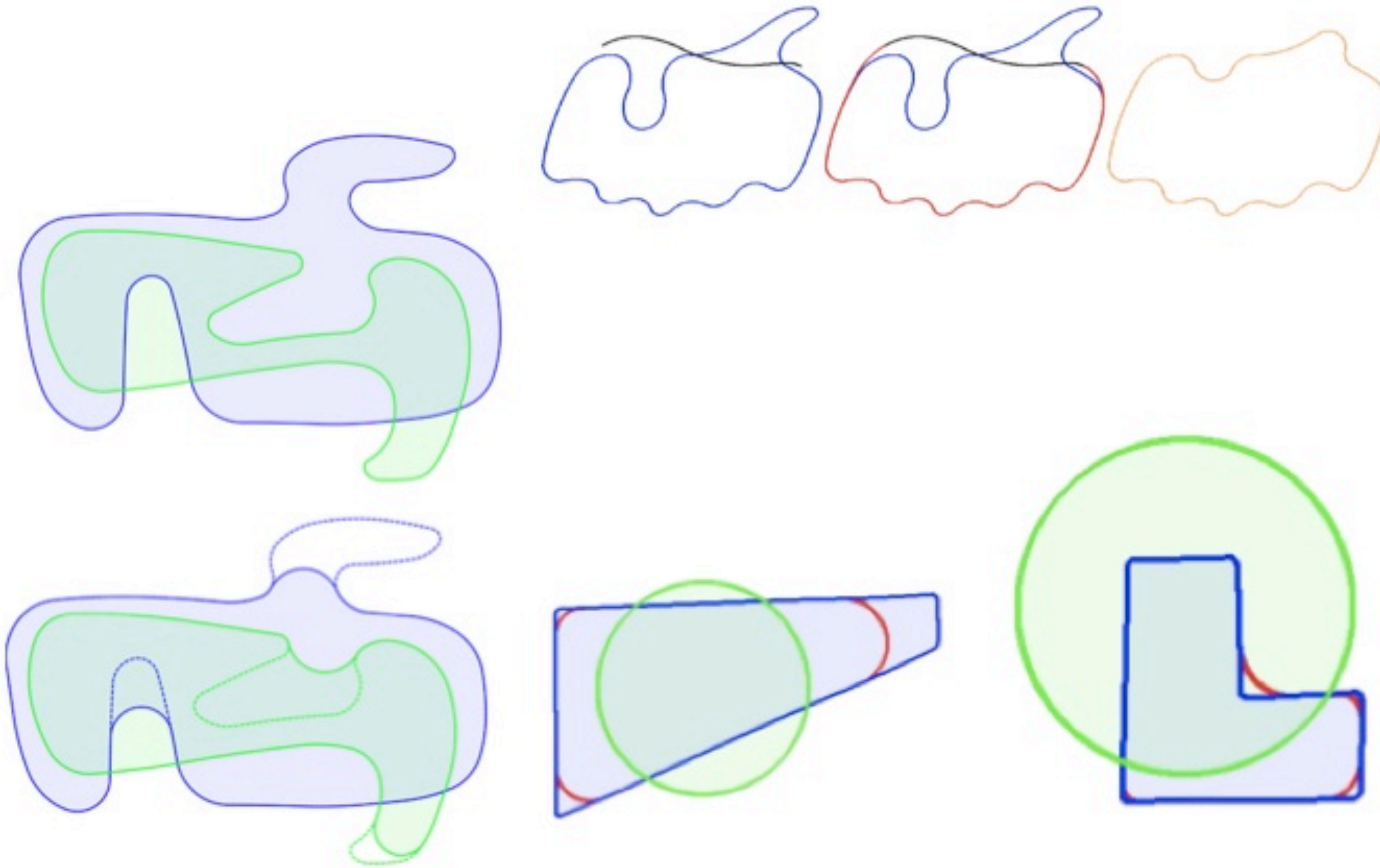
- X=A xor B
- C=balls that touch both,
- Y=their centers,
- M=interior of Y

$$R_B(A) = (A \cap (M \cup C)) \cup (M - C)$$

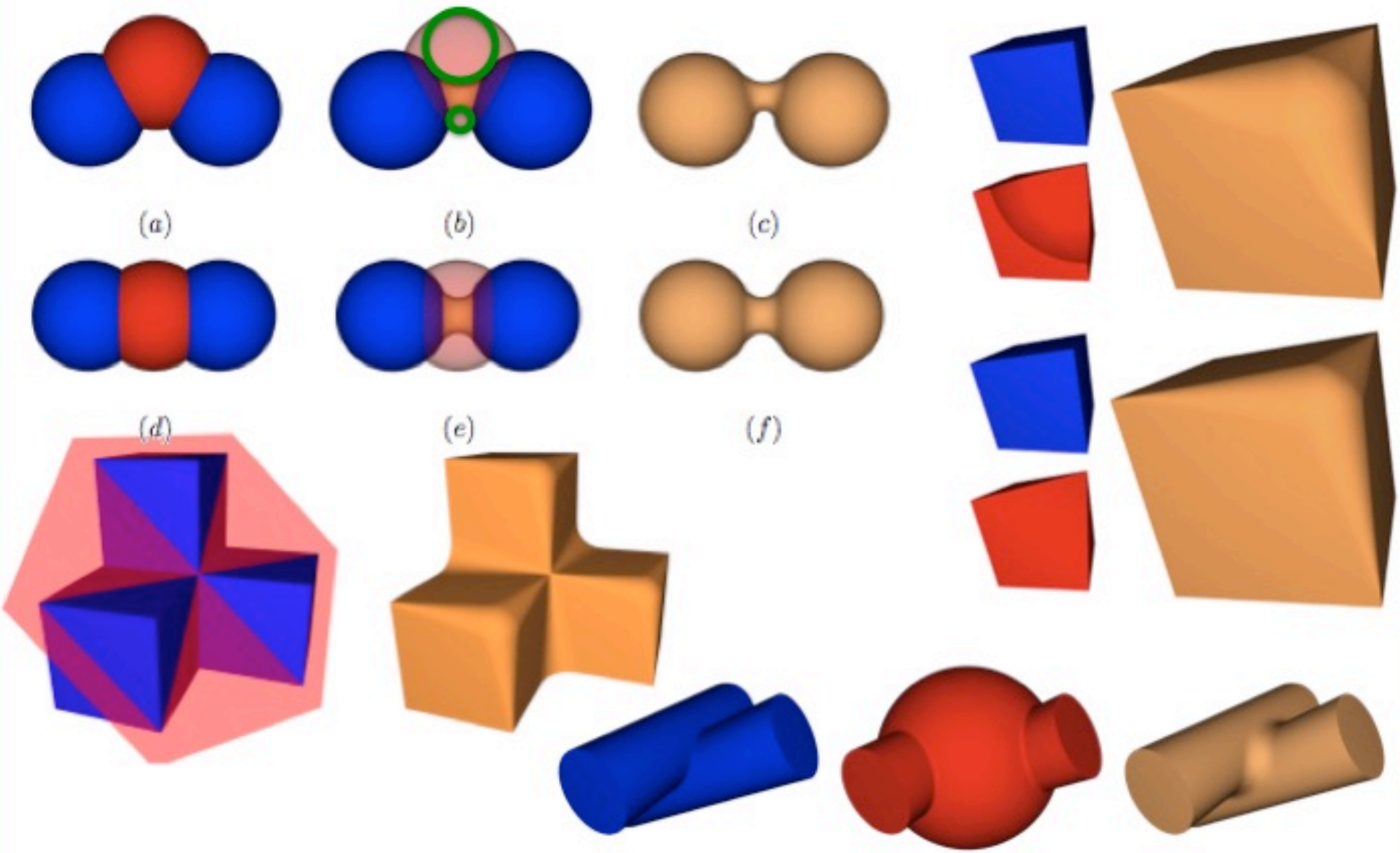
$$R_A(B) = (B \cap (M \cup C)) \cup (M - C)$$



