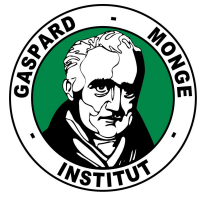
Isthmus-Based 6-Directional Parallel Thinning Algorithms

Benjamin Raynal and Michel Couprie

April 6, 2011



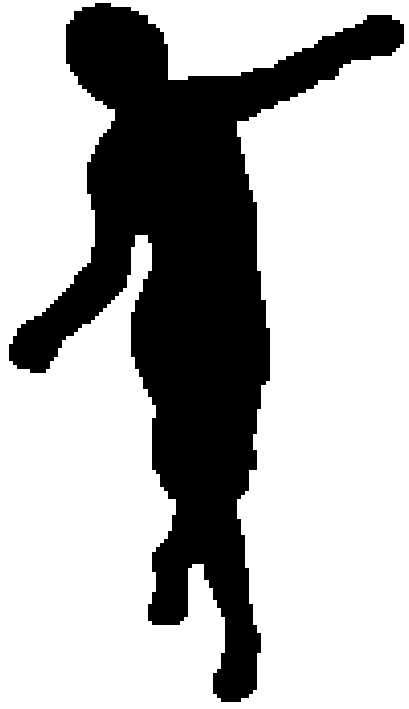
Overview

Introduction

Skeleton?

Blum's grass fire analogy (in 2D):

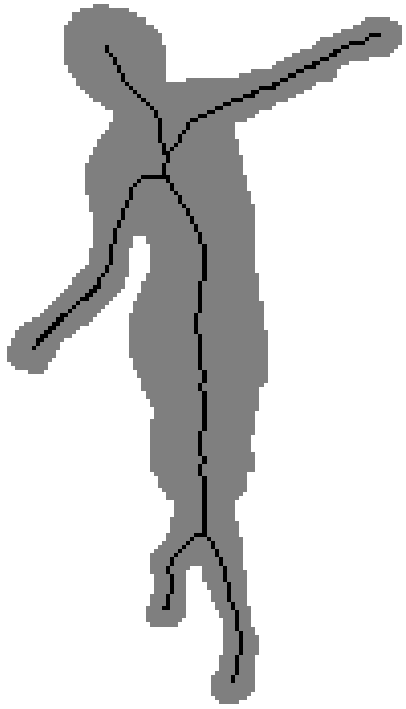
- imagine the shape as a dry grass field
- set on fire the contour of the field
- meeting points of the flame fronts = *skeleton* of the shape



Skeleton?

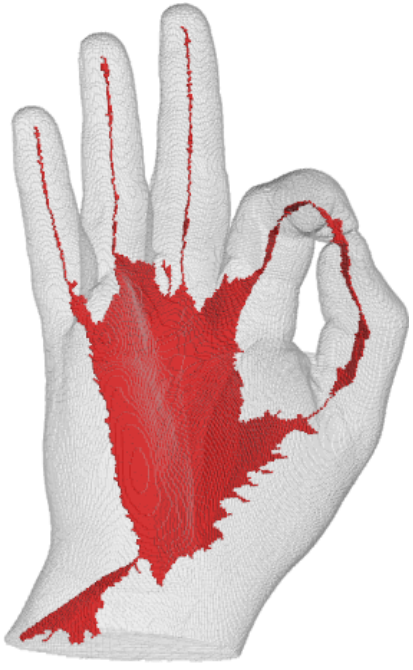
Skeleton properties :

- homotopic to the original object
- thin (lower dimension than the object)
- centered in the original object

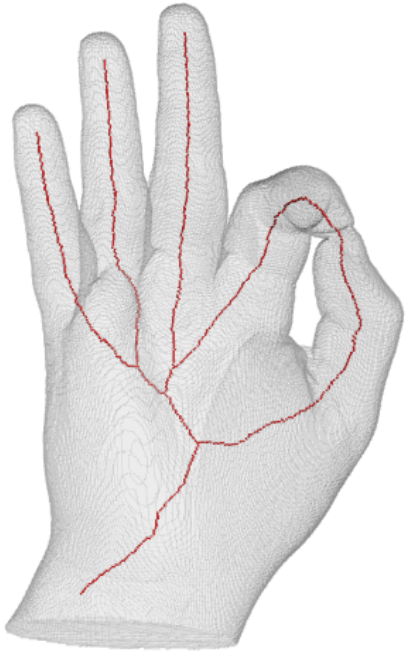


3D Skeleton

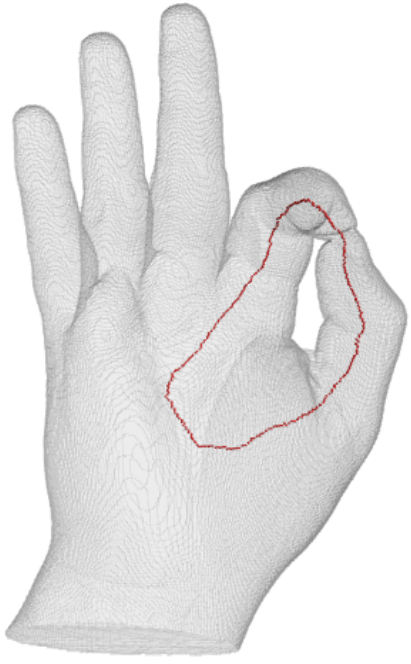
In 3D, 3 kinds of skeletons



Surface Skeleton



Curvilinear Skeleton



Ultimate Skeleton

Thinning

Simple point

- point which can be removed without changing the topology
- locally characterized

Thinning

Simple point

- point which can be removed without changing the topology
- locally characterized

Thinning

Iteratively removing simple points, until stability.

Thinning

Simple point

- point which can be removed without changing the topology
- locally characterized

Thinning

Iteratively removing simple points, until stability.

- without *constraints* $\rightarrow$ ultimate skeleton

Thinning

Simple point

- point which can be removed without changing the topology
- locally characterized

Thinning

Iteratively removing simple points, until stability.

- without *constraints* $\rightarrow$ ultimate skeleton
- with *constraints* $\rightarrow$ curvilinear or surface skeleton

Our goal

We are searching for:

- fast thinning algorithm
- skeleton with low amount of noise

Thinning strategies

Thinning strategies:

- sequential

Details

Only one simple point is removed at each iteration

Thinning strategies

Thinning strategies:

- sequential
- parallel

Details

All the simple points with same properties are removed at each iteration

Thinning strategies

Thinning strategies:

- sequential
- parallel

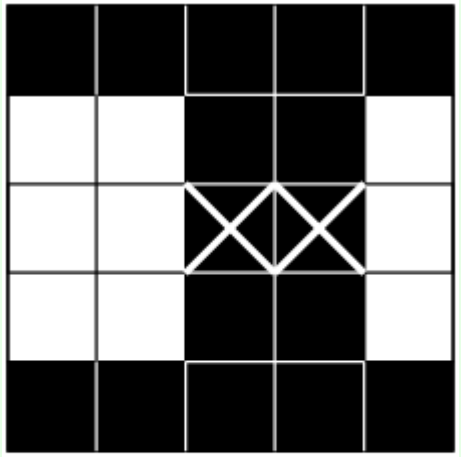
Details

Thinning strategies

Thinning strategies:

- sequential
- parallel

Details

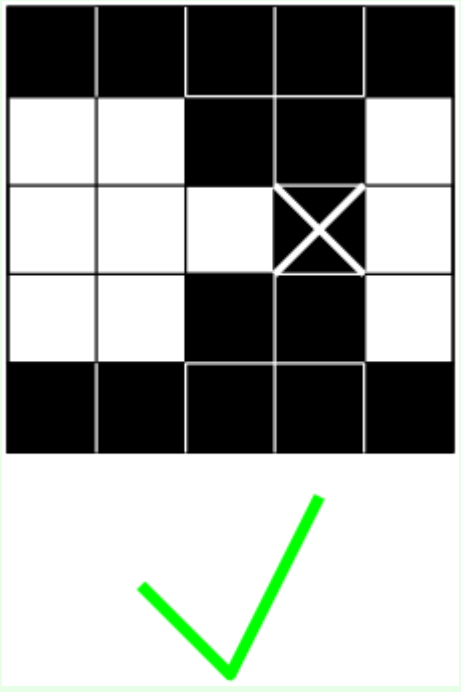


Thinning strategies

Thinning strategies:

- sequential
- parallel

Details

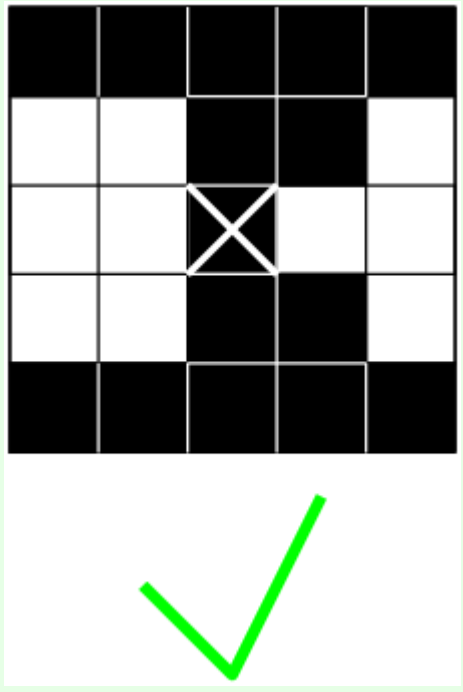


Thinning strategies

Thinning strategies:

- sequential
- parallel

Details

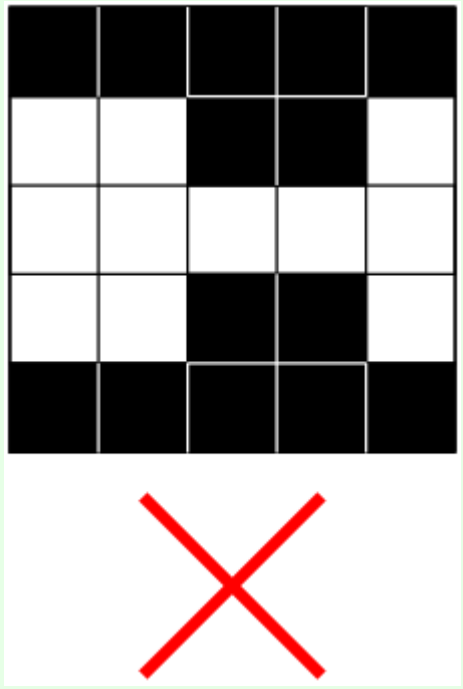


Thinning strategies

Thinning strategies:

- sequential
- parallel

Details



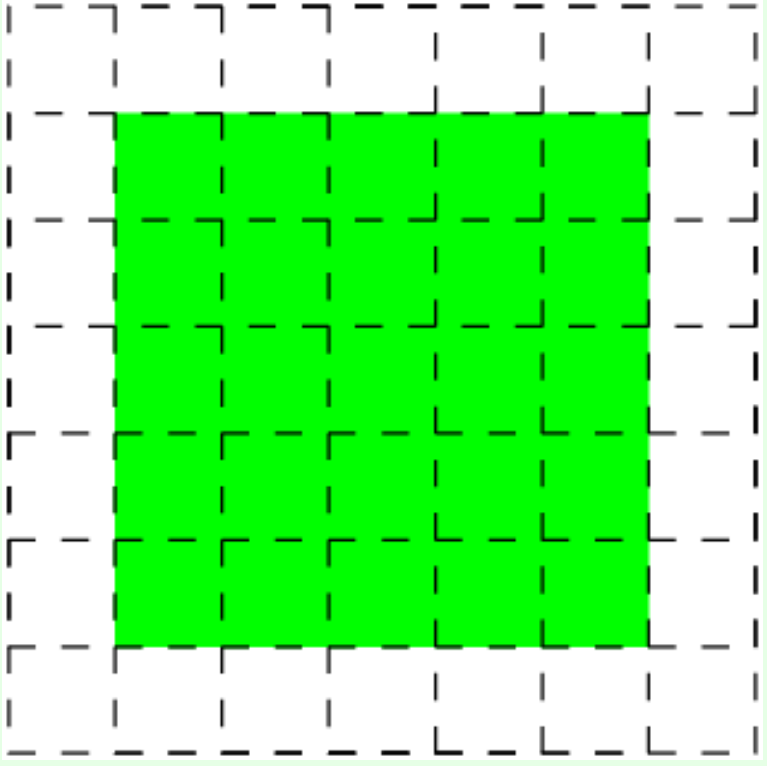
Thinning strategies

Thinning strategies:

- sequential
- parallel
  - directional

Details

Only points having a same border direction are considered



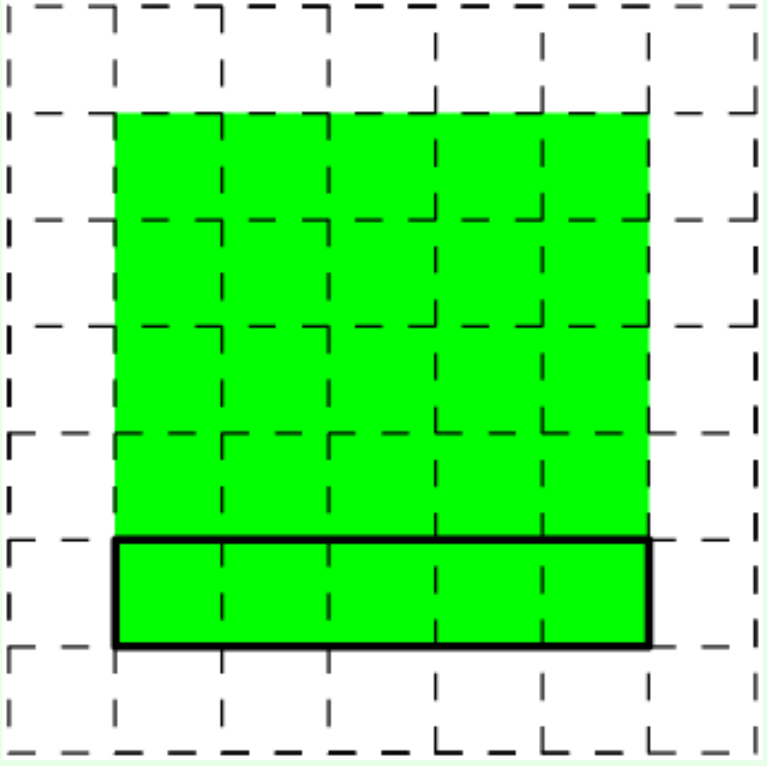
Thinning strategies

Thinning strategies:

- sequential
- parallel
  - directional

Details

Only points having a same border direction are considered



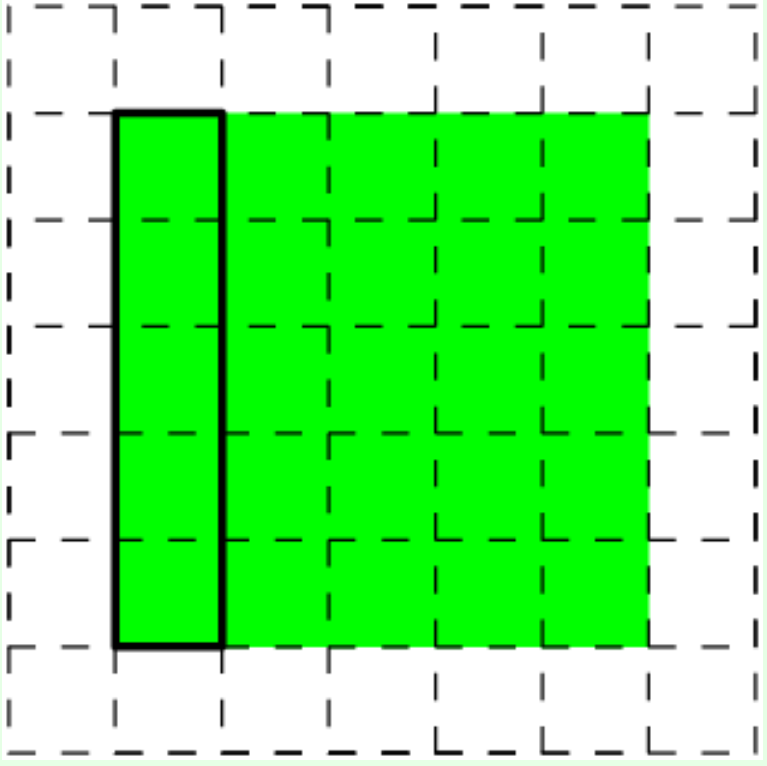
Thinning strategies

Thinning strategies:

- sequential
- parallel
  - directional

Details

Only points having a same border direction are considered



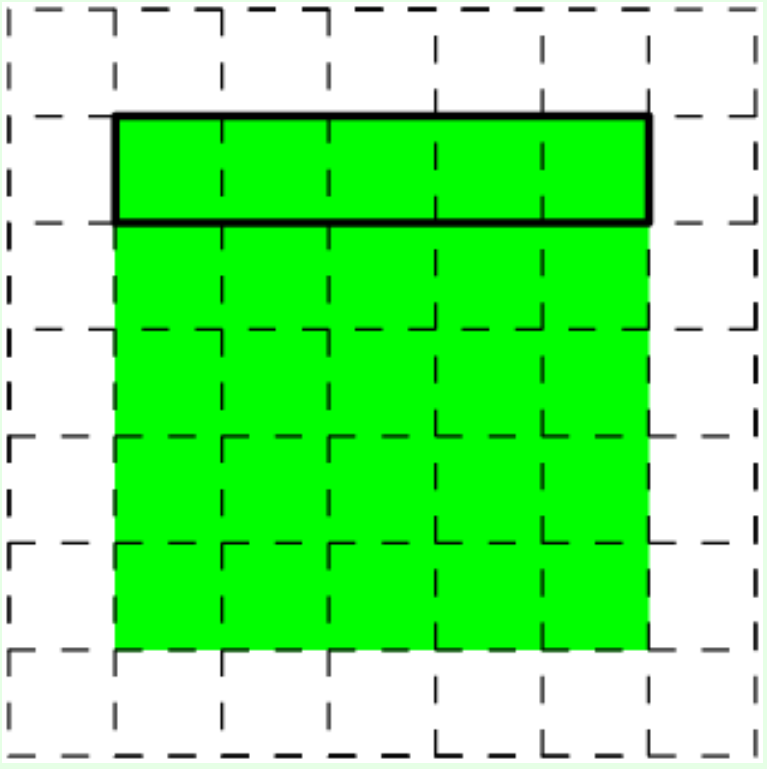
Thinning strategies

Thinning strategies:

- sequential
- parallel
  - directional

Details

Only points having a same border direction are considered



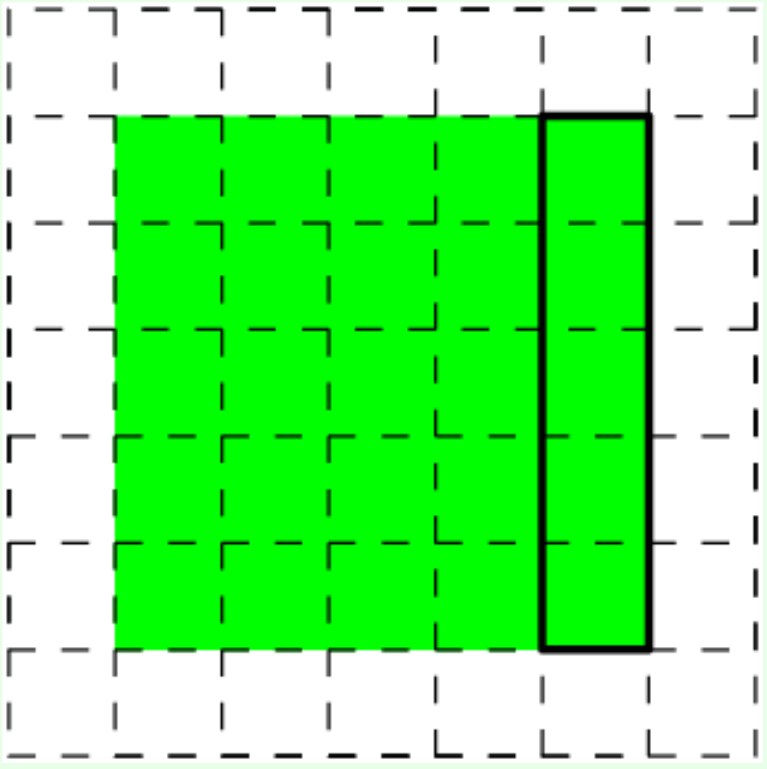
Thinning strategies

Thinning strategies:

- sequential
- parallel
  - directional

Details

Only points having a same border direction are considered



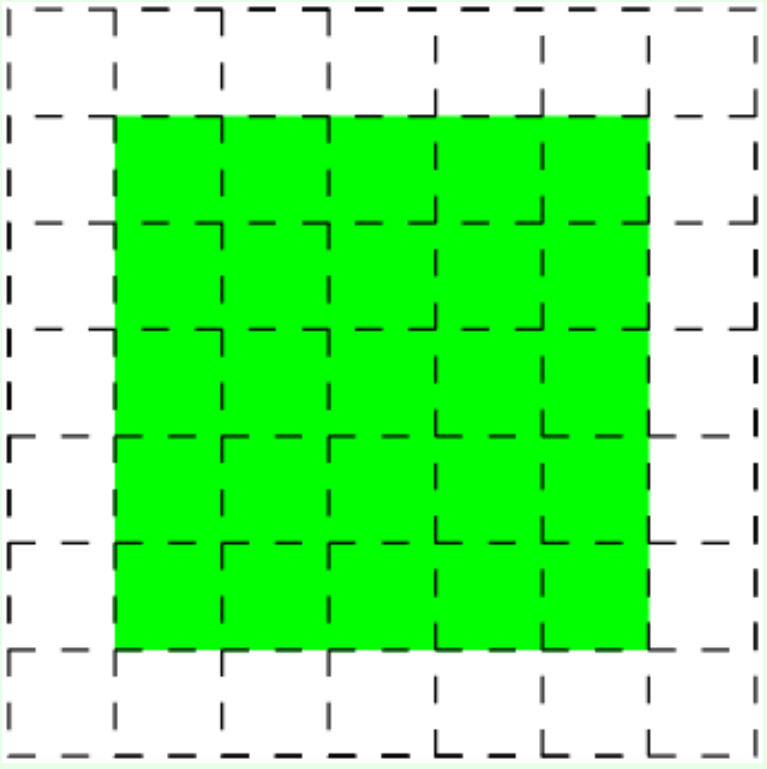
Thinning strategies

Thinning strategies:

- sequential
- parallel
  - directional
  - subfield-based

Details

Only points belonging to a same sub grid are considered



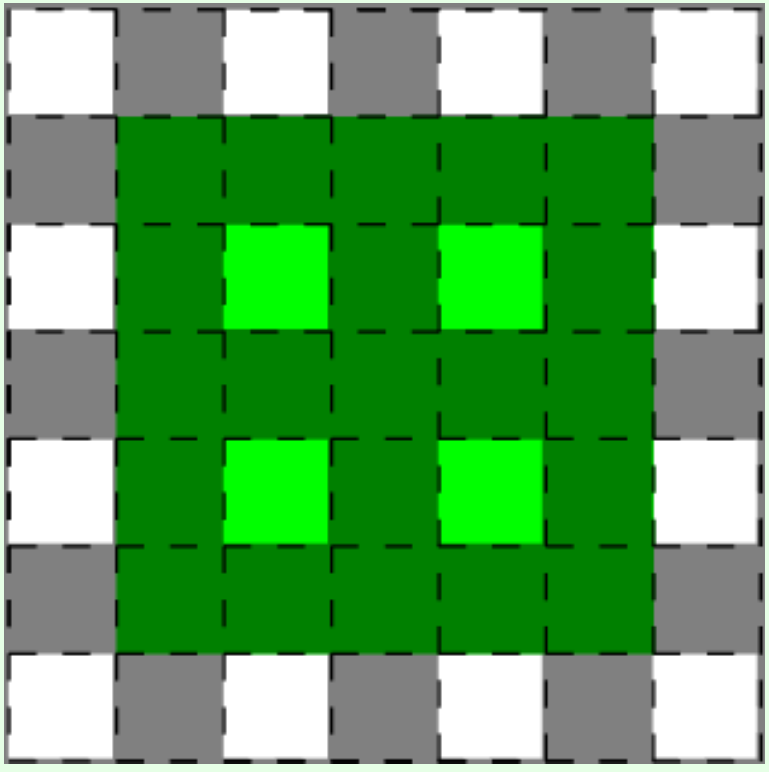
Thinning strategies

Thinning strategies:

- sequential
- parallel
  - directional
  - subfield-based

Details

Only points belonging to a same sub grid are considered



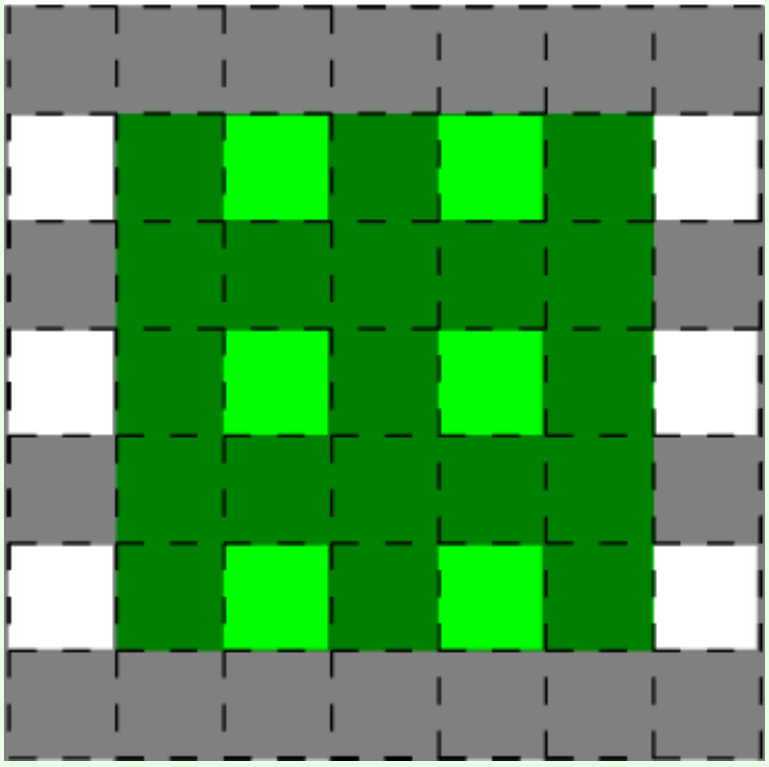
Thinning strategies

Thinning strategies:

- sequential
- parallel
  - directional
  - subfield-based

Details

Only points belonging to a same sub grid are considered



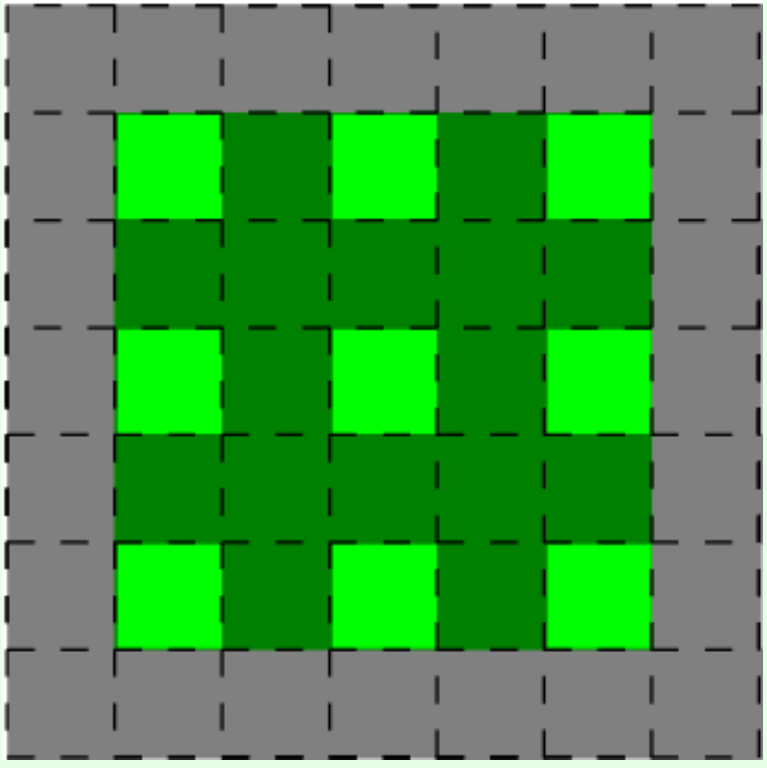
Thinning strategies

Thinning strategies:

- sequential
- parallel
  - directional
  - subfield-based

Details

Only points belonging to a same sub grid are considered



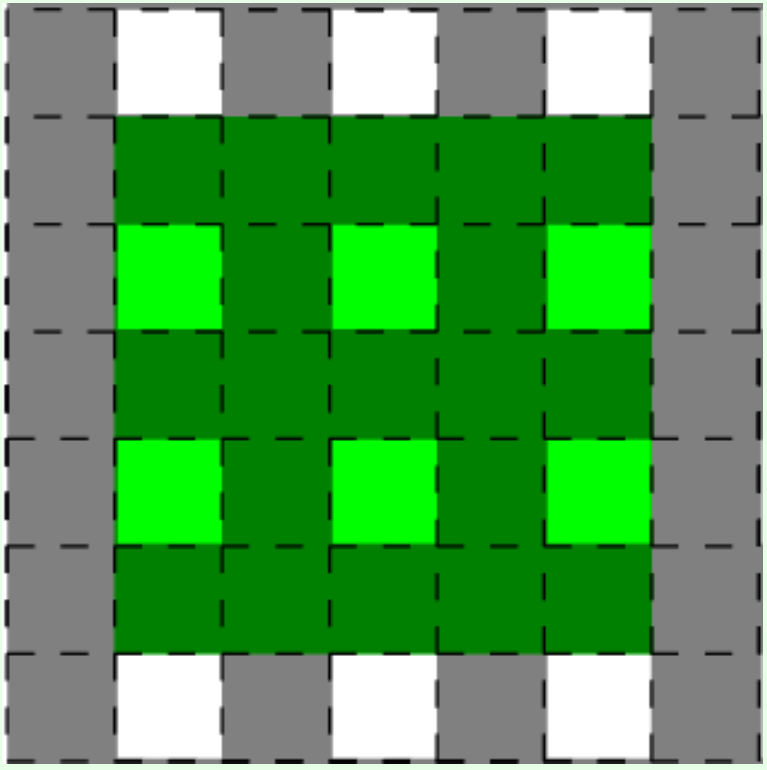
Thinning strategies

Thinning strategies:

- sequential
- parallel
  - directional
  - subfield-based

Details

Only points belonging to a same sub grid are considered



Thinning strategies

Thinning strategies:

- sequential
- parallel
  - directional
  - subfield-based
  - fully parallel

Details

- no sub iterations
- checking use a bigger neighborhood

Thinning strategies

Thinning strategies:

- sequential $\rightarrow$ noisy
- parallel
  - directional
  - subfield-based $\rightarrow$ noisy
  - fully parallel

Thinning strategies

Thinning strategies:

- sequential $\rightarrow$ noisy
- parallel
  - directional
  - subfield-based $\rightarrow$ noisy
  - fully parallel $\rightarrow$ slow

Thinning strategies

Thinning strategies:

- sequential $\rightarrow$ noisy
- parallel
  - **directional**
  - subfield-based $\rightarrow$ noisy
  - fully parallel $\rightarrow$ slow

PKD6 algorithm

Palágyi and Kuba 6-directional thinning algorithm

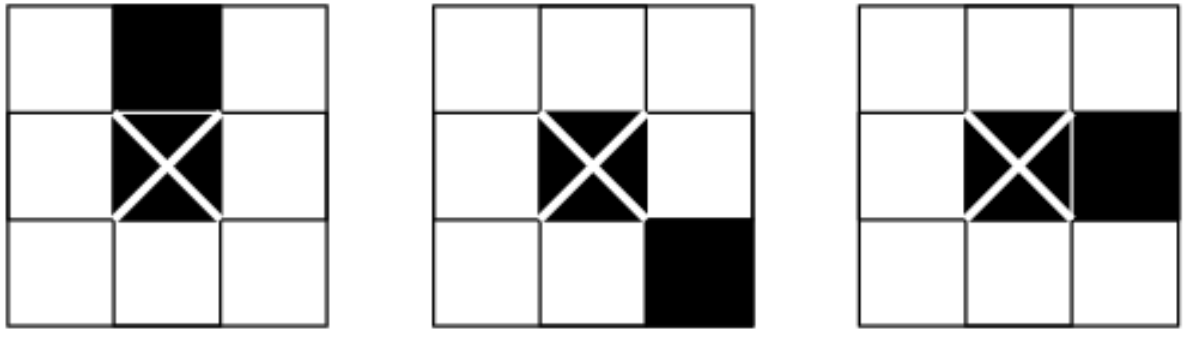
PKD6 (*Palágyi and Kuba, 1998*):

- 6-directional thinning algorithm
- using *ending points* as constraints
- constraint set updated at each sub iteration

Ending Points

Points ending a curve:

- locally defined: one and only one neighbor
- sub-case of simple point



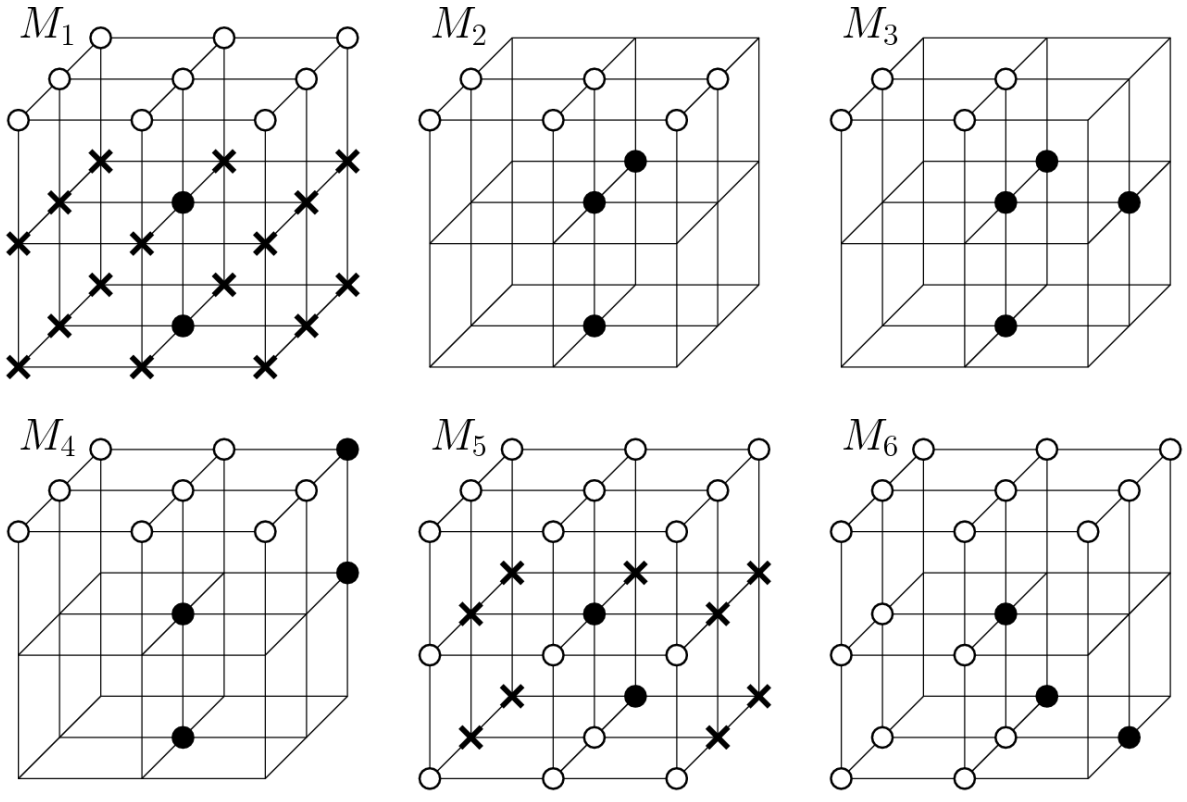
PKD6

Particularities of PKD6:

- integrate the ending point preservation in the deletion test
- use of masks for the deletion tests

PKD6

Masks $\mathcal{M}_{UP}$ for "UP" direction for detection of removable points



Preserve the topology

PKD6

Algorithm

input An object $X \subseteq \mathbb{Z}^3$

result A skeleton of X

. Do until stability:

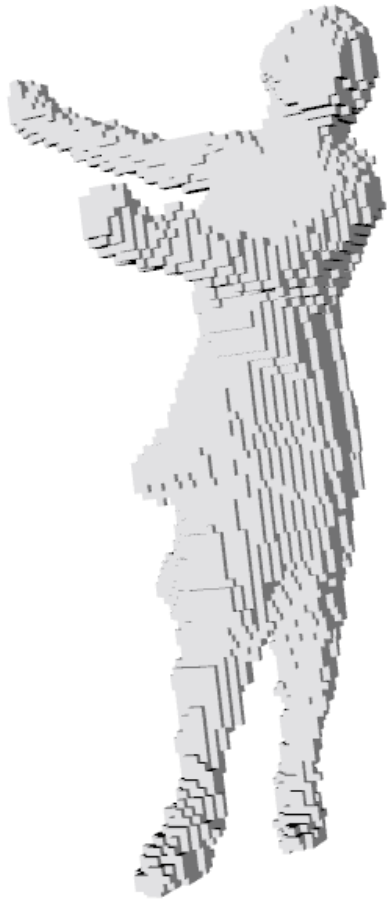
- for each direction d:
  - $X = X \setminus \{p \in X; p \text{ matching in } \mathcal{M}_d\}$

. return X

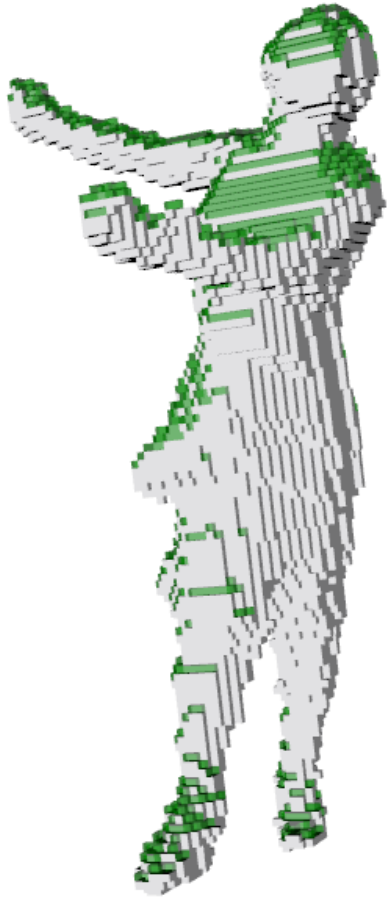
Notations

- X : the object
- $\mathcal{M}_d$: set of masks representing removable points for d

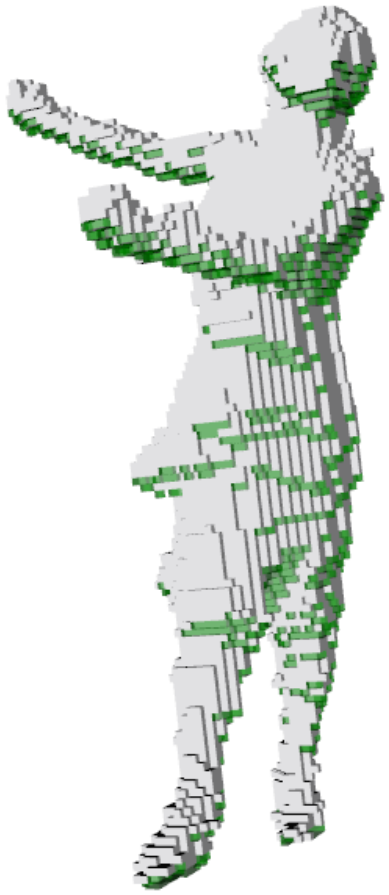
PKD6



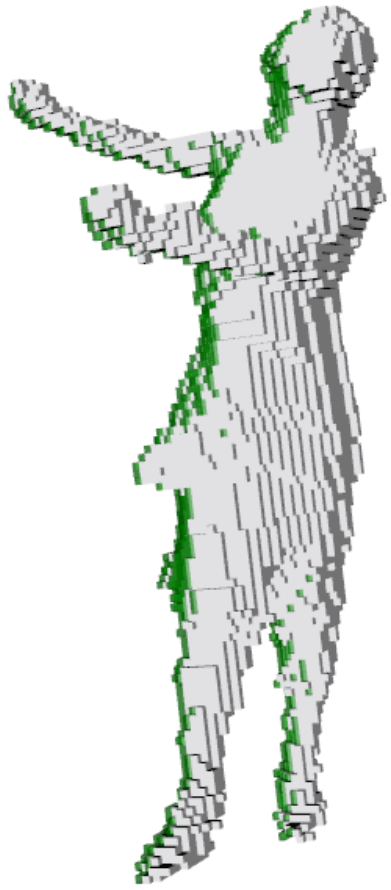
PKD6



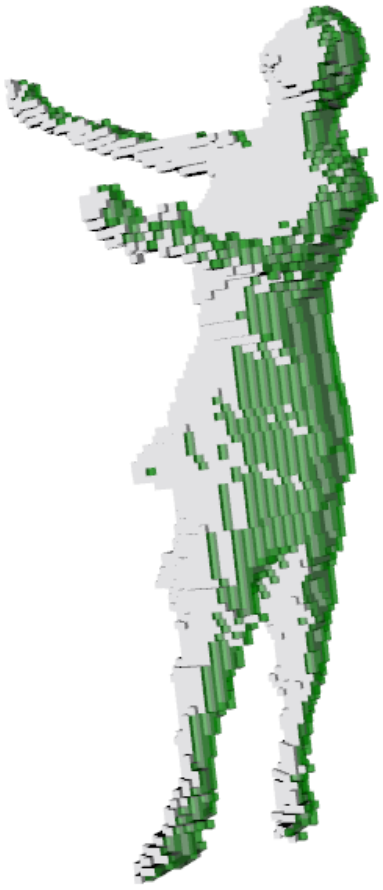
PKD6



PKD6

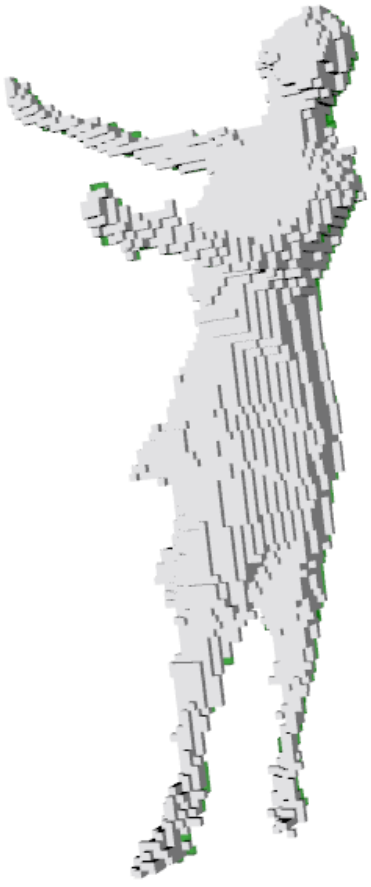


PKD6

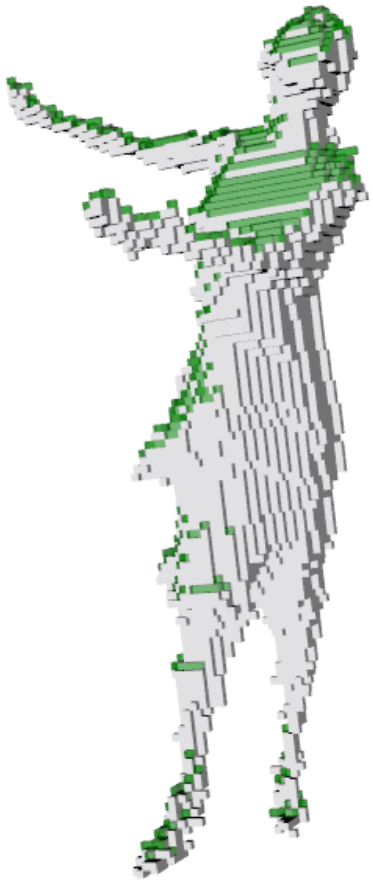


PKD6

PKD6



PKD6



Our algorithm: D6I1D

D6I1D

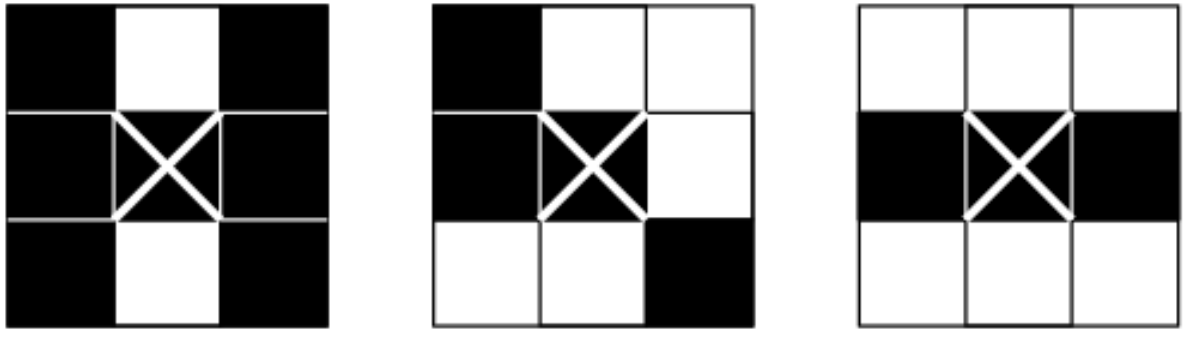
Our new thinning scheme D6I1D:

- 6-directional thinning algorithm (based on *PKD6*)
- using *1D-isthmus* constraints (*Bertrand and Couprie, 2007*)
- constraint set updated at each iteration

1D-Isthmuses

1D-isthmus: point which cannot be removed without breaking connectivity of neighborhood.

- locally defined: more than one CC of (object ∩ neighborhood)
- non simple point



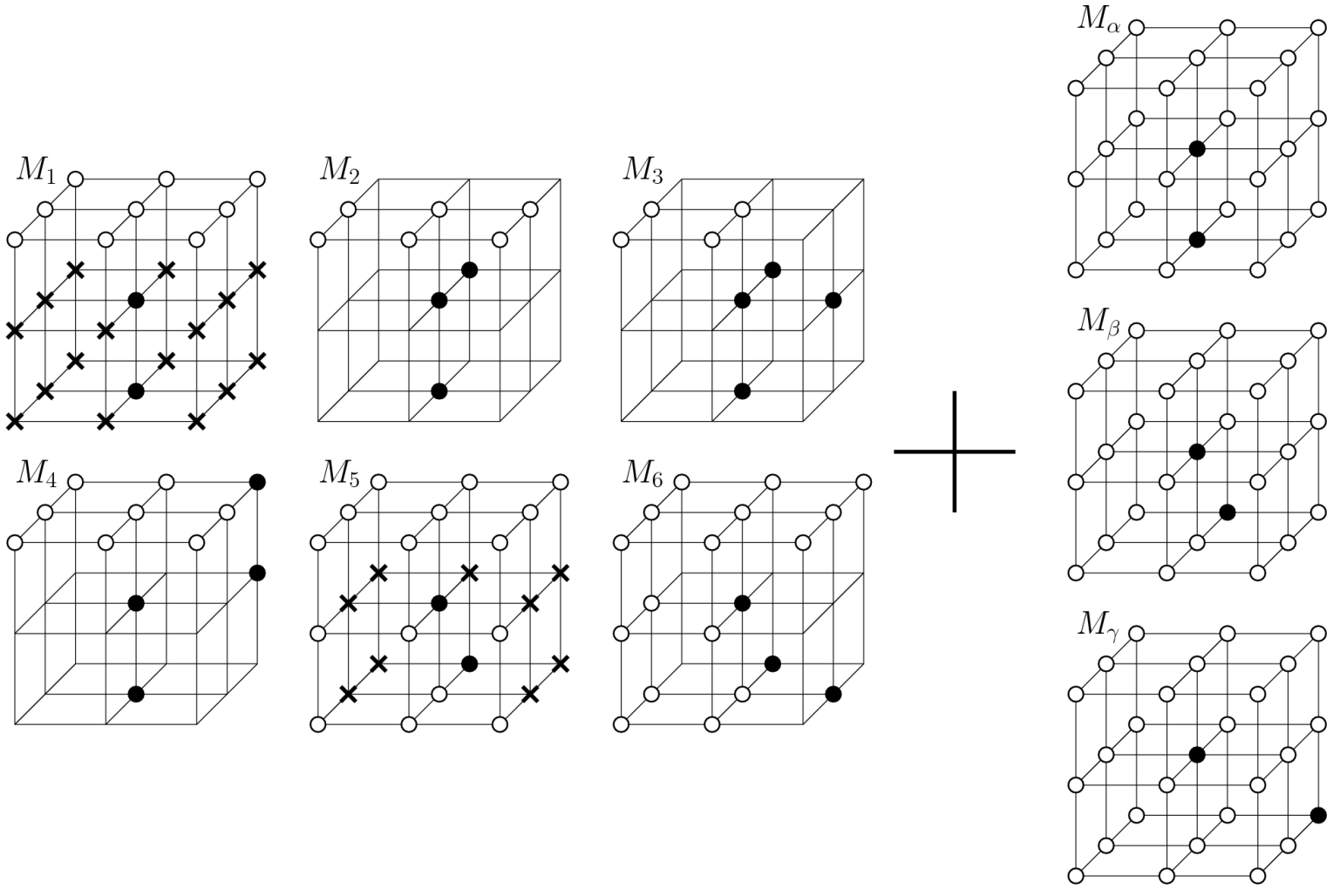
D6I1D

Particularities of D6I1D:

- cannot integrate the isthmuses preservation in the deletion test
- use a modification of PKD6 masks for the deletion tests
- use a separate detection of the constraint set

D6I1D

Masks $\mathcal{M}'_{UP}$ for "UP" direction for detection of removable points



Preserve the topology

D6I1D

Algorithm

input $X \subseteq \mathbb{Z}^3, K \subseteq X$

result A skeleton of X

. Do until stability:

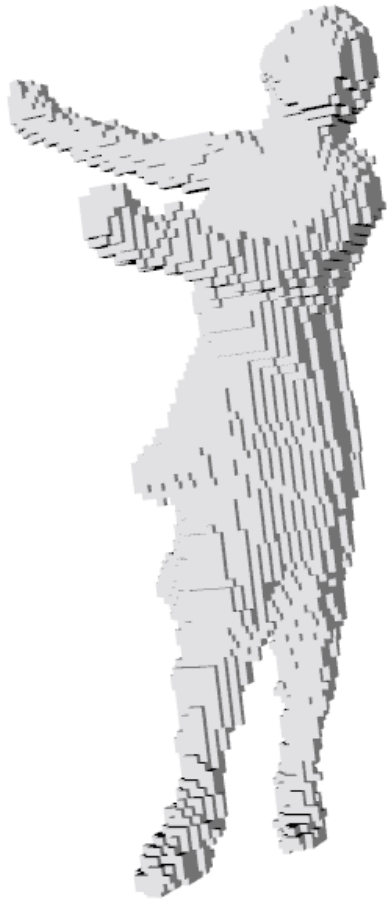
- $K = K \cup \Psi(X)$
- for each direction d:
  - $X = X \setminus \{p \in X \setminus K; p \text{ matching in } \mathcal{M}'_d\}$

. return X

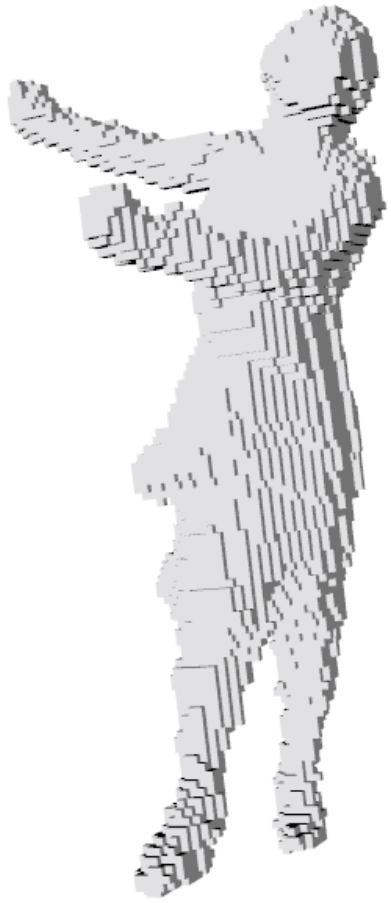
Notations

- K : set of points which can not be removed
- $\Psi(X)$: set of 1D-isthmuses in X

