Well-composed cell complexes

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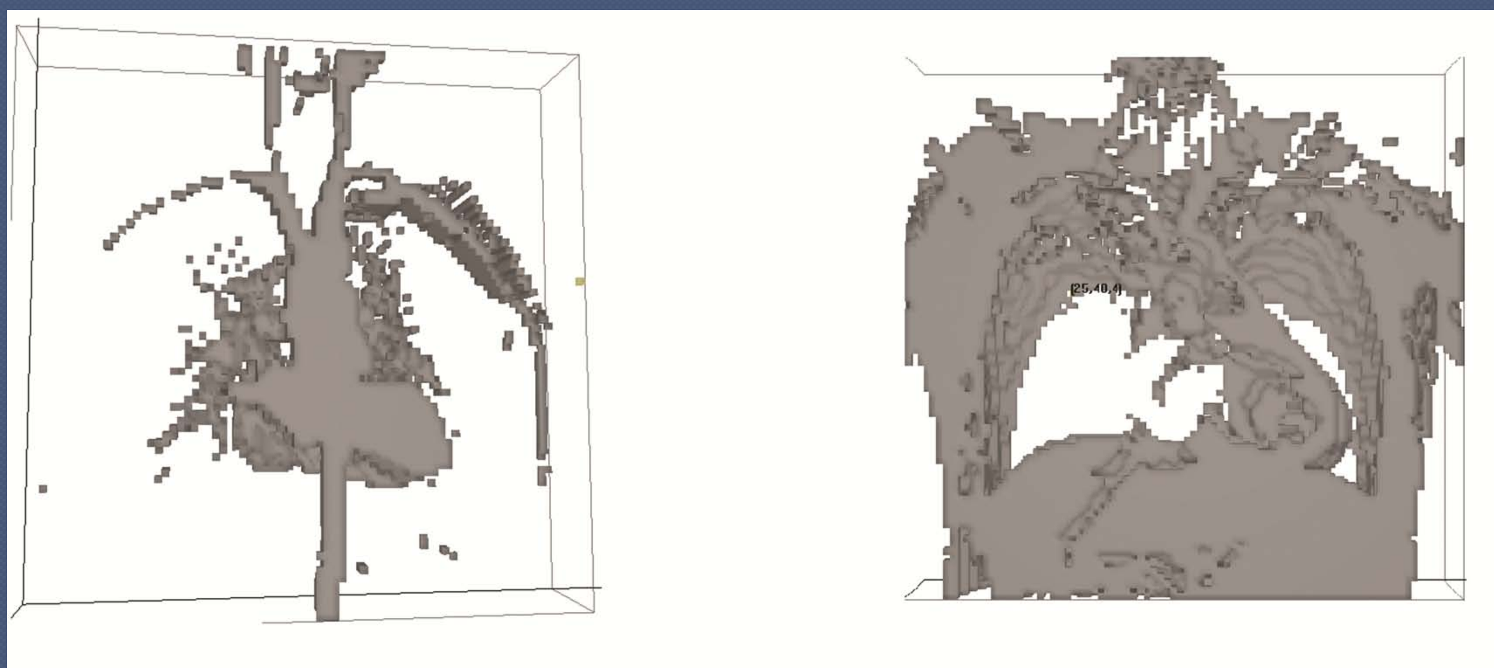
Motivation

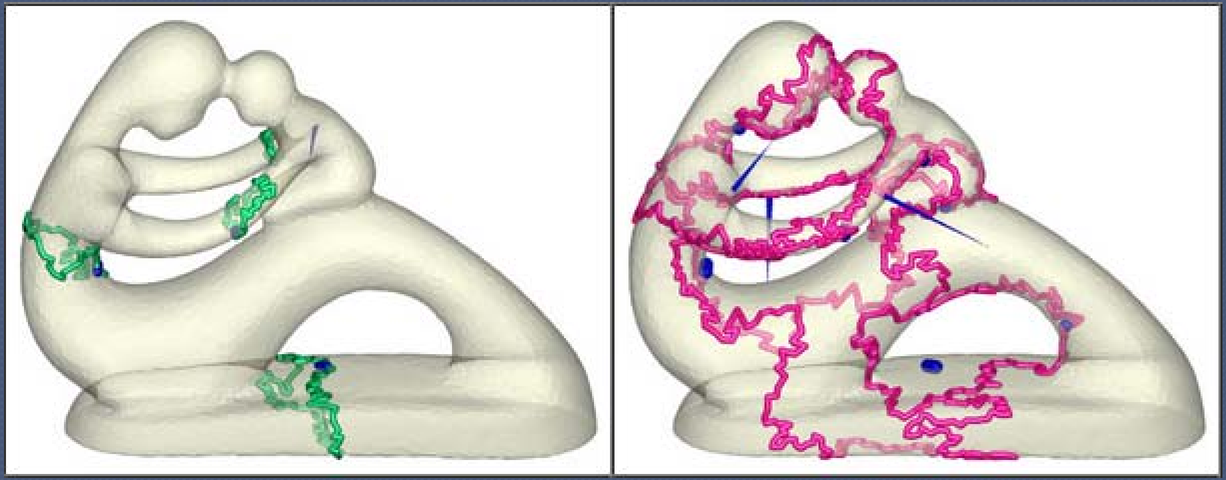
- Our ultimate purpose:

  To extract (co)homological information of a 3D model.



Motivation

For this aim, first we need to compute representative (co)cycles of (co)homology generators of dimension 1 in the model.

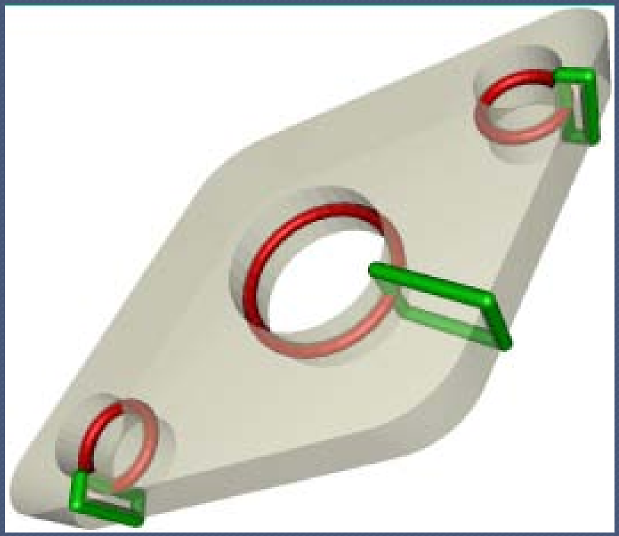


Extracted from Dey et al., Computing Geometry-aware Handle and Tunnel Loops in 3D Models. ACM Transactions on Graphics, Vol. 27, No. 3, Article 45, (2008).