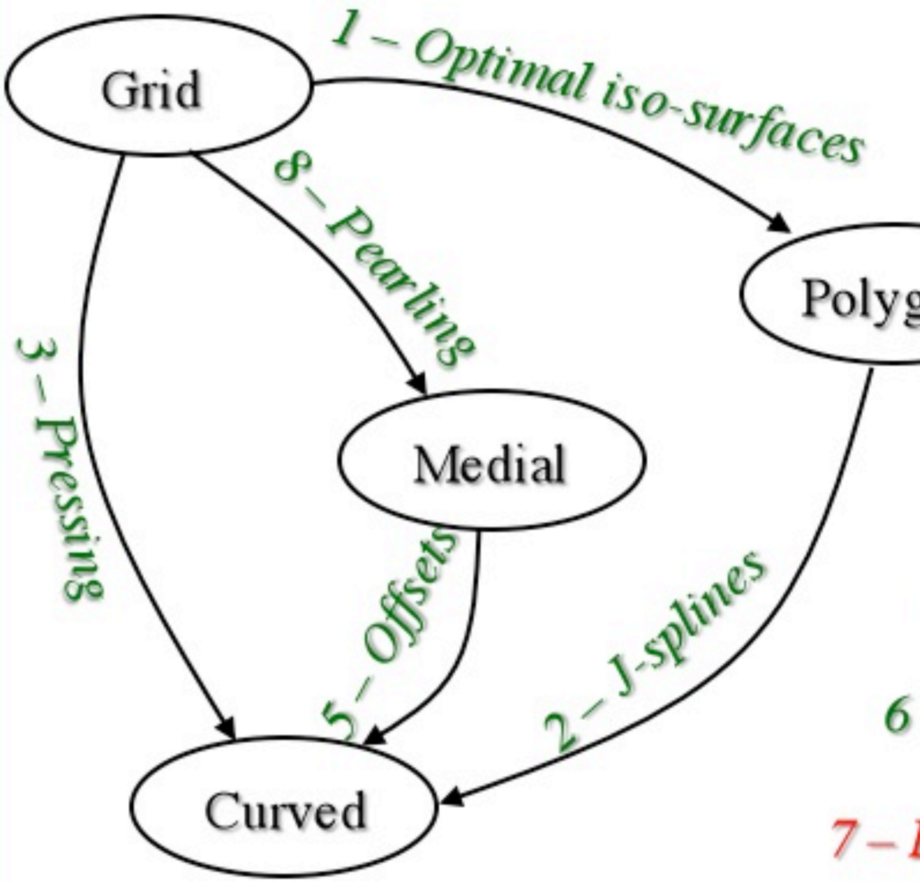
Relative Blending: 2D examples



Relative Blending: 3D examples



MORPHING



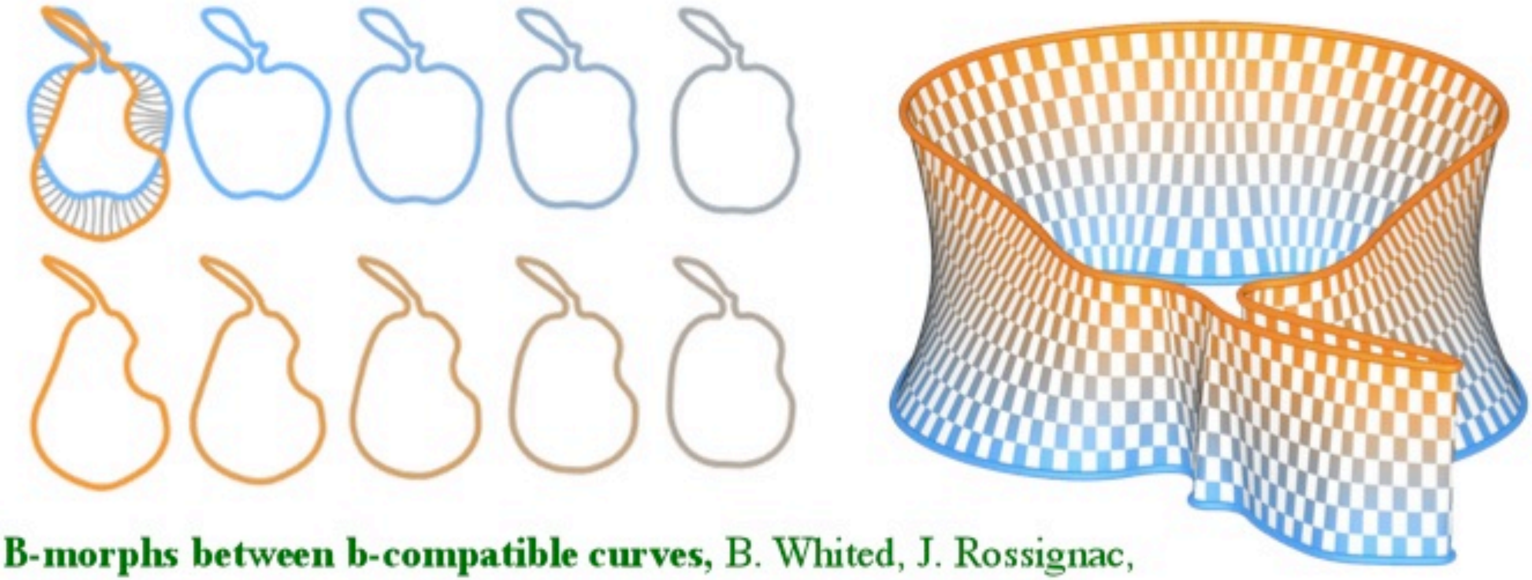
4 – Tightening

6 – Relative blending

7 – Ball-morphing

Ball morph

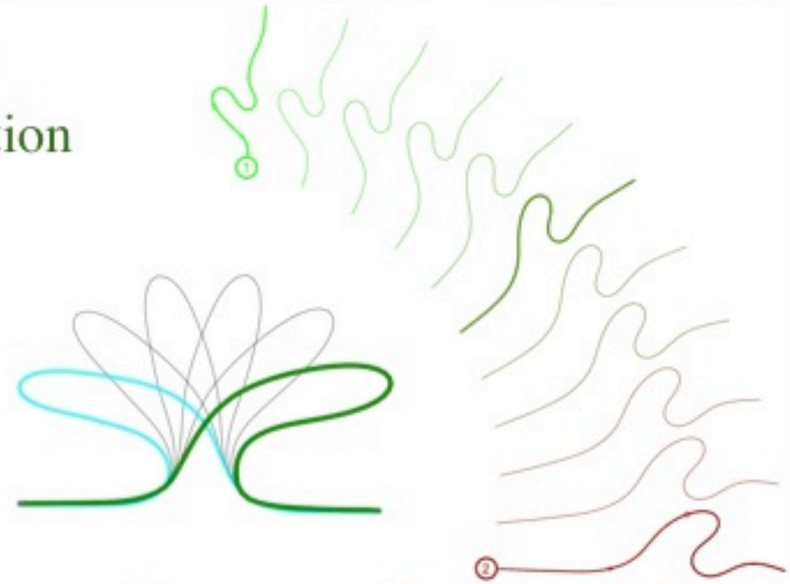
Goal: Specify morph between two curves or two surfaces

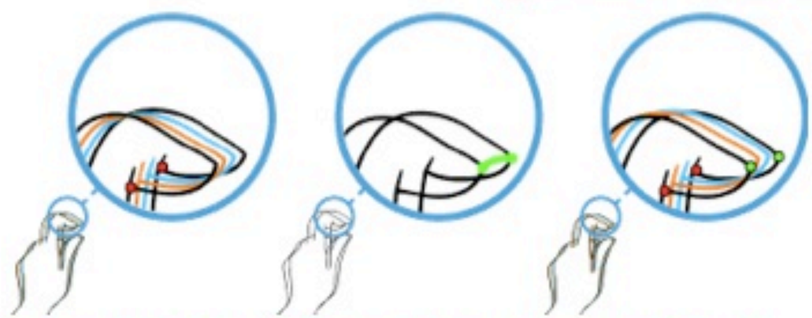


B-morphs between b-compatible curves, B. Whited, J. Rossignac, ACM Symposium on Solid and Physical Modeling (SPM), 2009

BetweenIT

- Automate tight inbetweening
  - for feature animation production
- Issues:
  - Segmentation into strokes
  - Correspondence
  - Morphing strokes
  - Occlusion
  - Enable artists' tweaks

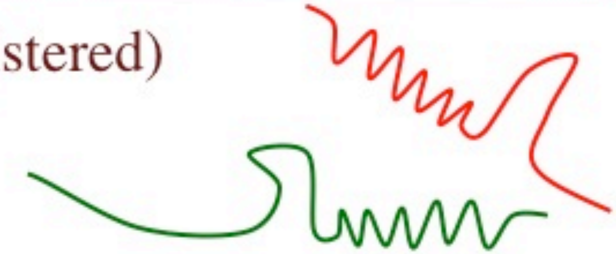




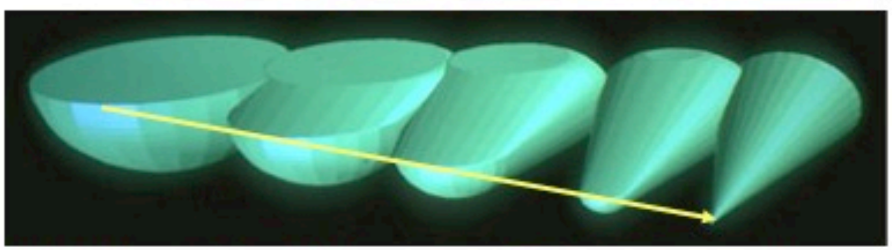
"*BetweenIT: An Interactive Tool for Tight Inbetweening*", B. Whited, G. Noris, M. Simmons, R. Sumner, M. Gros, J. Rossignac. 2009.