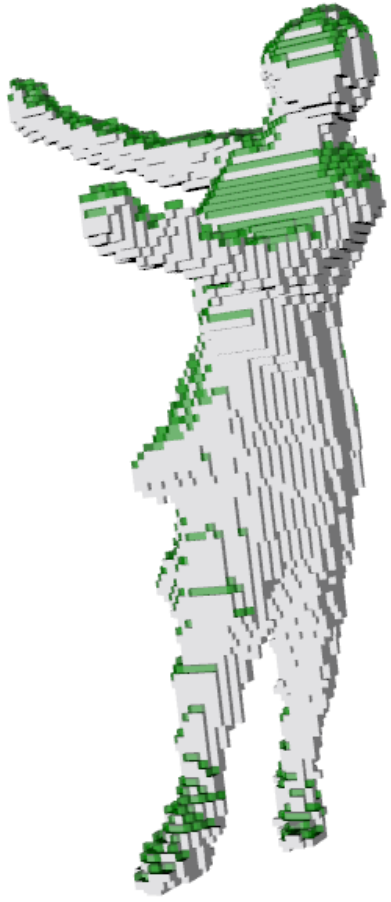
D6I1D



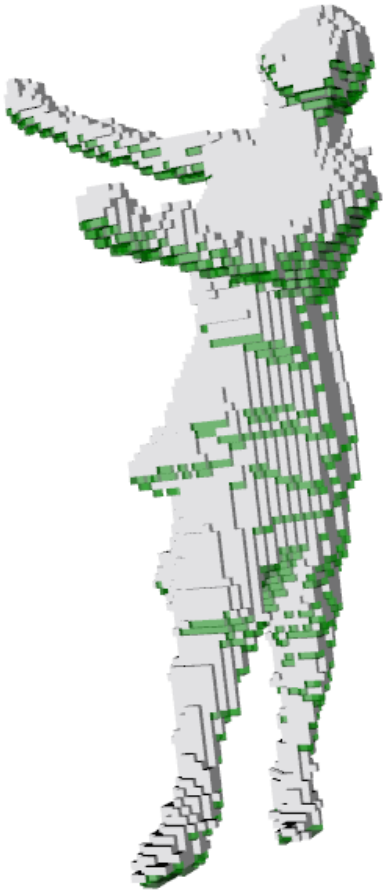
D6I1D



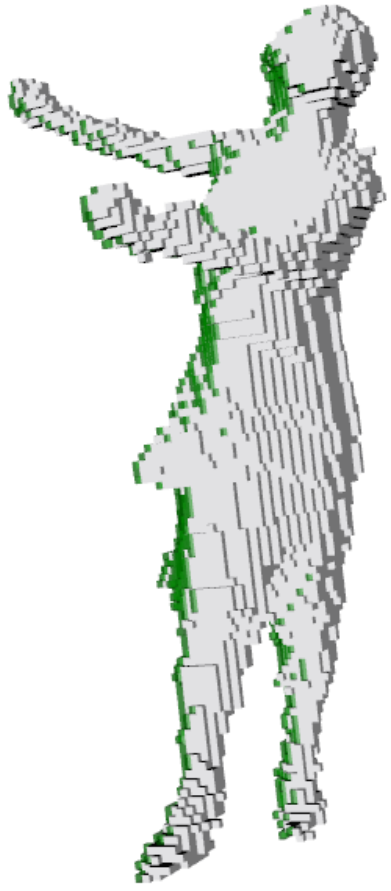
D6I1D



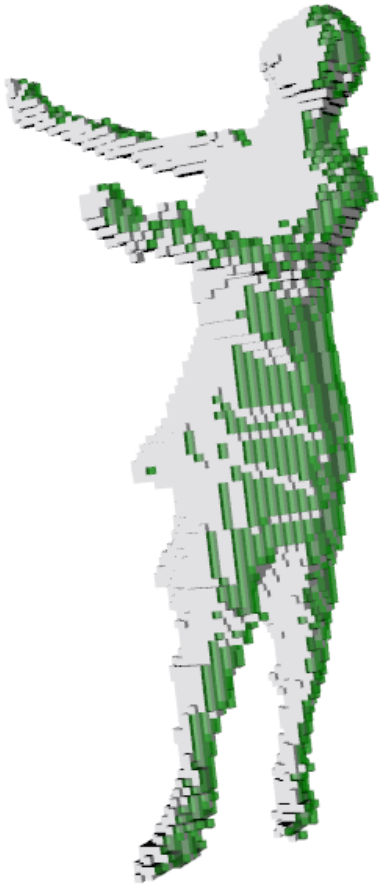
D6I1D



D6I1D

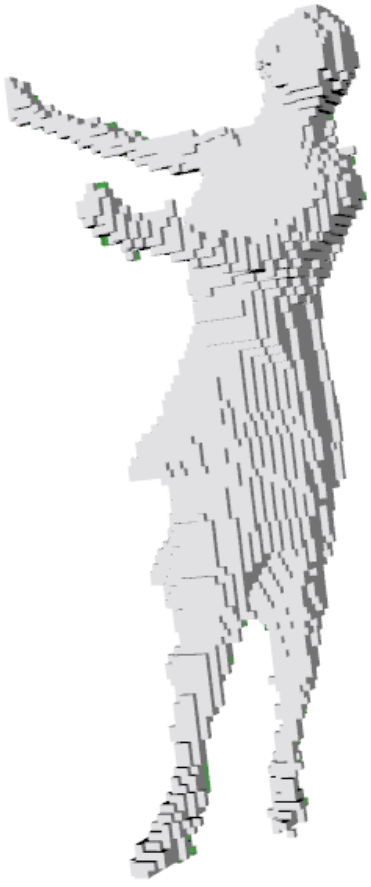


D6I1D

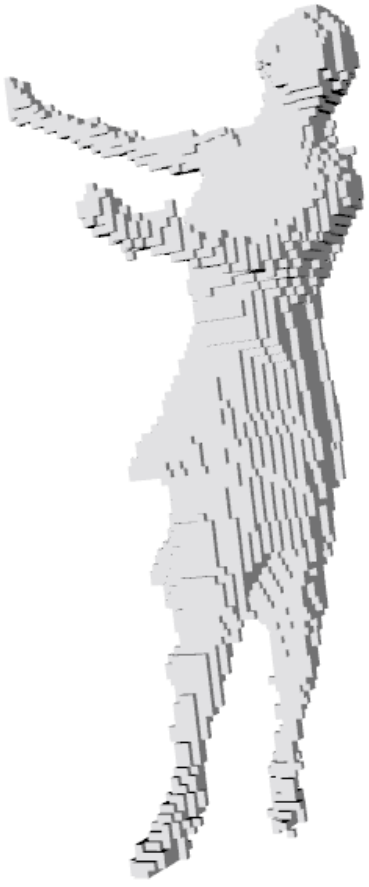


D6I1D

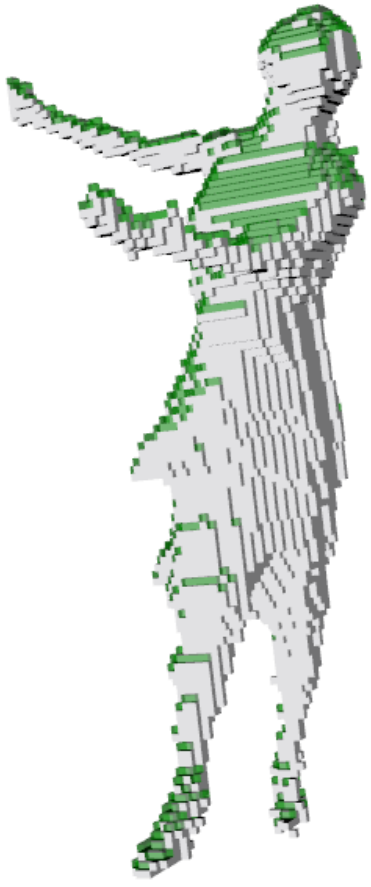
D6I1D



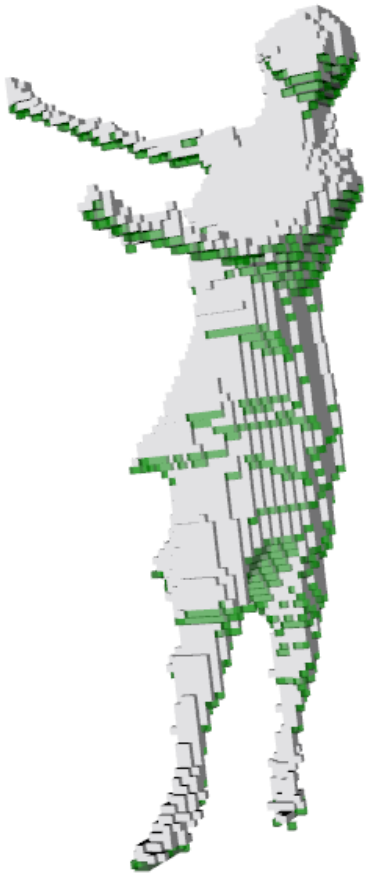
D6I1D



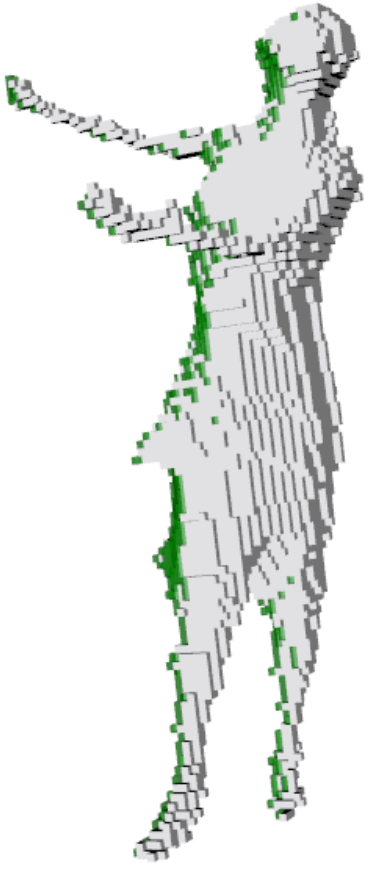
D6I1D



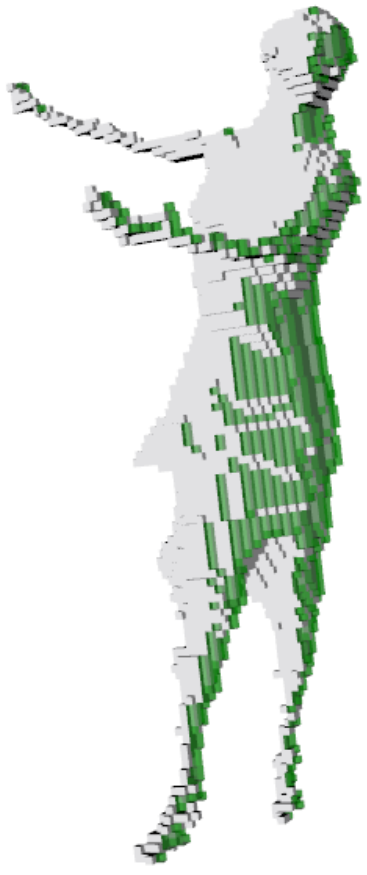
D6I1D



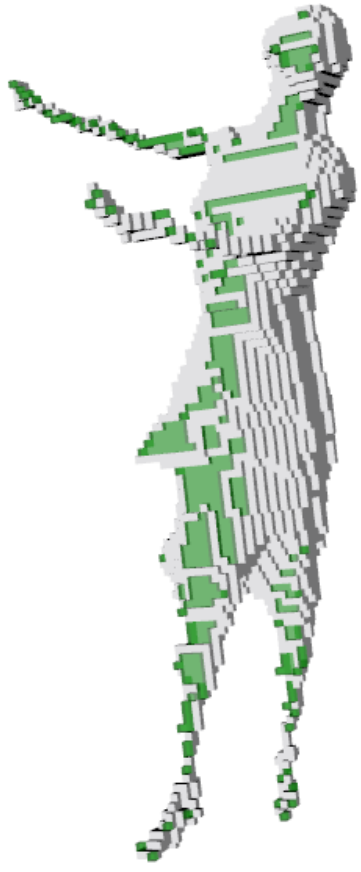
D6I1D



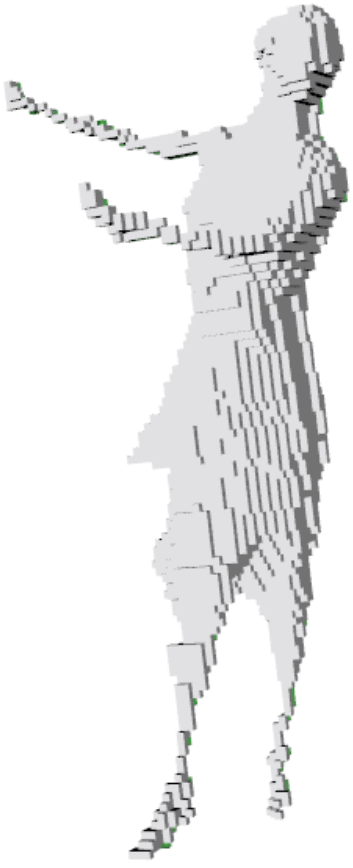
D6I1D



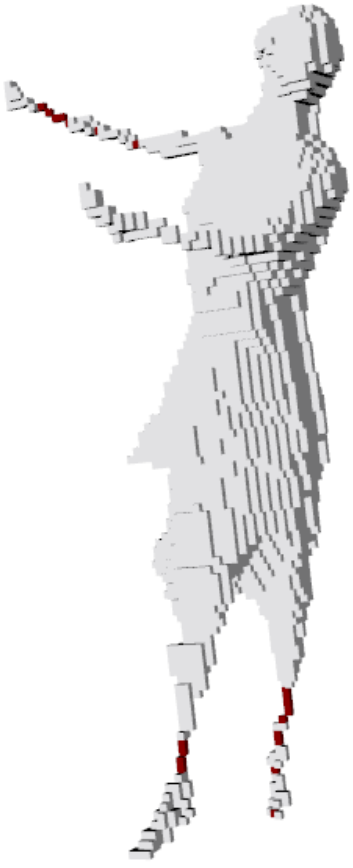
D6I1D



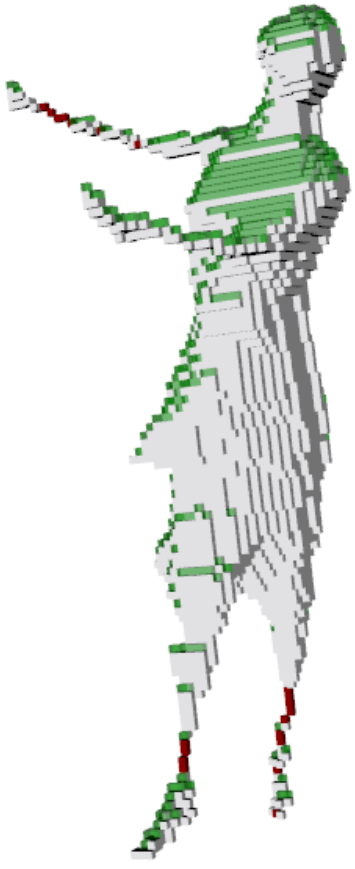
D6I1D



D6I1D



D6I1D



Variations

By modifying the constraints of D6I1D, we can obtain other kinds of skeletons:

Variations

By modifying the constraints of D6I1D, we can obtain other kinds of skeletons:

- no constraints $\rightarrow$ ultimate skeleton

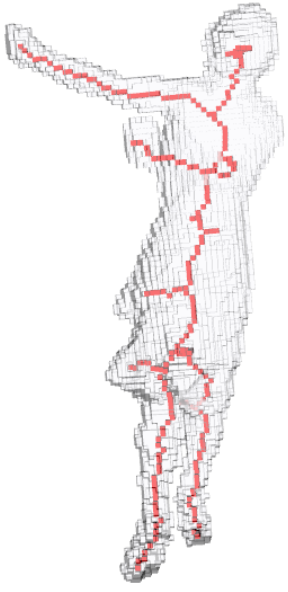
Variations

By modifying the constraints of D6I1D, we can obtain other kinds of skeletons:

- no constraints $\rightarrow$ ultimate skeleton
- 2D-isthmuses (D6I2D) $\rightarrow$ surface skeleton