Introduction

- ***T. Dey et al*:**

Computing loops on the surface that wraps around their 'handles' and 'tunnels'.



Applications:

- Feature detection
- Topological simplification
- …

Dey T. K., Li K., Sun J.: On computing handle and tunnel loops. IEEE Proceedings of the international conference on cyberworlds; 2007.p.357-66.

Introduction

- All the computations are carried out over a connected closed surface in R^3.

- Can we start from a similar scenario if we consider the cubical complex associated to a 3D digital picture?

⇓

Our answer: Well-composed cell complexes

Introduction

- Well-composed images enjoy important topological and geometric properties:

1. There is only one type of connected component.

2. Some algorithms used in computer vision, computer graphics and image processing are simpler.

Introduction

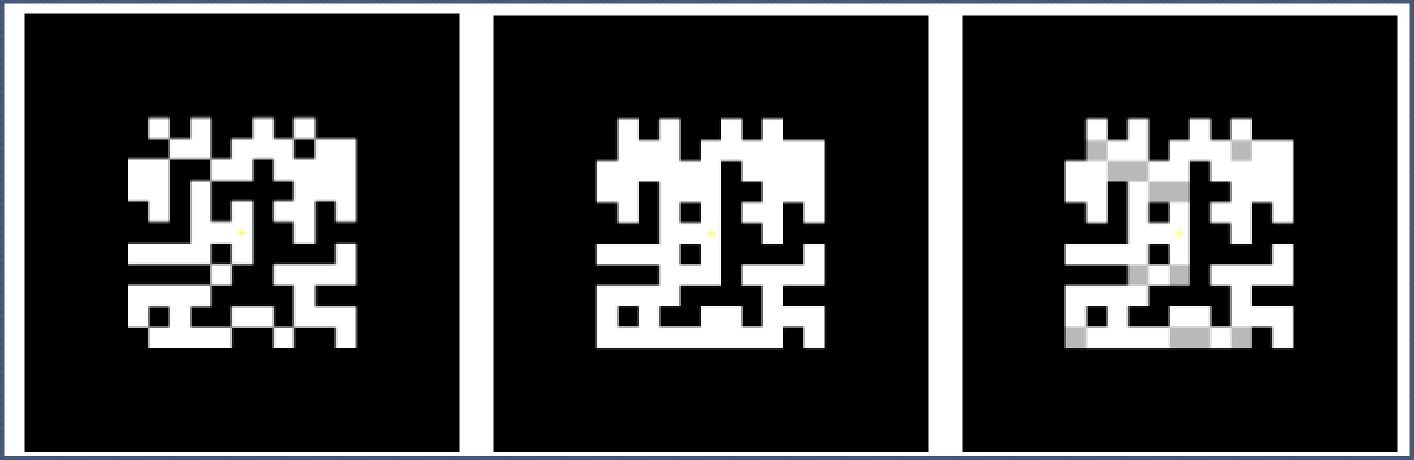
- Well-composed images enjoy important topological and geometric properties:

3. Thinning algorithms can be simplified and naturally made parallel if the input image is well-composed.

4. Some algorithms for computing surface curvature or extracting adaptive triangulated surfaces assume that the input image is well-composed.

Introduction

❑ There are several methods for turning binary digital images that are not well-composed into well-composed ones:



Original image Well-composed Differences in gray

…but these methods "destroy the topology".

Introduction

- Our goal in this paper:

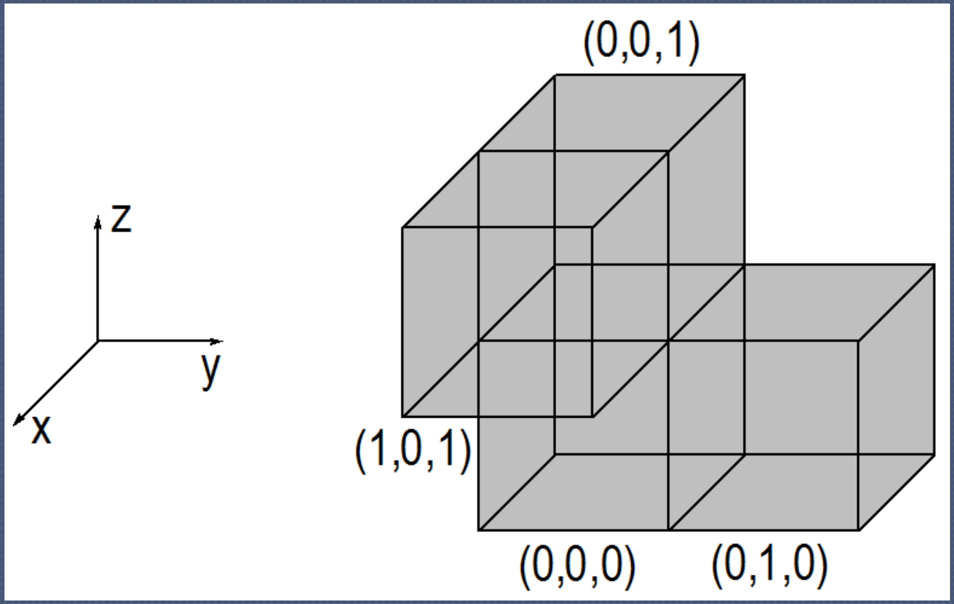
To “transform’’ the cubical complex induced by a 3D binary digital picture into a homotopy equivalent cell complex, whose boundary is made up by 2-manifolds:

WELL –COMPOSED CELL COMPLEX.

3D digital images:

- **I = (Z^3, B):** set of unit cubes (***voxels***) centered at the points of B together with all the faces.