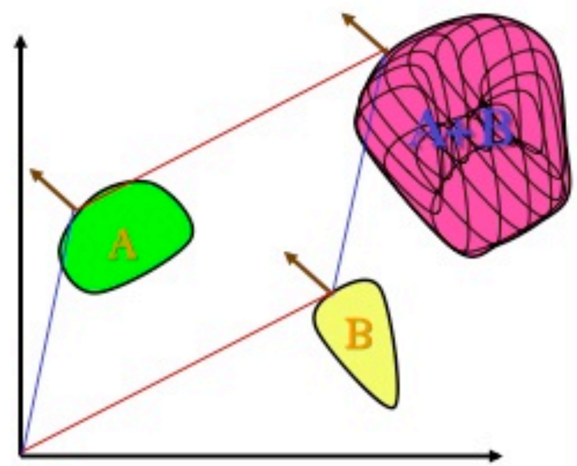
The correspondence problem

- Intrinsic (assume curves are not registered)
  - Uniform arc-length sampling
  - Curvature sensitive sampling
  - Smooth sampling constrained by matched salient feature pairs



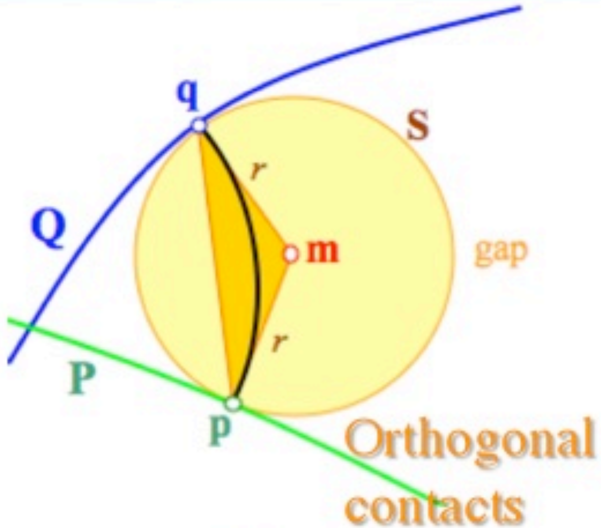
- Extrinsic (assume curves are registered)
  - Closest Point: **distance**
  - Minkowski morph: **normal**
  - Ball map: **distance** and **normal**

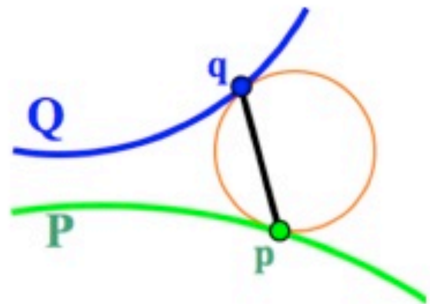


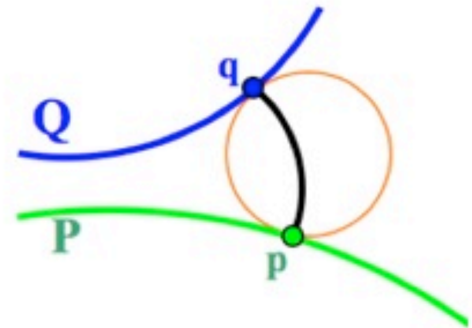


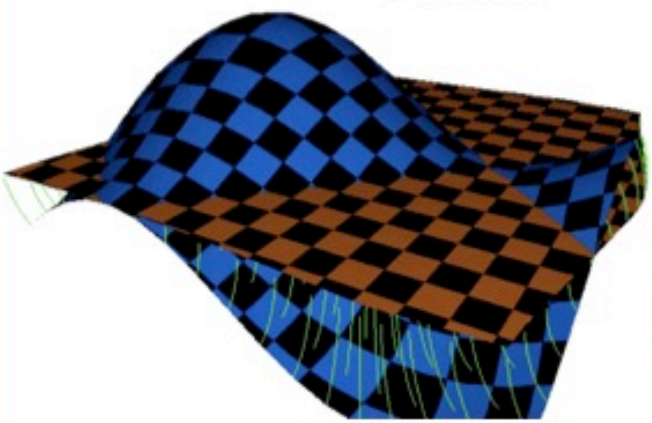
"Solid-Interpolating Deformations: Construction and Animation of PIPs", A. Kaul and J. Rossignac, Computers&Graphics, Vol. 16, No. 1, pp. 107-115, 1992.