Results

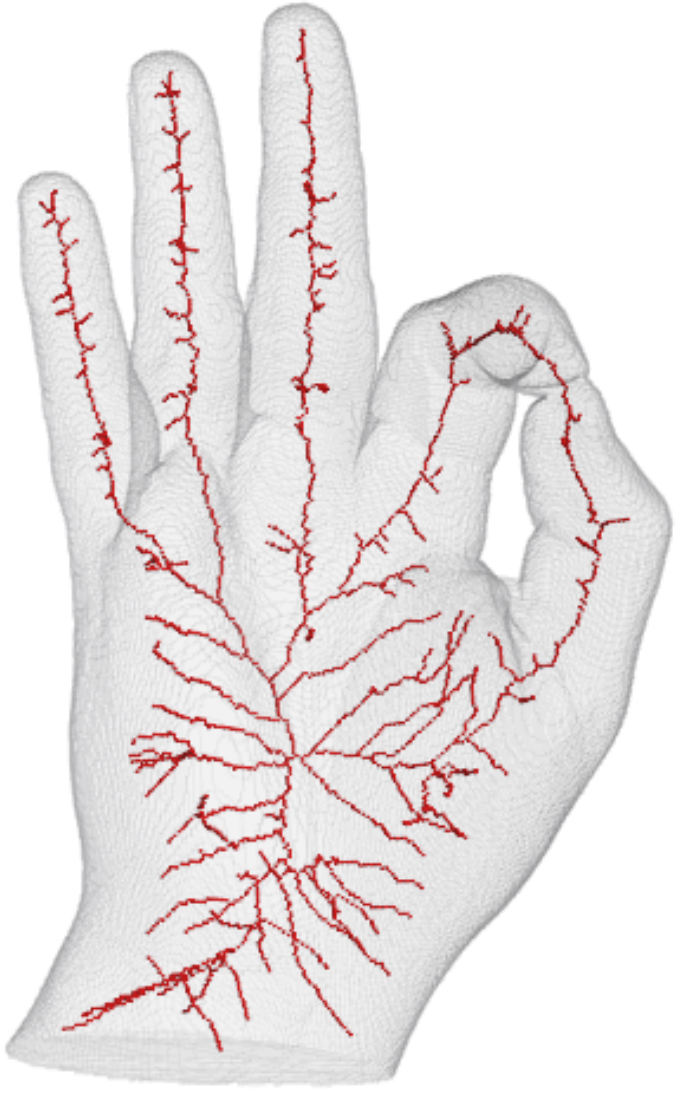
Curvilinear Skeleton with D6I1D



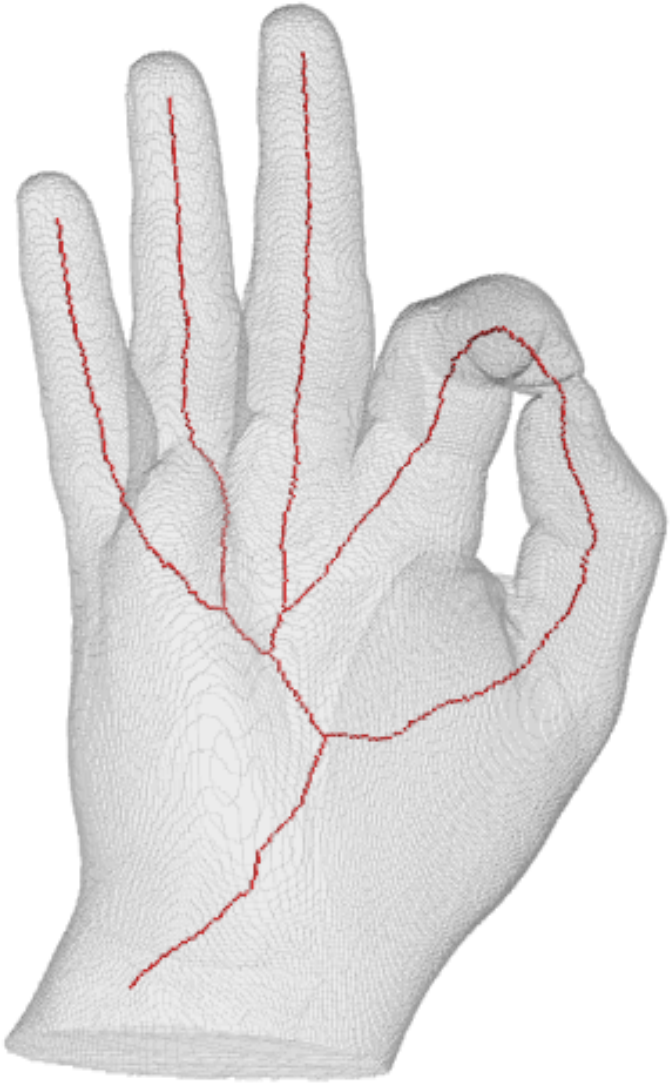
PKD6

D6I1D

Curvilinear Skeleton with D6I1D

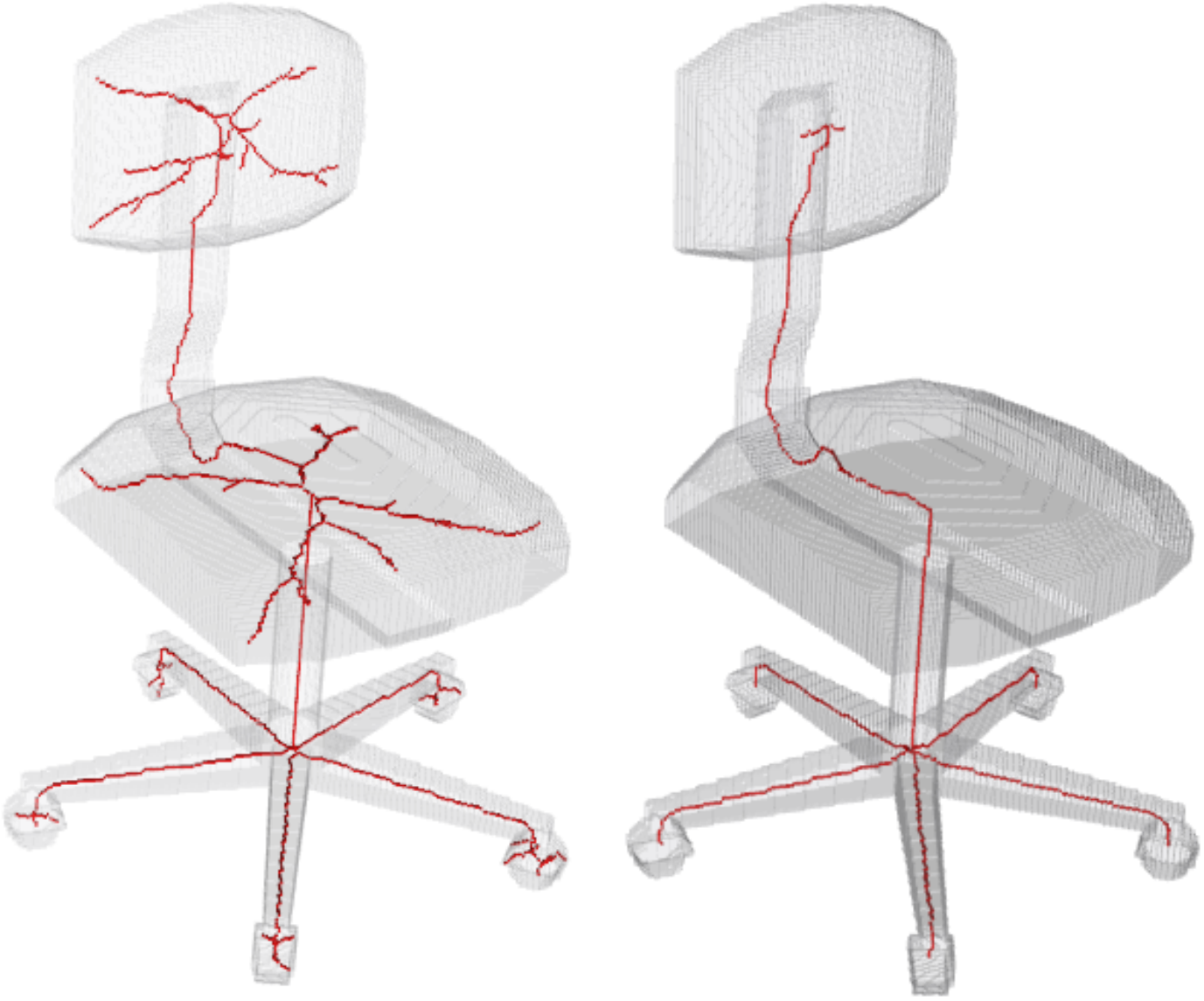


PKD6



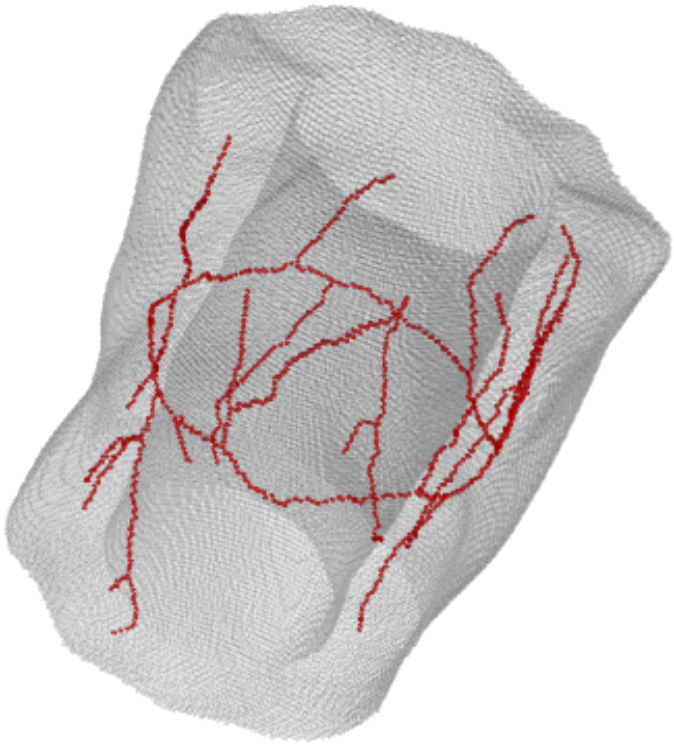
D6I1D

Curvilinear Skeleton with D6I1D

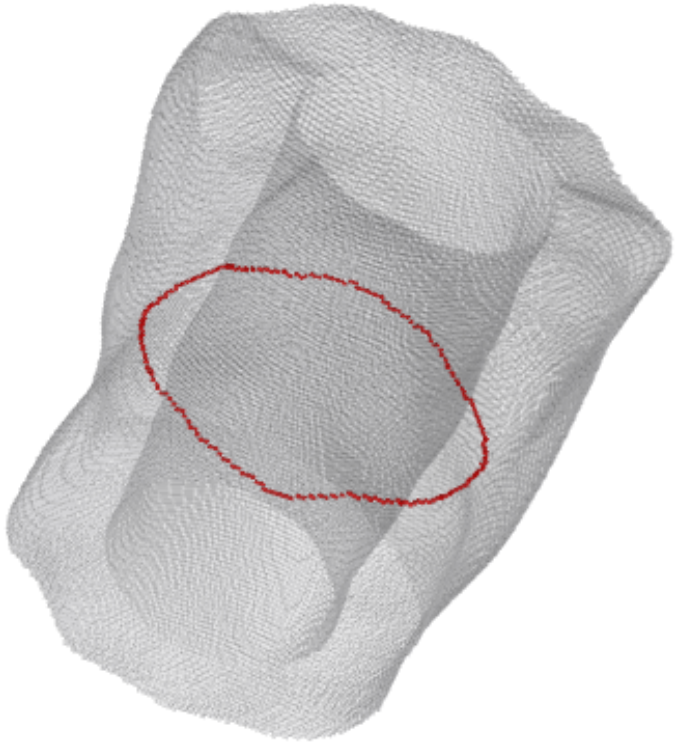


PKD6 D6I1D

Curvilinear Skeleton with D6I1D



PKD6



D6I1D

Surface Skeleton with D6I2D

We compare the results of D6I2D with those obtained from two other thinnings:

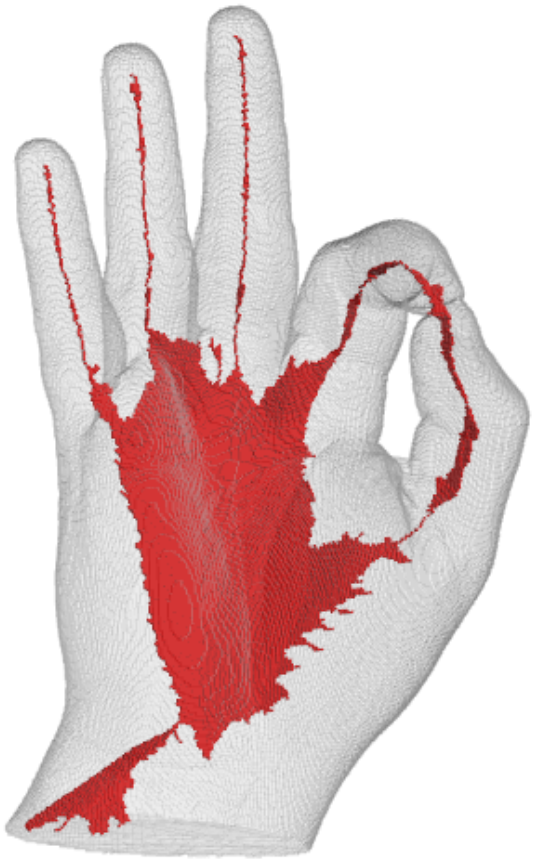
Pal02 described by Palagyi in:
"A 3-subiteration 3D thinning algorithm for extracting medial surfaces"

Pal08 described by Palagyi in:
"A 3D fully parallel surface-thinning algorithm"