- Example: I = (Z^3, B), B = {(0,0,0), (0,1,0), (0,0,1), (1,0,1)}



B = ***foreground***

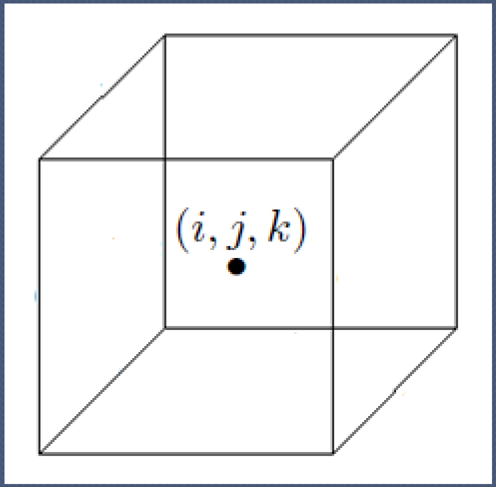
$B^c = Z^3 \setminus B$ = ***background***

Cubical complex:

{ Voxels } ⟷ { 3D cubes in R^3 }

⇓

Combinatorial struture: **CUBICAL COMPLEX**



Voxel

⟷

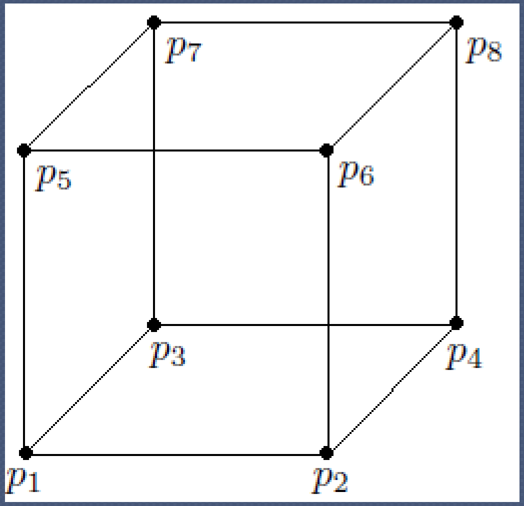
- 0-cells = vertices
- 1-cells = edges
- 2-cells = squared faces
- 3-cells = cubes

Cubical complex:

{ Voxels } ⟷ { 3D cubes in R^3 }

⇩

Combinatorial struture: **CUBICAL COMPLEX**



0-cells notation:

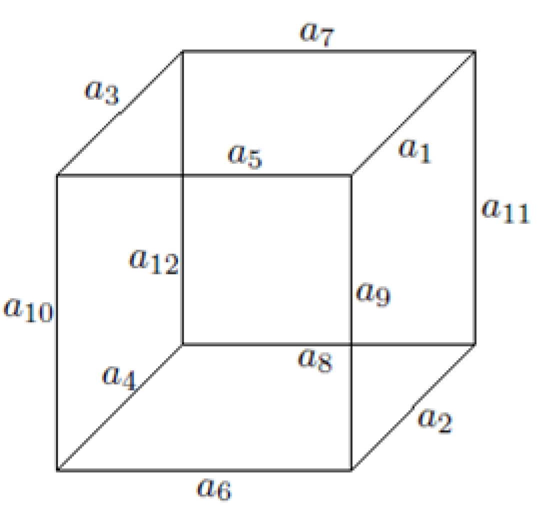
$$p_l = (i \pm \frac{1}{2}, j \pm \frac{1}{2}, k \pm \frac{1}{2})$$

Cubical complex:

{ Voxels } ⟷ { 3D cubes in R^3 }

⇓

Combinatorial struture: **CUBICAL COMPLEX**



1-cells notation:

$$a_l = (i, j \pm \frac{1}{2}, k \pm \frac{1}{2})$$

$$a_l = (i \pm \frac{1}{2}, j, k \pm \frac{1}{2})$$

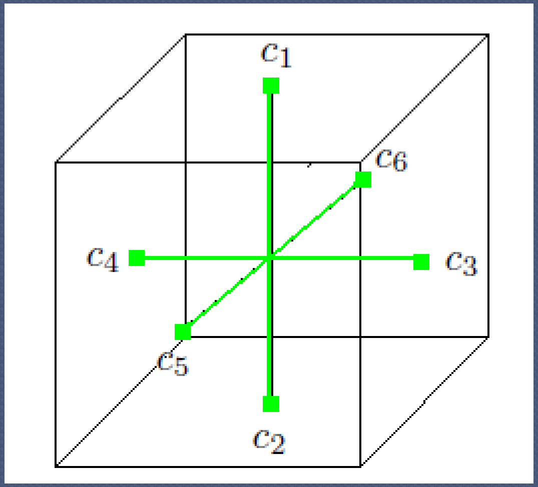
$$a_l = (i \pm \frac{1}{2}, j \pm \frac{1}{2}, k)$$

Cubical complex:

{ Voxels } ⟷ { 3D cubes in R^3 }

⇓

Combinatorial struture: **CUBICAL COMPLEX**



2-cells notation:

$$c_l = (i \pm \frac{1}{2}, j, k)$$

$$c_l = (i, j \pm \frac{1}{2}, k)$$

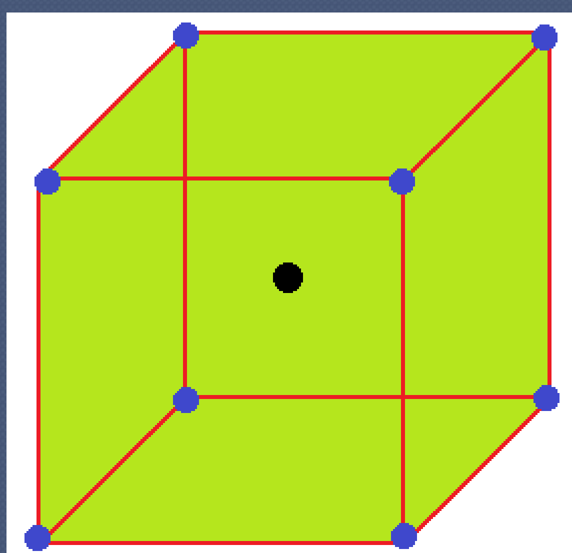
$$c_l = (i, j, k \pm \frac{1}{2})$$

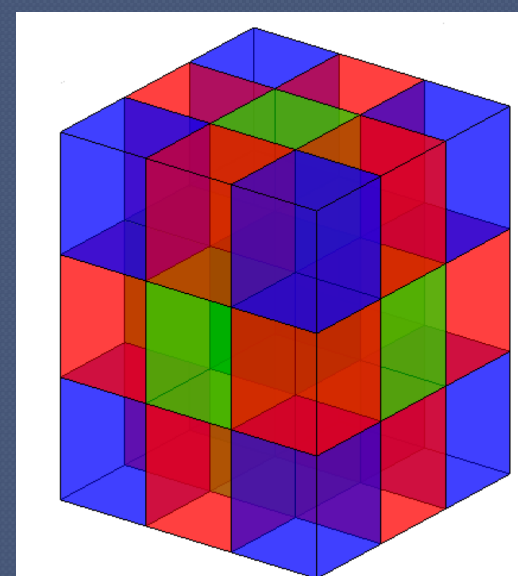
Cubical complex:

{ Voxels } ⟷ { 3D cubes in R^3 }

⇓

Combinatorial struture: **CUBICAL COMPLEX**

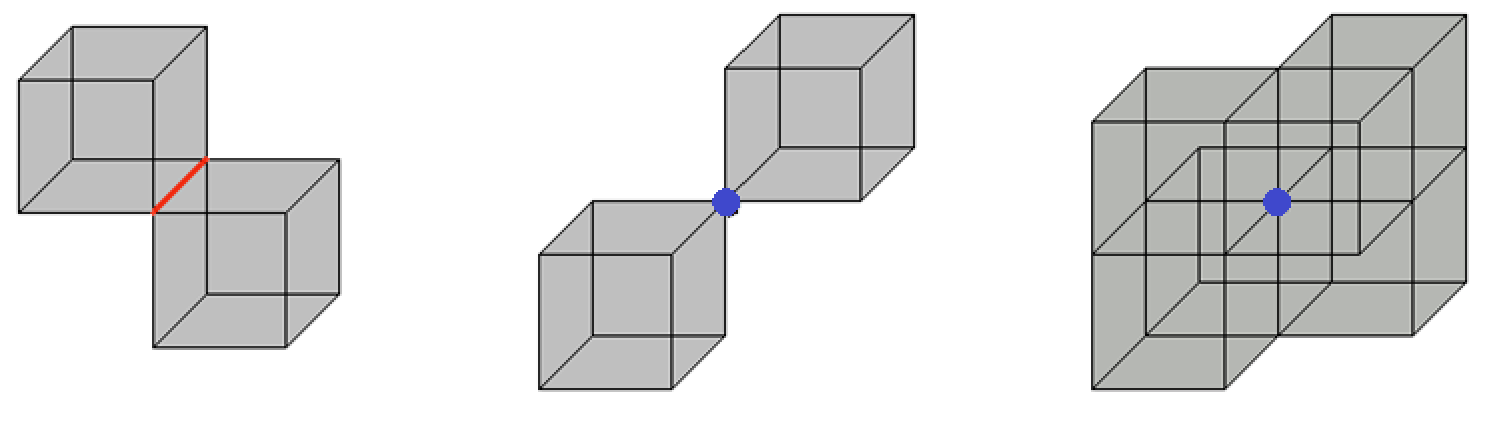




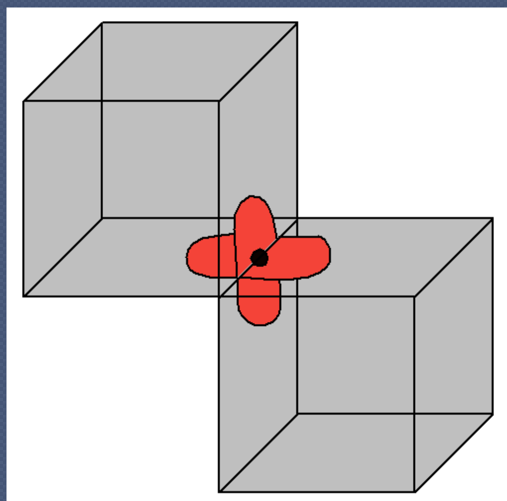
Well-composed images:

- I = (Z^3, B) is a **well-composed image** if the boundary of the cubical complex associated, ∂Q(I), is a 2D-manifold.

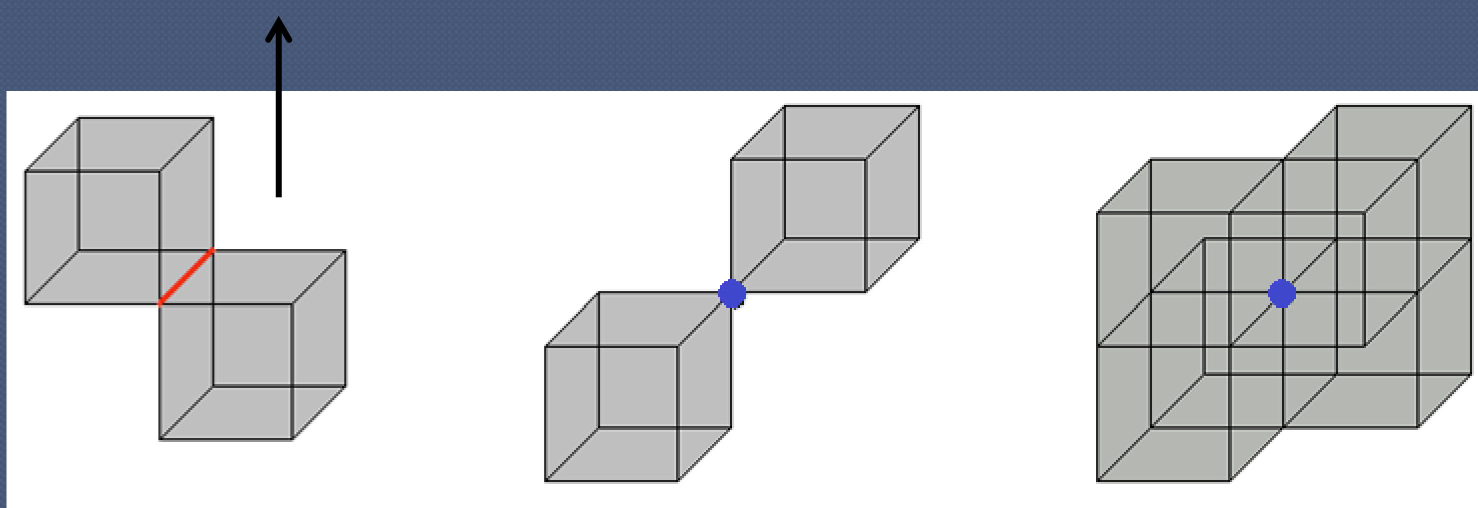
- [Latecki97] A 3D binary digital image is well-composed iff the configuractions C_1, C_2 and C_3 do not occur in Q(I).