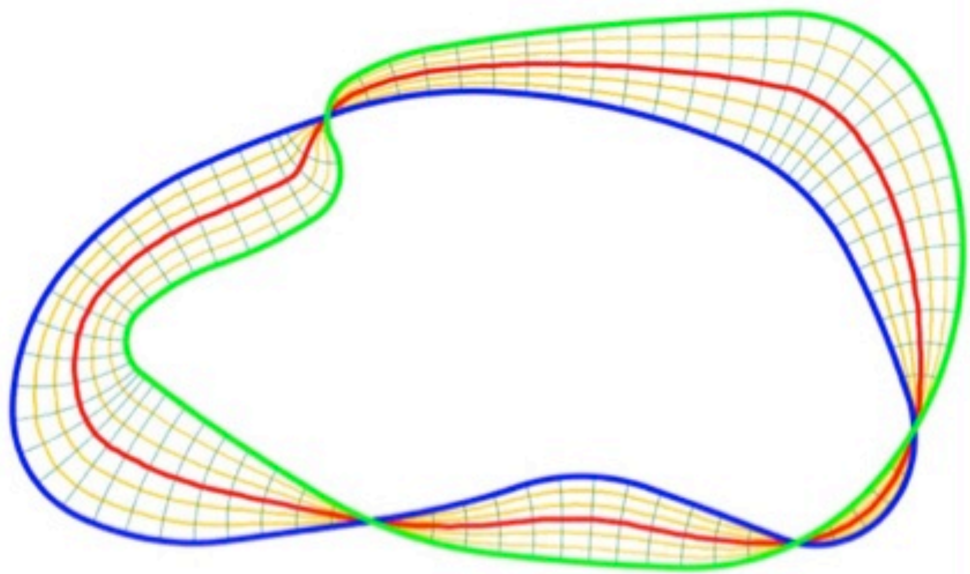
Ball morph: Circular trajectories





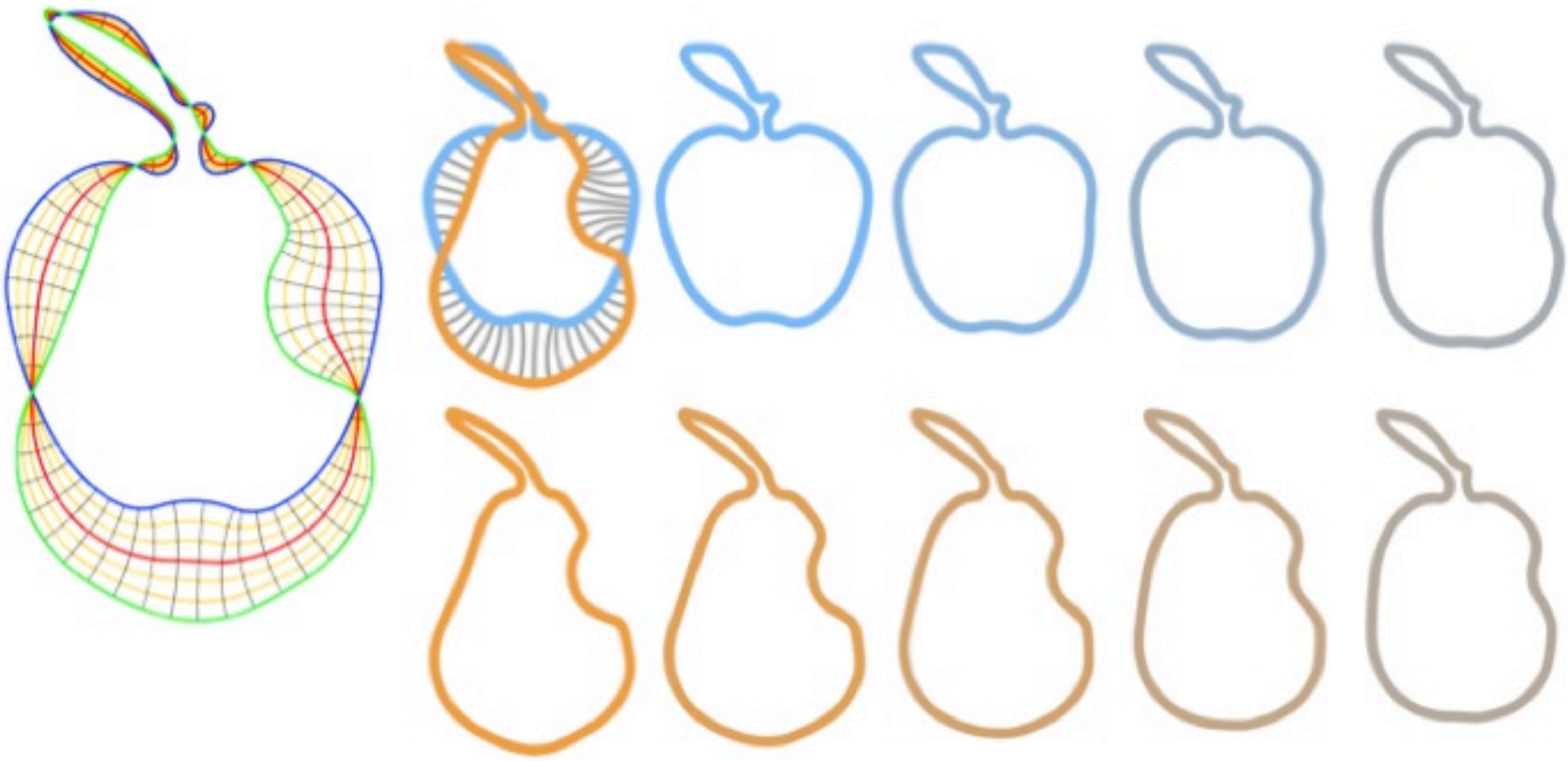




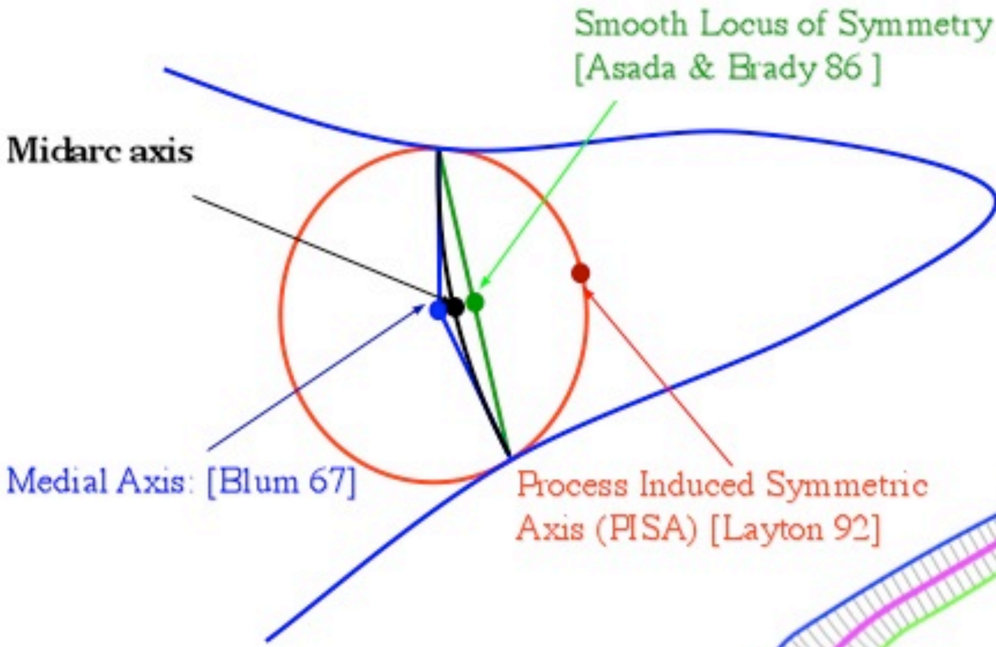


Ball morph example

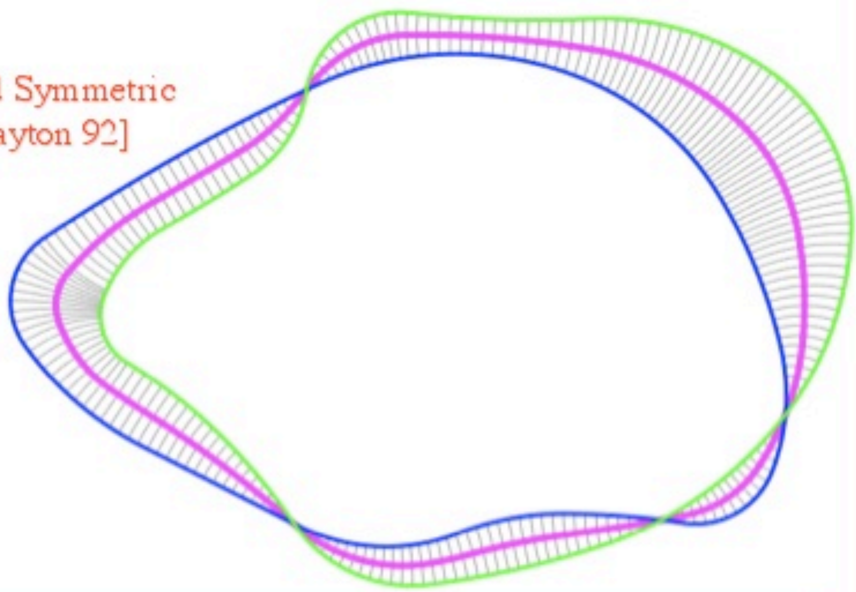
- Samples move at constant speed along ball-map arcs



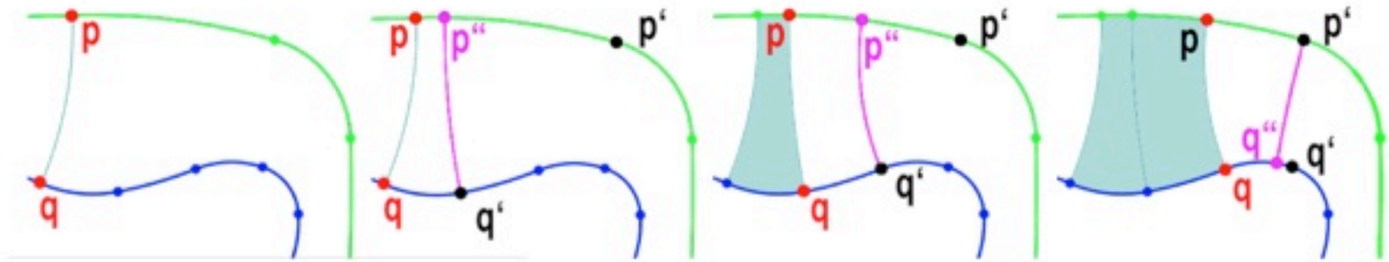
Midarc axis:

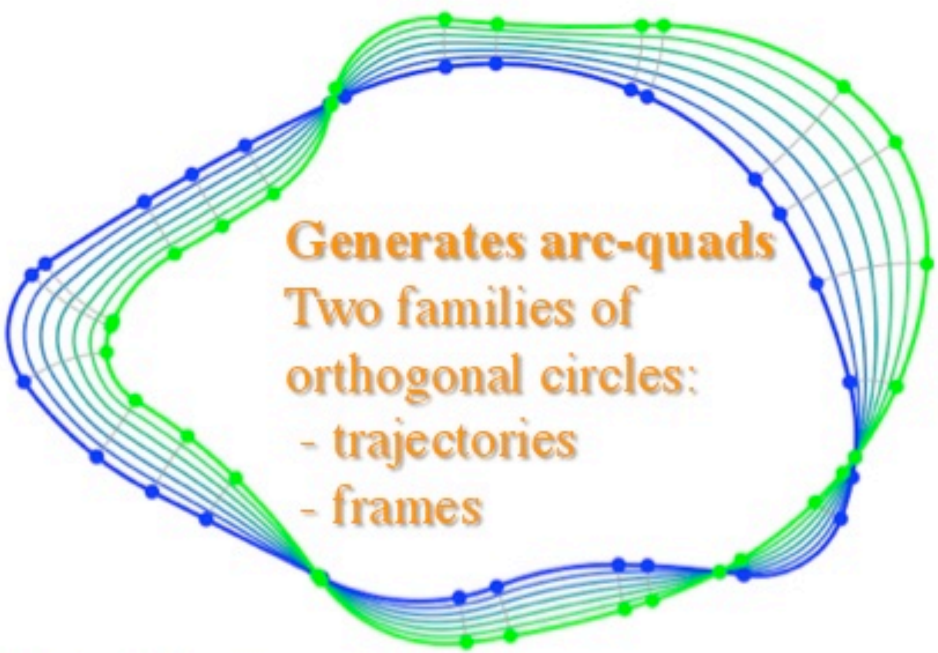


Smooth Locus of Symmetry
[Asada & Brady 86]
Midarc axis
Medial Axis: [Blum 67]
Process Induced Symmetric
Axis (PISA) [Layton 92]