Surface Skeleton with D6I2D

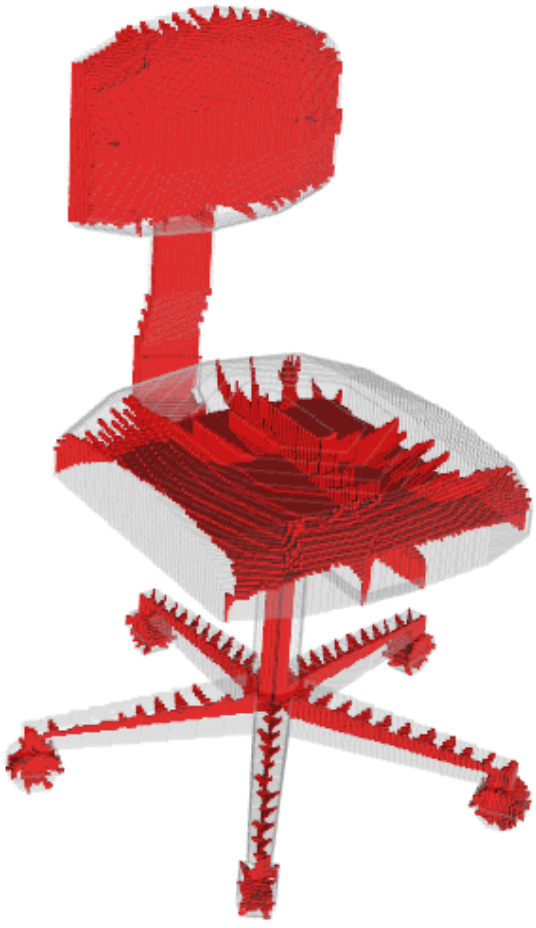
Pal02

Pal08



D6I2D

Surface Skeleton with D6I2D

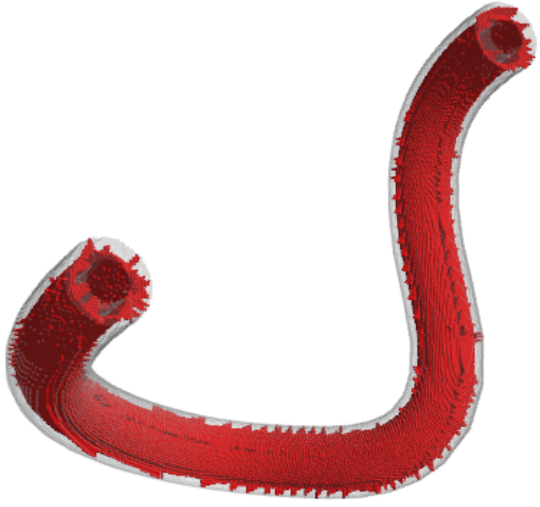


PaI02

PaI08

D6I2D

Surface Skeleton with D6I2D

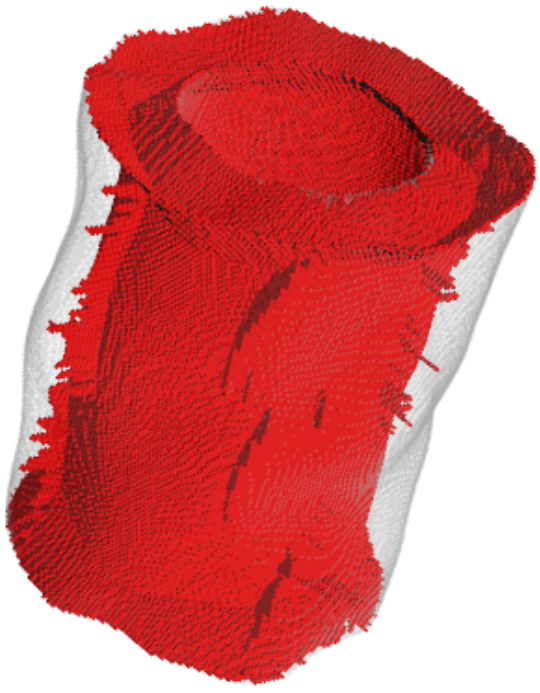


Pal02

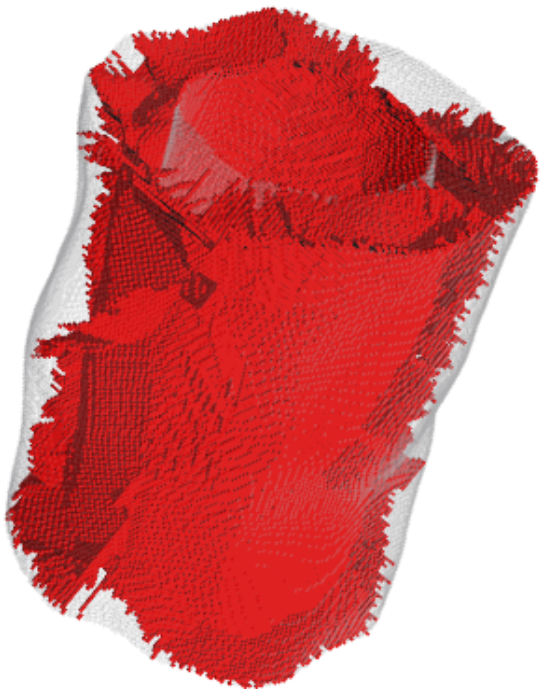
Pal08

D6I2D

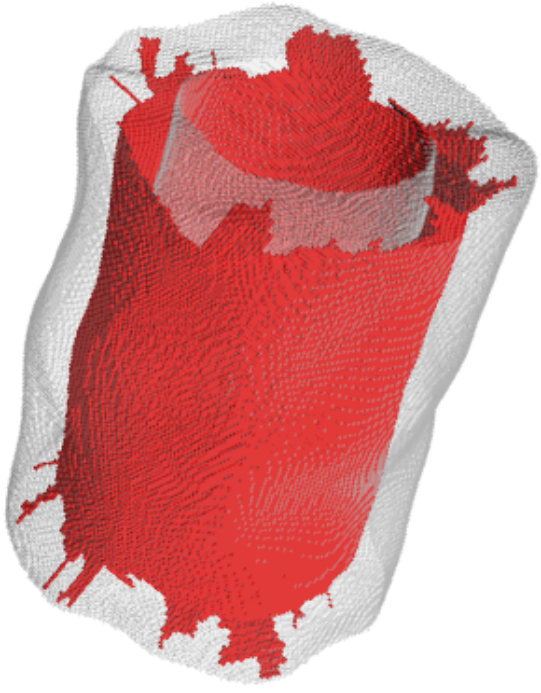
Surface Skeleton with D6I2D



Pal02

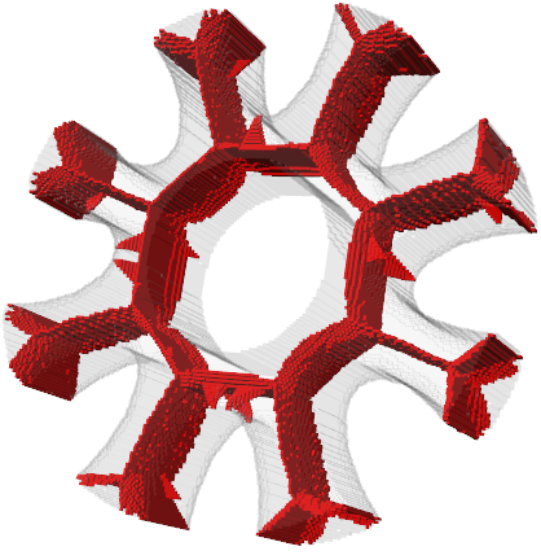


Pal08

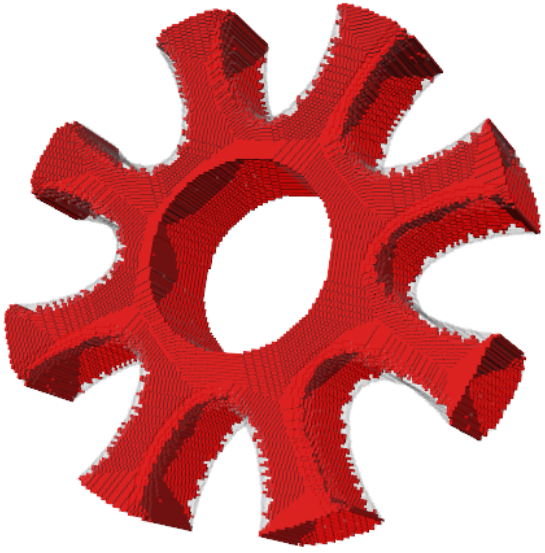


D6I2D

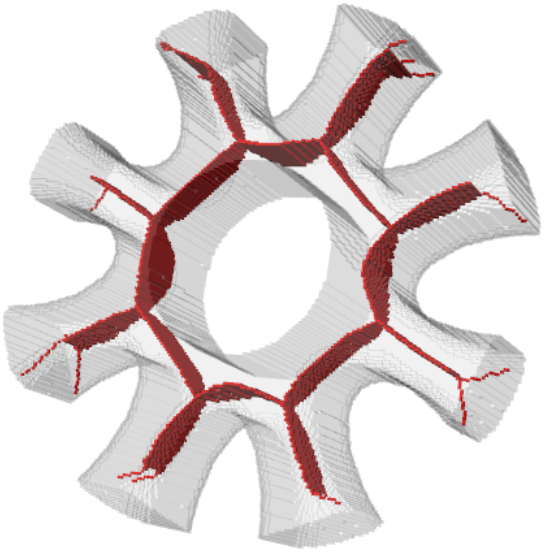
Surface Skeleton with D6I2D



Pal02



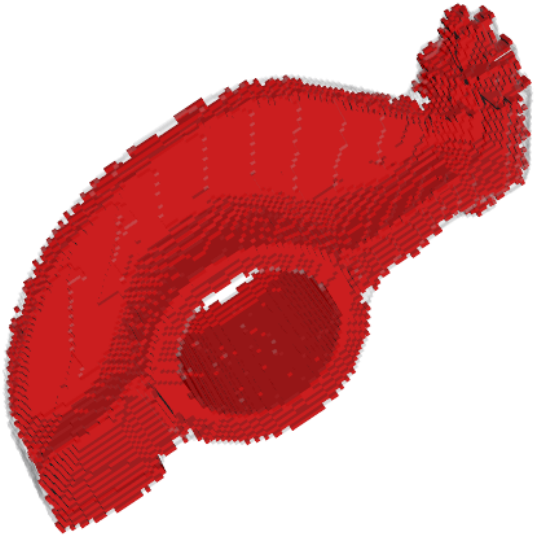
Pal08



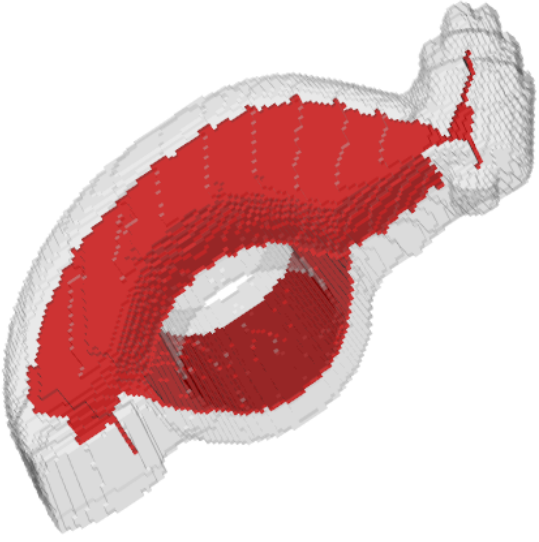
D6I2D

Surface Skeleton with D6I2D

Pal02

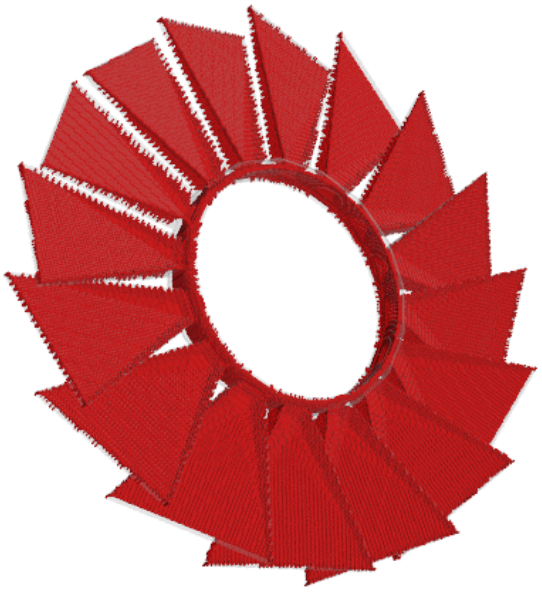


Pal08

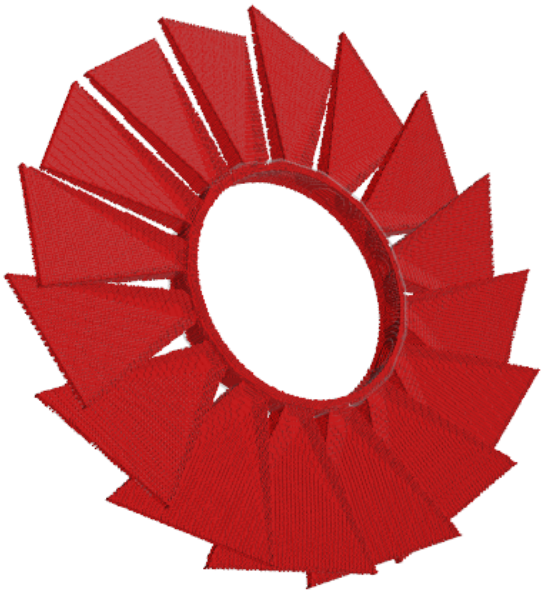


D6I2D

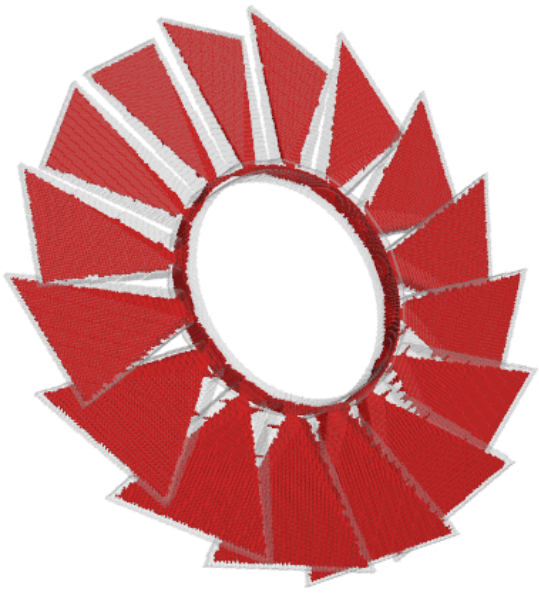
Surface Skeleton with D6I2D



PaI02



PaI08



D6I2D

Conclusion

Achieved work

We have proposed a new thinning scheme based on isthmuses:

Achieved work

We have proposed a new thinning scheme based on isthmuses:

- fast

Achieved work

We have proposed a new thinning scheme based on isthmuses:

- fast
- providing different kind of skeletons

Achieved work

We have proposed a new thinning scheme based on isthmuses:

- fast
- providing different kind of skeletons
- with good visual aspect preservation

Future work

Now we are working on:

Future work

Now we are working on:

- speed optimizations

Future work

Now we are working on:

- speed optimizations
- design of new constraints

Future work

Now we are working on:

- speed optimizations
- design of new constraints
- finding a postdoc position :)

Questions?