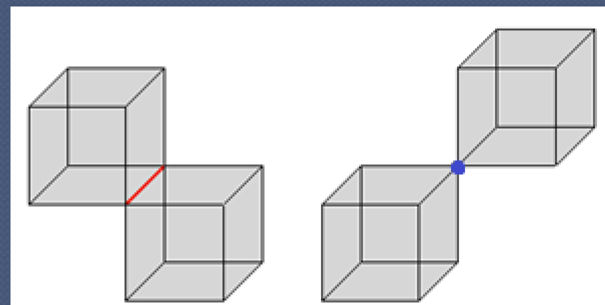
Well-composed images:

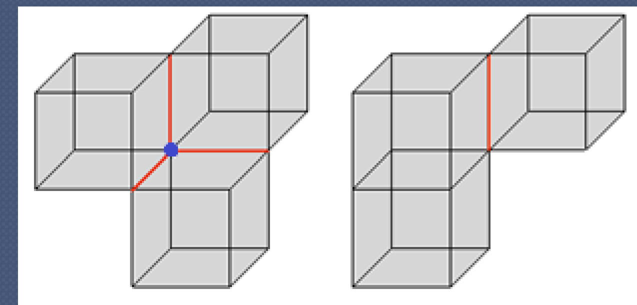


This point has not a neighborhood homeomorphic to R^2.



Well-composed images:



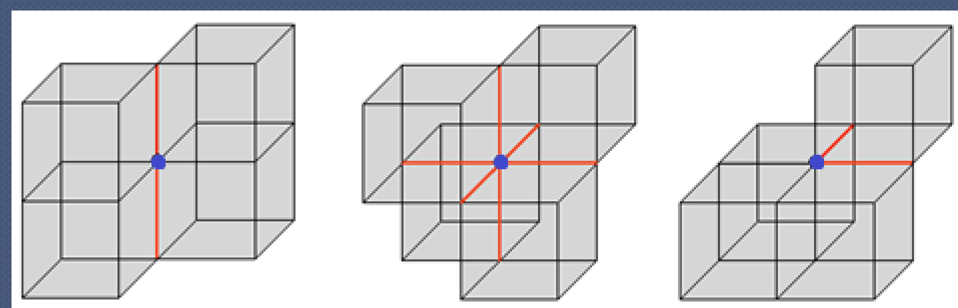


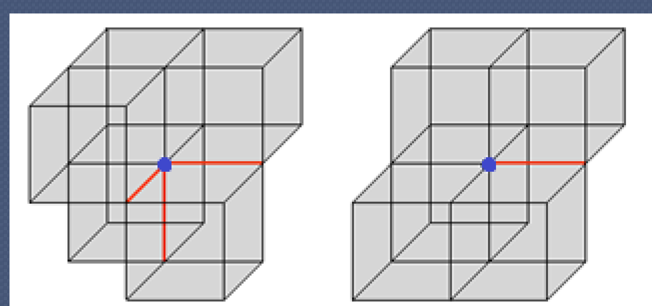
Critical

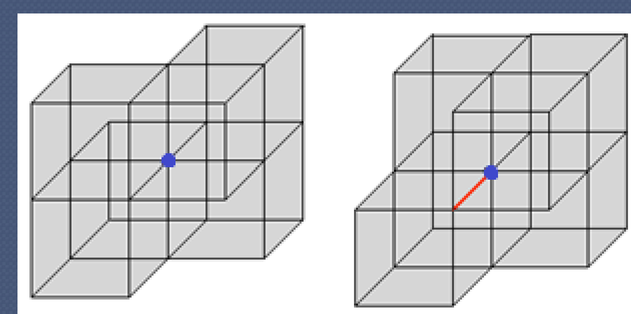
configurations

within a block of

eight cubes







From a cubical complex to a well-composed cell complex:

A 3D digital image is not generally a well-composed image

Cubical complex Q(I)

Homotopy equivalent

Cell complex K(I) such that ∂K(I) is composed by 2D-manifolds: a *well-composed cell complex*.

From a cubical complex to a well-composed cell complex:

Key idea: to create a true face adyacency to avoid the critical configurations

Cubical complex Q(I)