Ball morph: Defines arc-quads

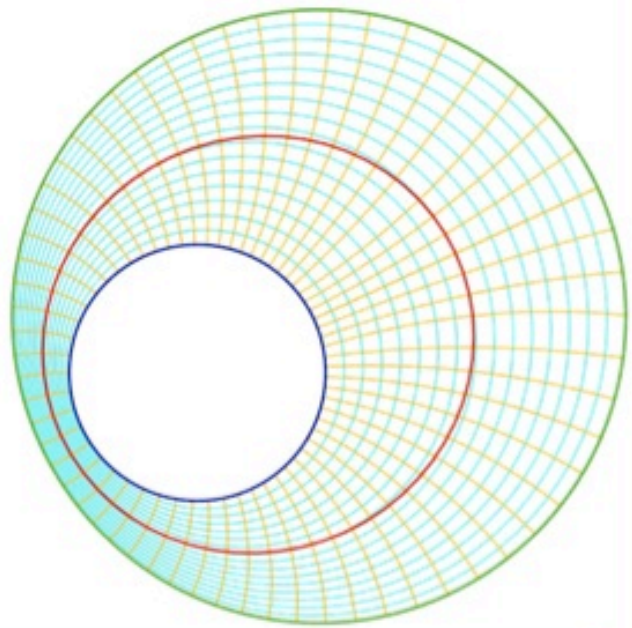




Generates arc-quads

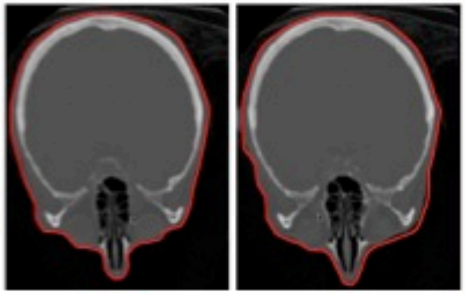
Two families of orthogonal circles:

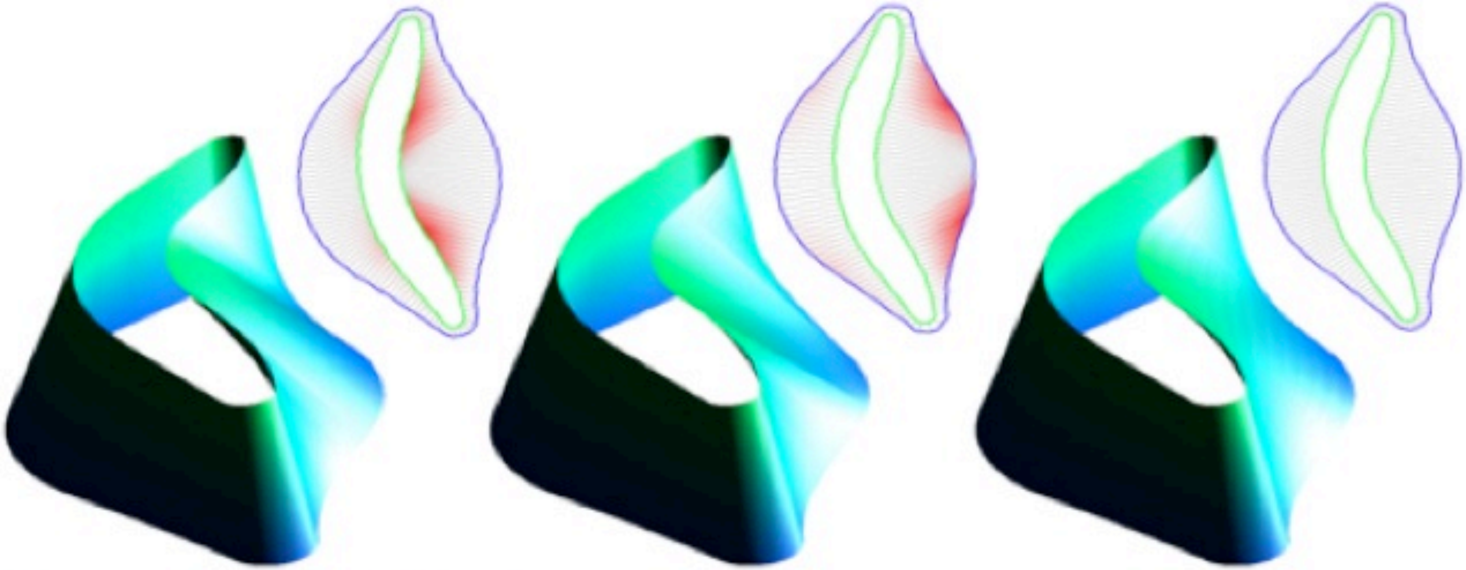
- **trajectories**
- **frames**



Ball morph for 3D reconstruction

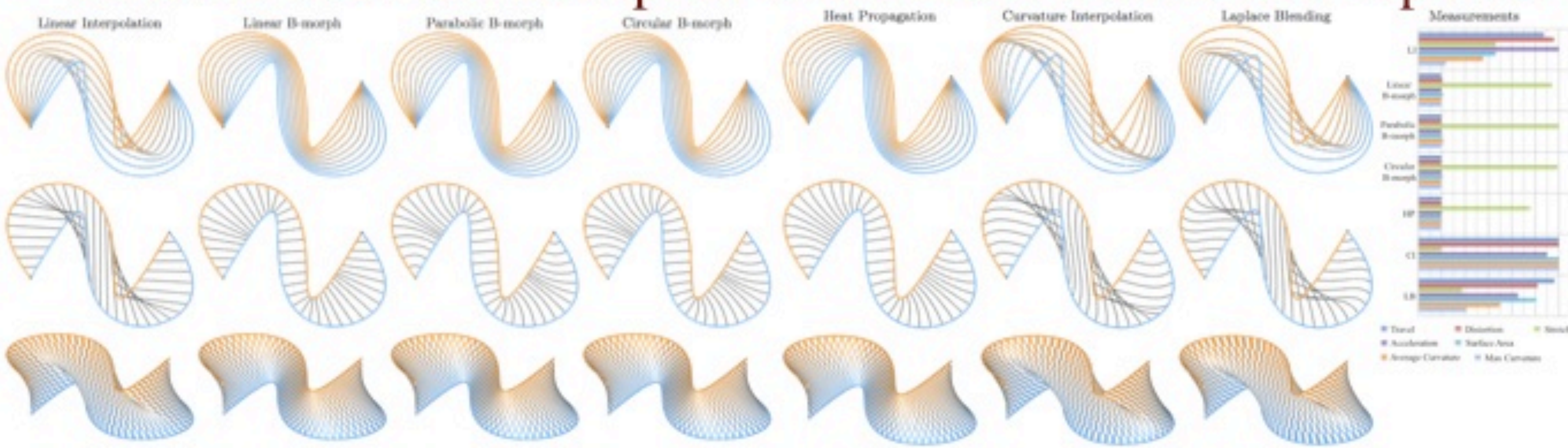
- Reconstruct surface from cross-sections
  - Ball morph with z=t
- Surface = Wall of **helices**
  - Trajectory orthogonal to section
  - Smoother than other interpolants