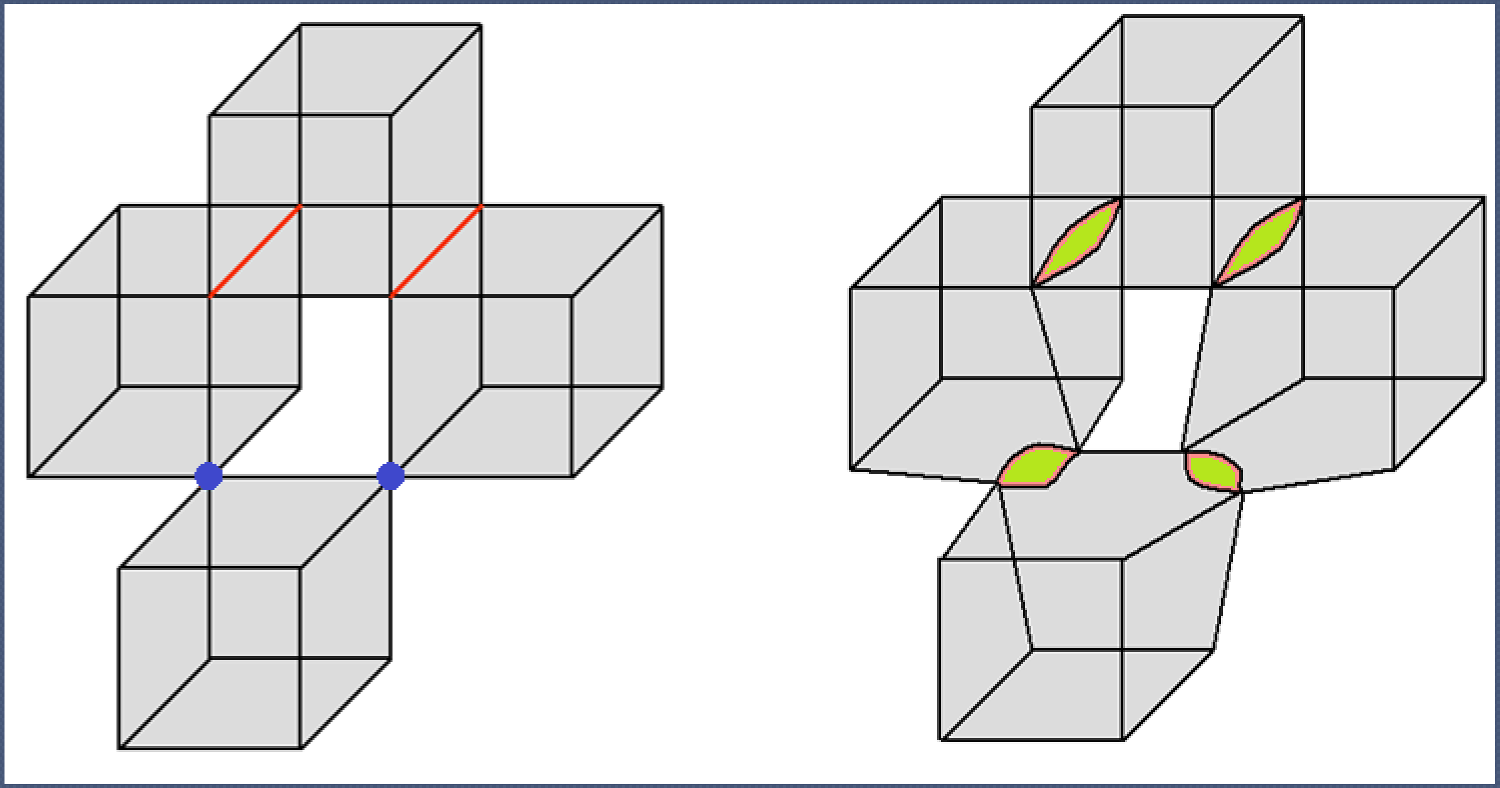
Homotopy equivalent

Cell complex K(I) such that ∂K(I) is composed by 2D-manifolds: a *well-composed cell complex*.

From a cubical complex to a well-composed cell complex:

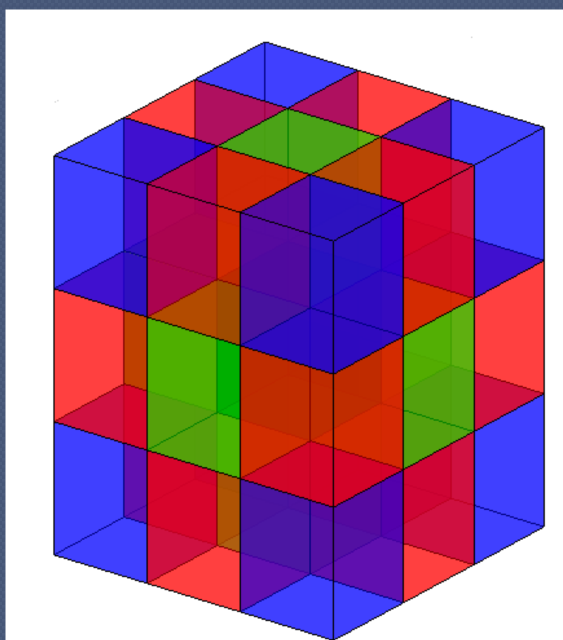
INPUT
Cubical complex Q(I)

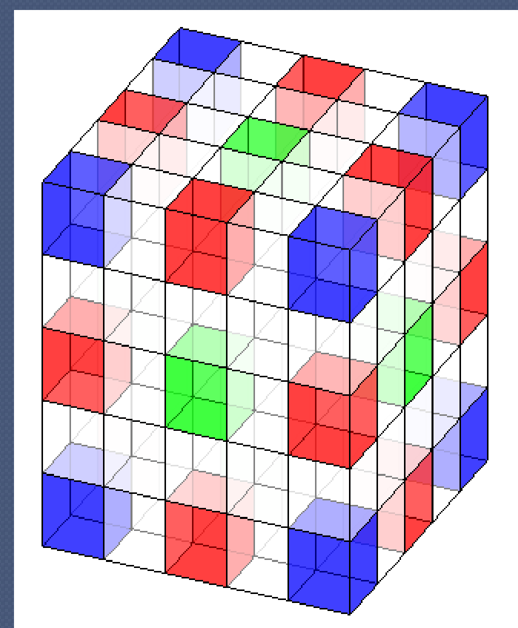
OUTPUT
Well-composed cell complex K(I)

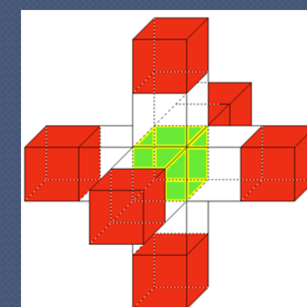


From a cubical complex to a well-composed cell complex:

So, we need "more space" to add new cells:







Adjacency = face relation between cells.

From a cubical complex to a well-composed cell complex:

Step 1: Label critical edges and critical vertices of Q(I).

Step 2: Repair critical edges of Q(I).

Step 3: Repair critical vertices of Q(I).

Cubical complex Q(I)

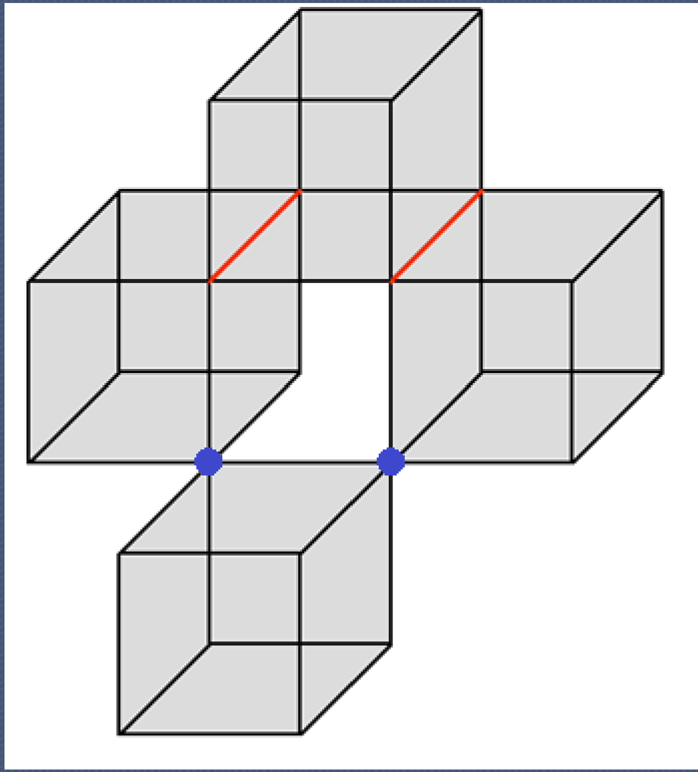
Homotopy equivalent

Cell complex K(I) such that ∂K(I) is composed by 2D-manifolds: a *well-composed cell complex*.

From a cubical complex to a well-composed cell complex:

Step 1: Label critical edges and critical vertices of Q(I).

A := set of critical edges of Q(I)
V^i := set of critical vertices of Q(I)



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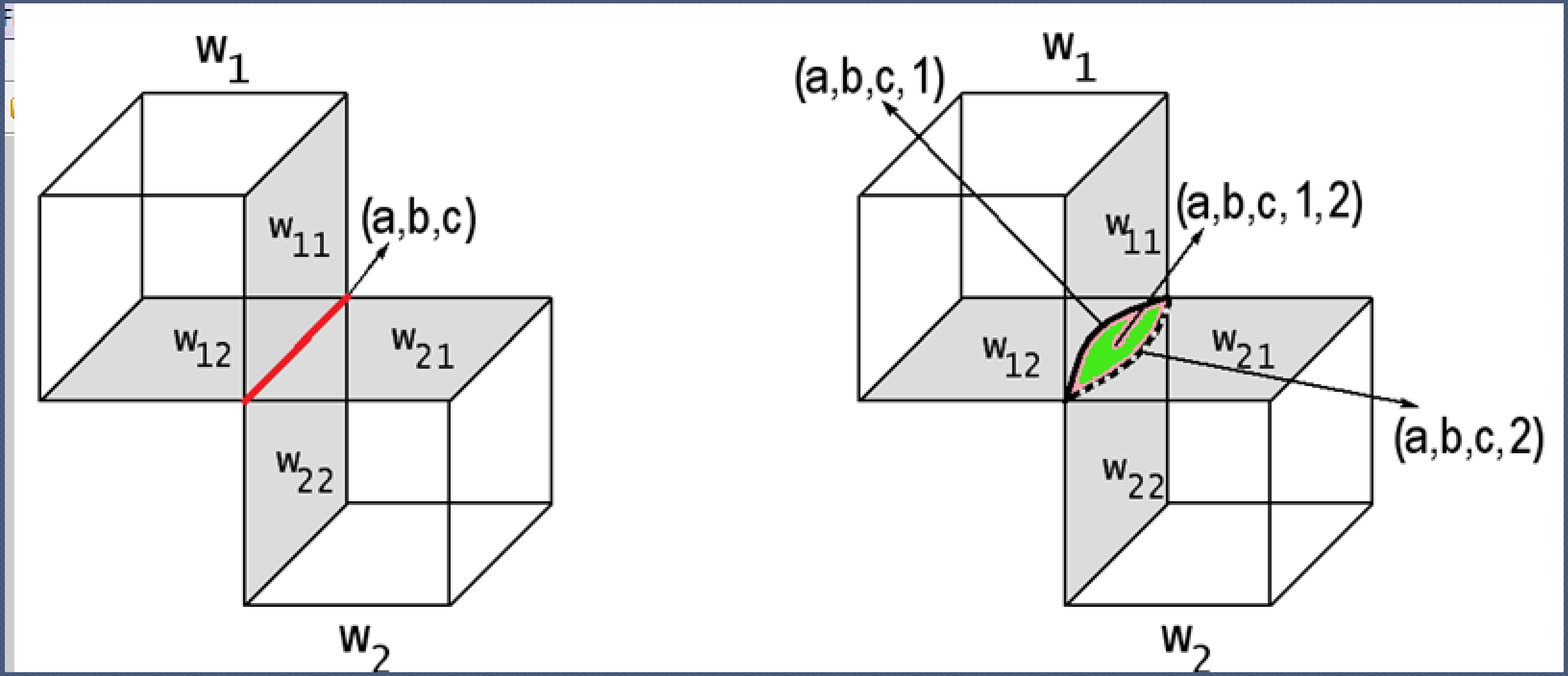
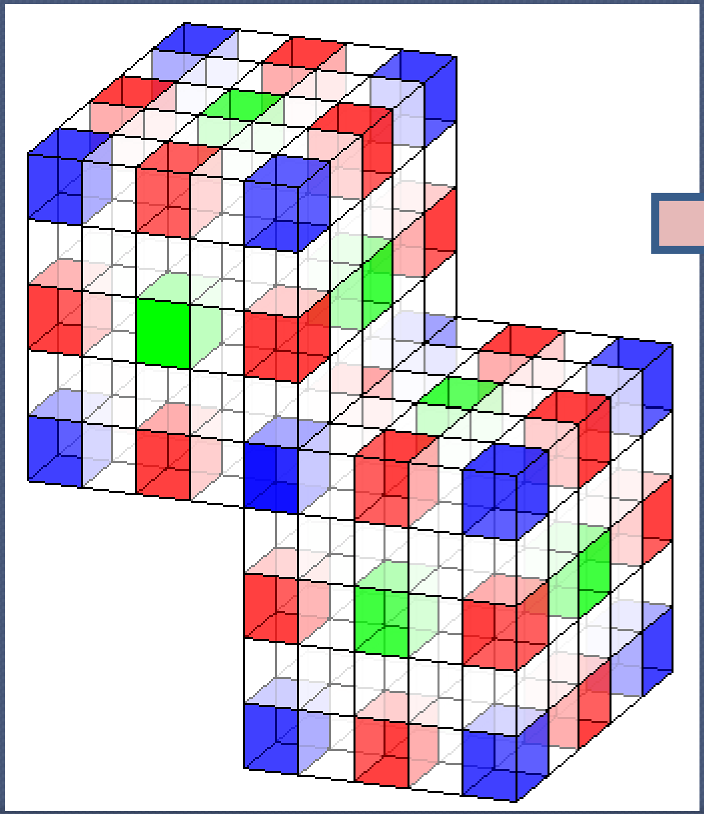
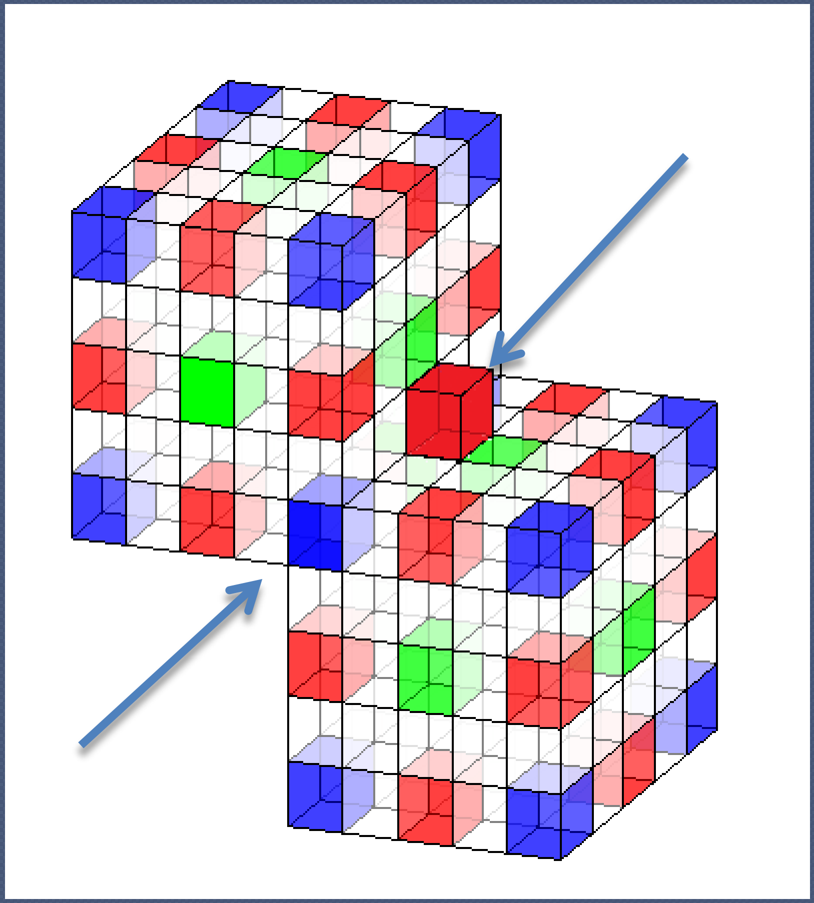