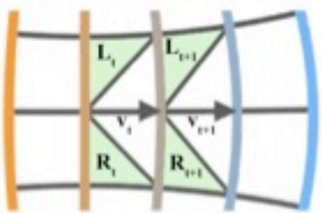


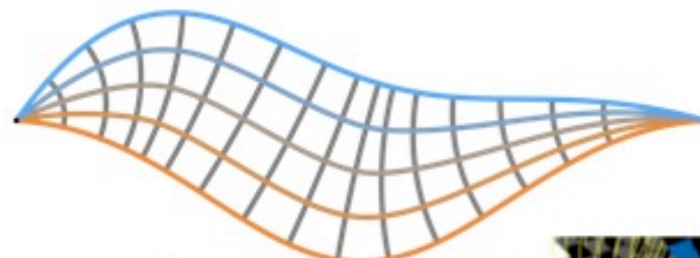
Morph: Comparison

- Assume two closed loops or two strokes with same endpoints

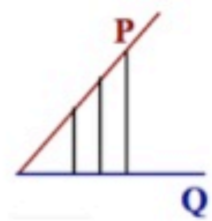


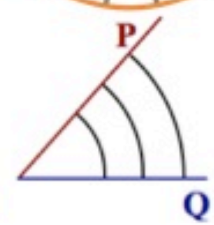
Travel distance
Stretch
Acceleration

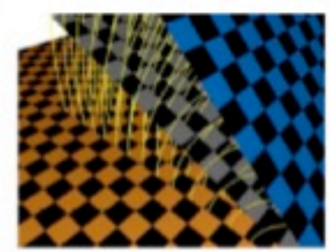




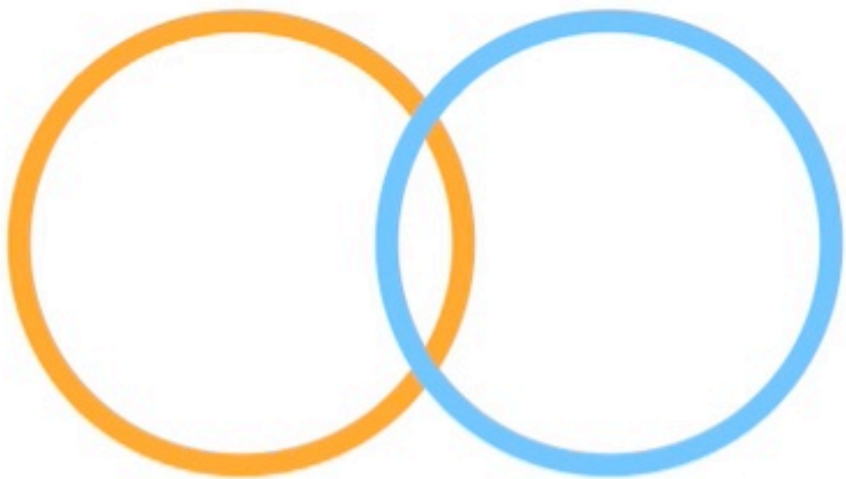
Surface area
Max square mean curvature





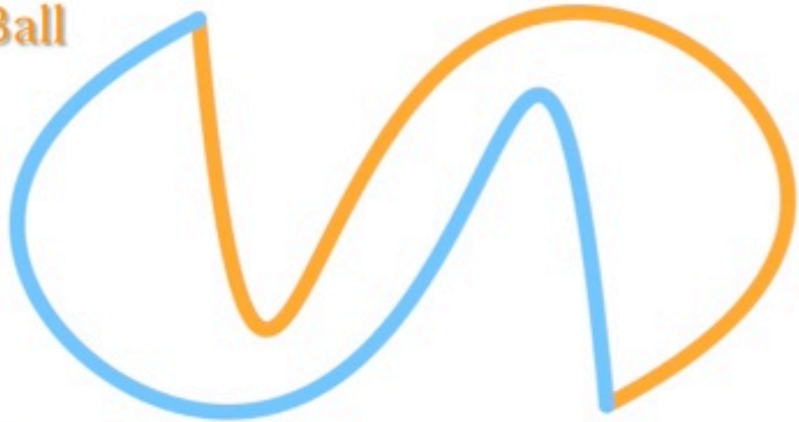


Example of ball-morph



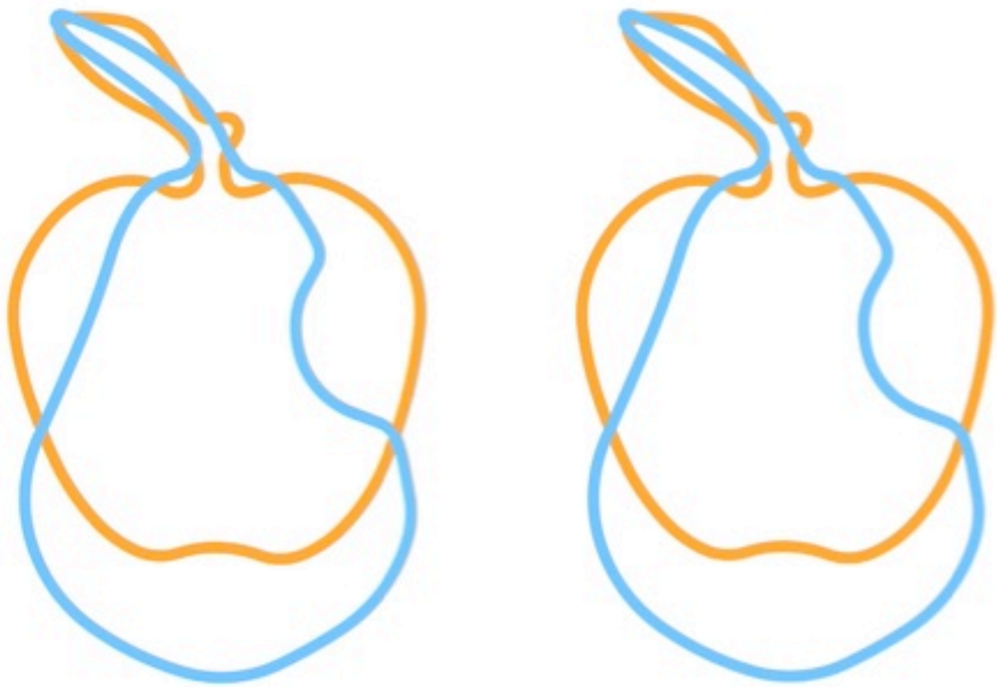
Comparison with curvature interpolation

Ball

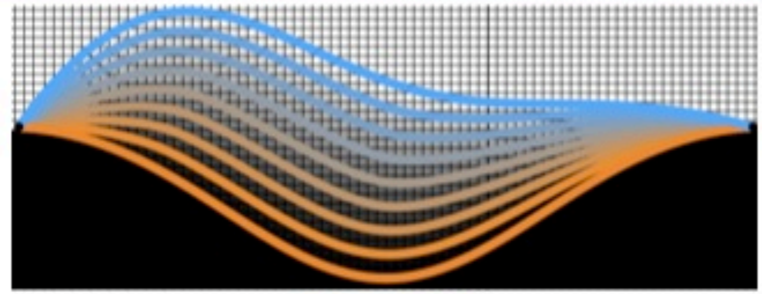


Curvature

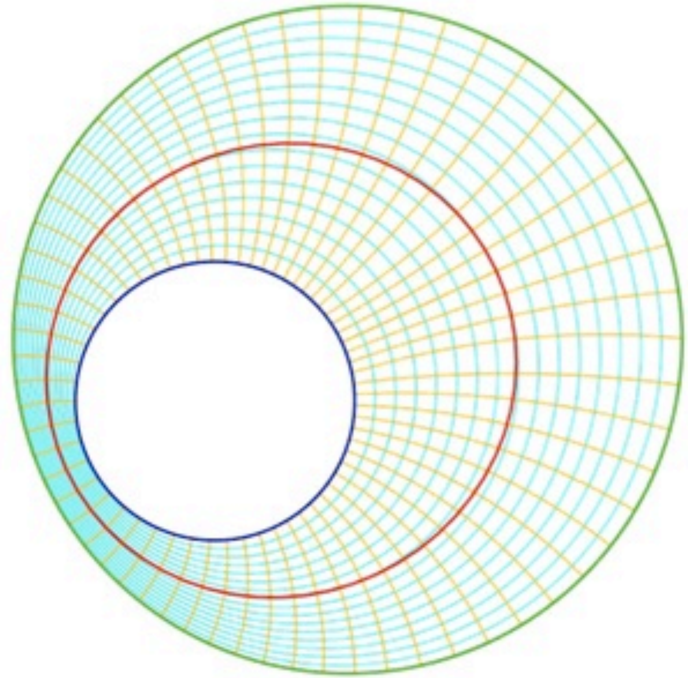
Comparison heat propagation



Ball morph approximates heat diffusion

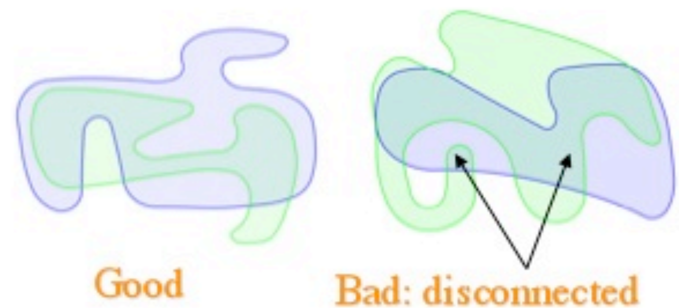


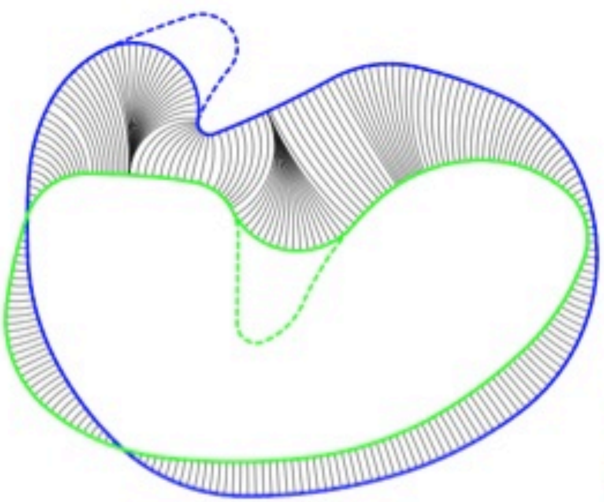
Very similar results, but ball-morph computation is much faster and independent of resolution (no grid)

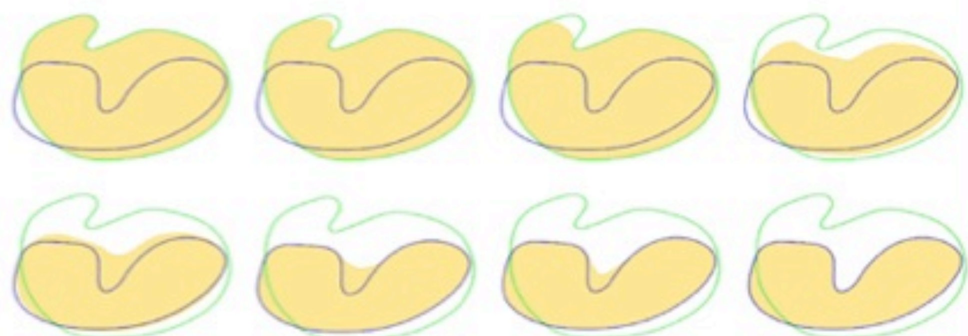


Extension to incompatible shapes

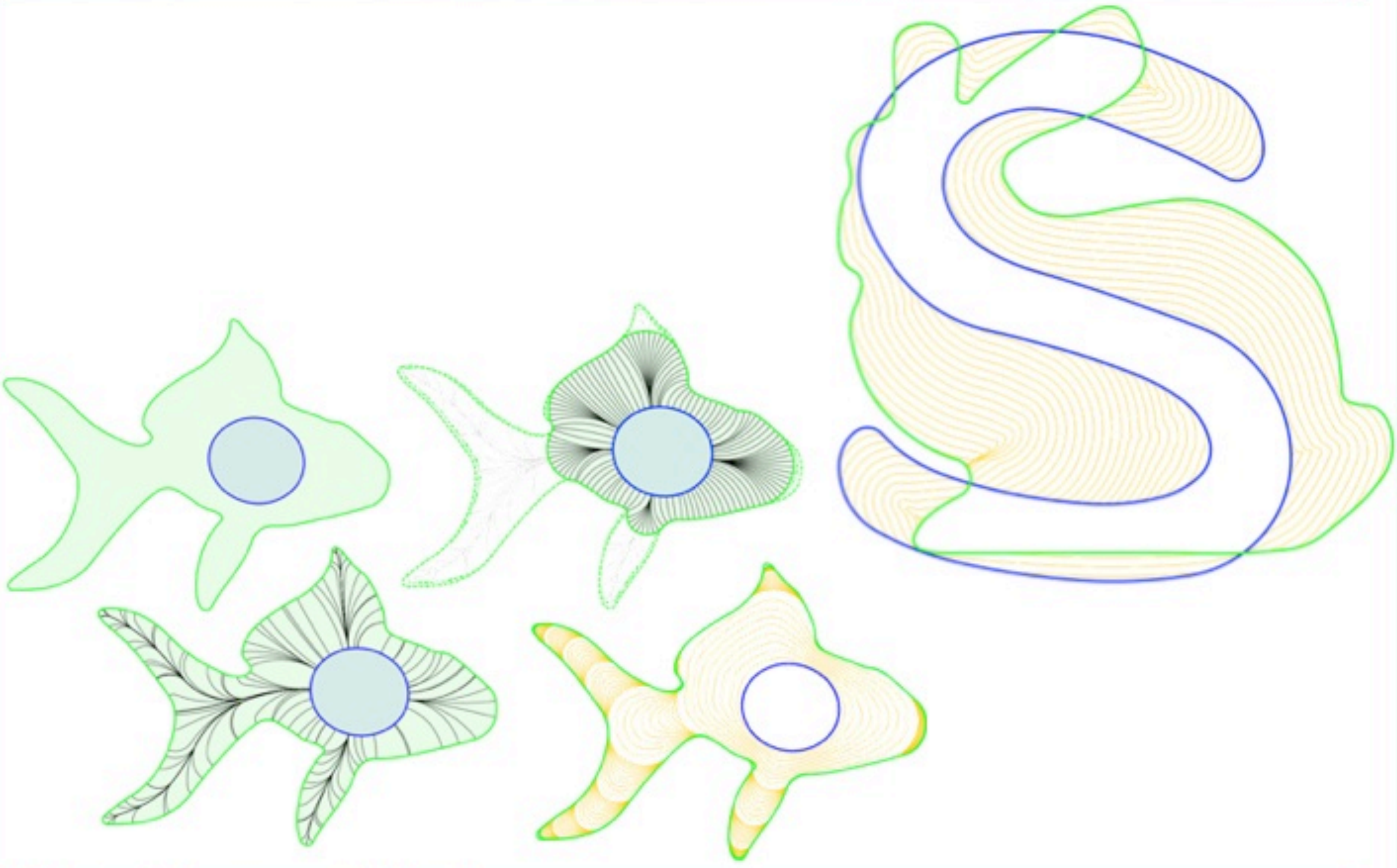
- Use relative blending to make them compatible
- Morph the compatible parts
- Extend the morph to the incompatible regions.. Iteratively
- Generates smooth trajectories
- Topological restrictions



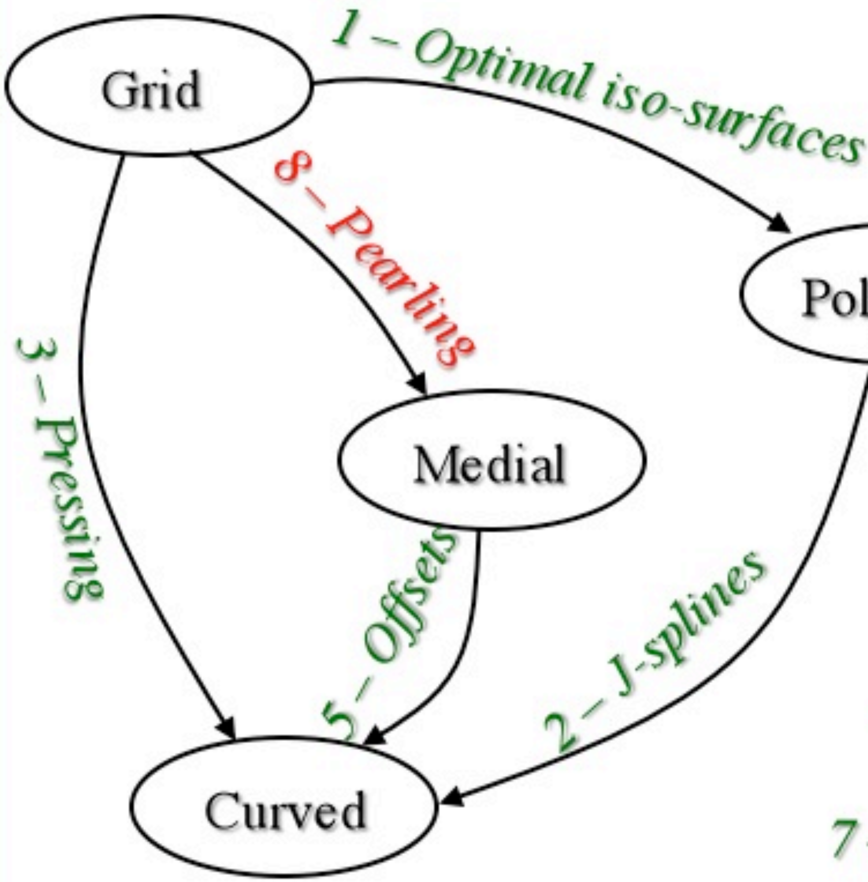




Ball-morph to incompatible shapes



PEARLING



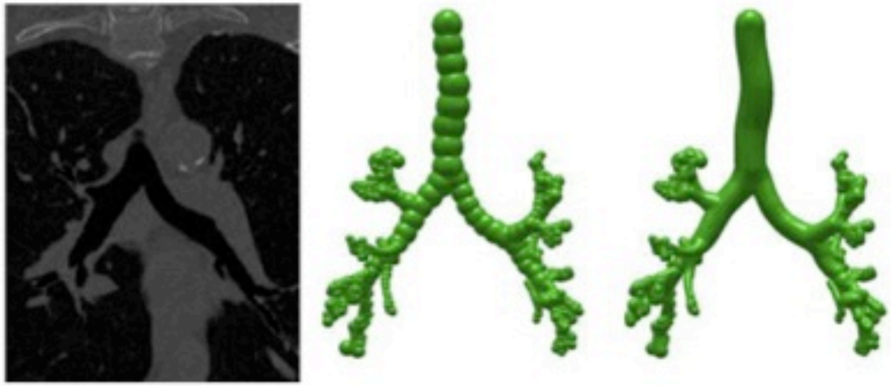
4 – Tightening

6 – Relative blending

7 – Ball-morphing

PEARLING

Goal: Extract tubular structures from 2D and from 3D (medical) images



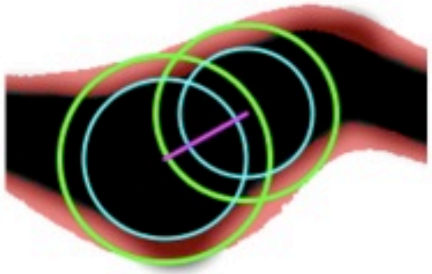
Pearling: 3D Interactive extraction of tubular structures from volumetric images, J. Rossignac, B. Whited, G. Slabaugh, T. Fang, G. Unal . MICCAI workshop on Interaction in Medical Image Analysis and Visualization, Nov. 2007

Pearling: Stroke segmentation with crusted pearl strings, B. Whited, J. Rossignac, G. Slabaugh, T. Fang, G. Unal, The First International Workshop on Image Mining Theory and Applications (IMTA), 2008. Also: Journal of Pattern Recognition and Image Analysis, Pleiades Publishing 19(2)277-283, 2009

3D Ball Skinning using PDEs for Generation of Smooth Tubular Surfaces, G. Slabaugh, J. Rossignac, B. Whited, T. Fang, G. Unal, Journal of Computer Aided-Design (JCAD). 2009

Variational Skinning of an Ordered Set of Discrete 2D Balls, G. Slabaugh, G. Unal, T. Fang, J. Rossignac, B. Whited, Geometric Modeling and Processing 2008. Springer

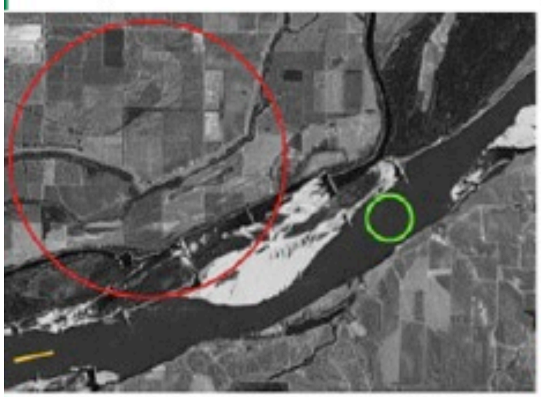
Pearling 2D (with Siemens)

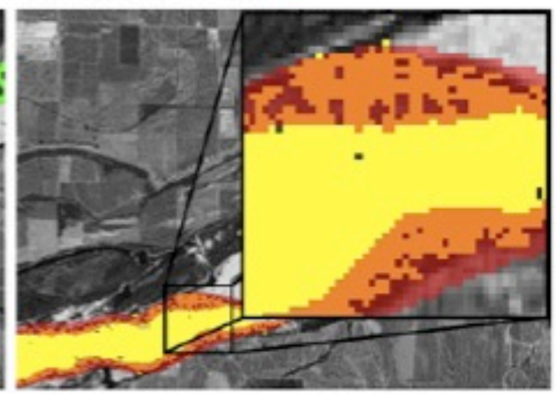


$$F(c_i,r_i)=\frac{9}{\pi r_i^3}\int_{x\in P_i}\Phi(x)(c_i-x)\left(1-\frac{\|c_i-x\|^2}{r_i^2}\right)dx$$

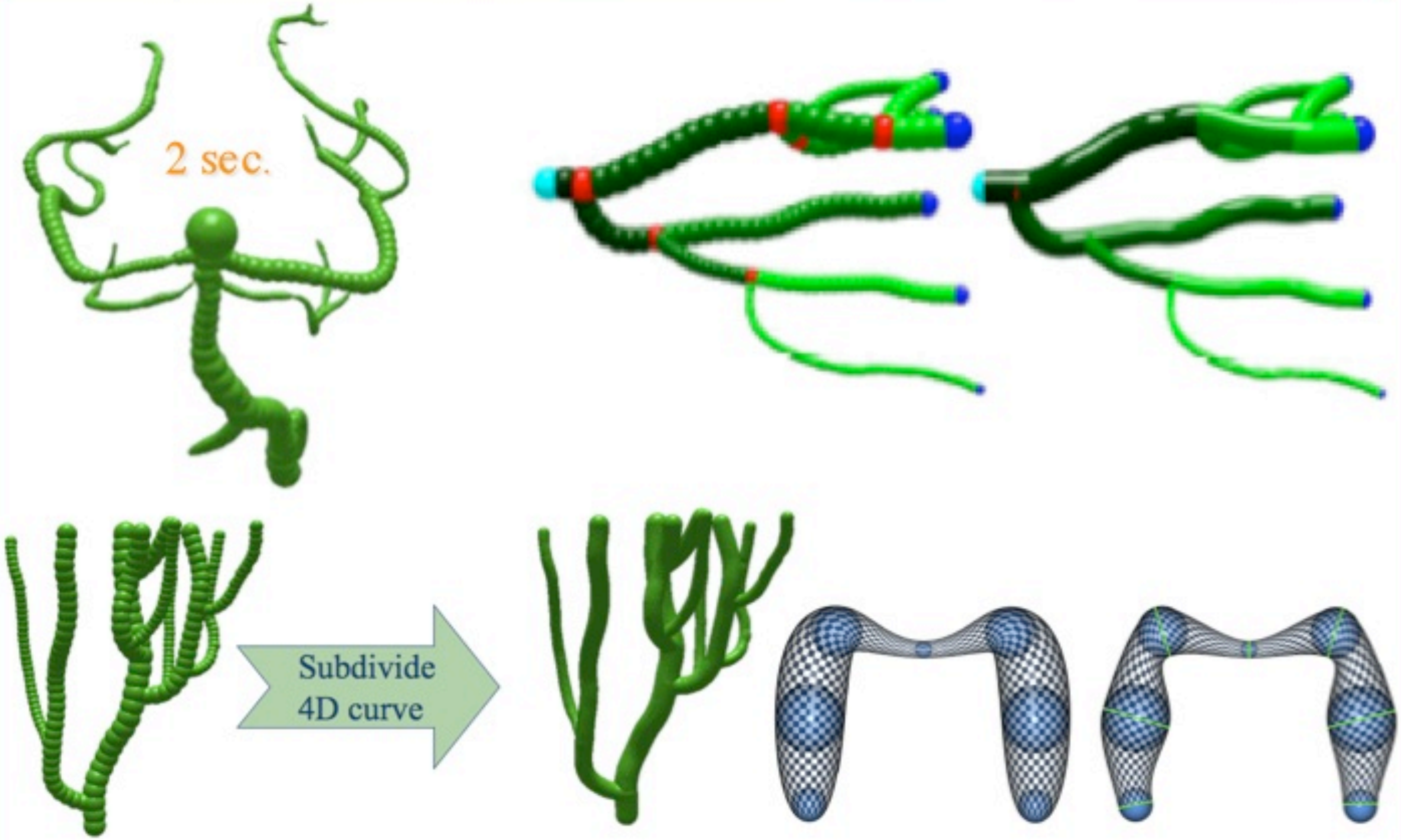
$$G=\frac{\int_{x\in P_i}\Phi(x)dx}{\int_{x\in P_i}dx}$$

$$\Phi(x)=\begin{cases}1, & \text{if } p_b(I(x)) > p_g(I(x))\\ 0, & \text{otherwise}\end{cases}$$

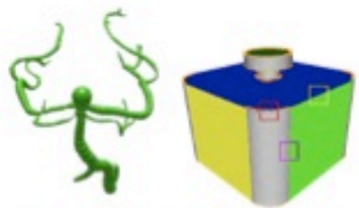




Pearling 3D

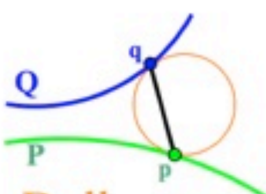


Questions

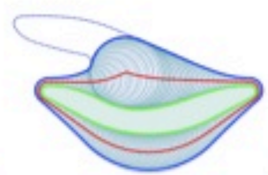


Pearling & Pressing

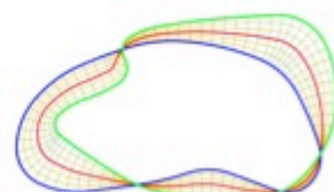
J-spline & ringing



Ball map



Relative blending



Ball morph