Computing well-composed cell complexes

Step 1: Label critical edges and vertices.

Step 2: Repair critical edges of Q(I).

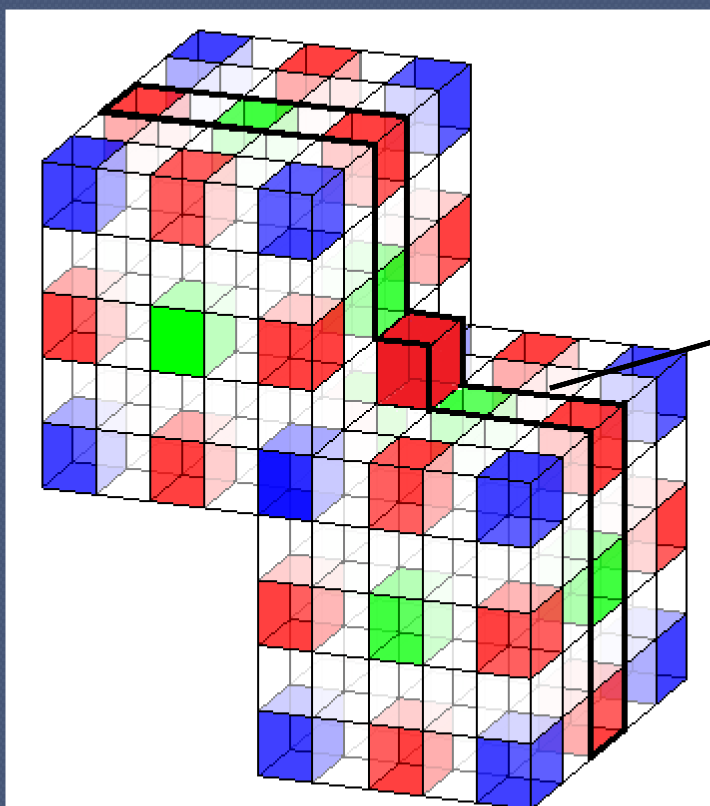


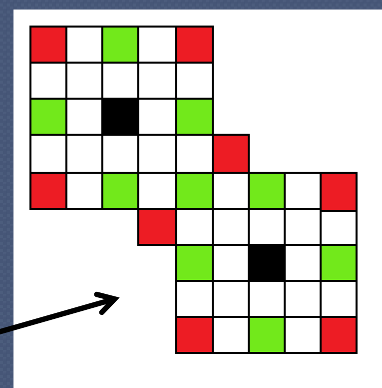


Computing well-composed cell complexes efficiently

Step 1: Label critical edges and vertices.

Step 2: Repair critical edges of Q(I).





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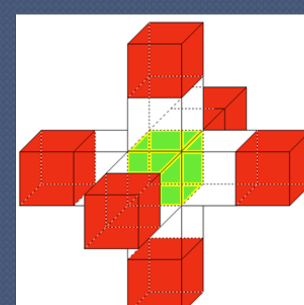

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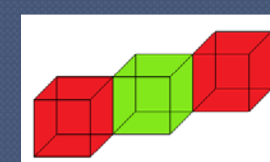

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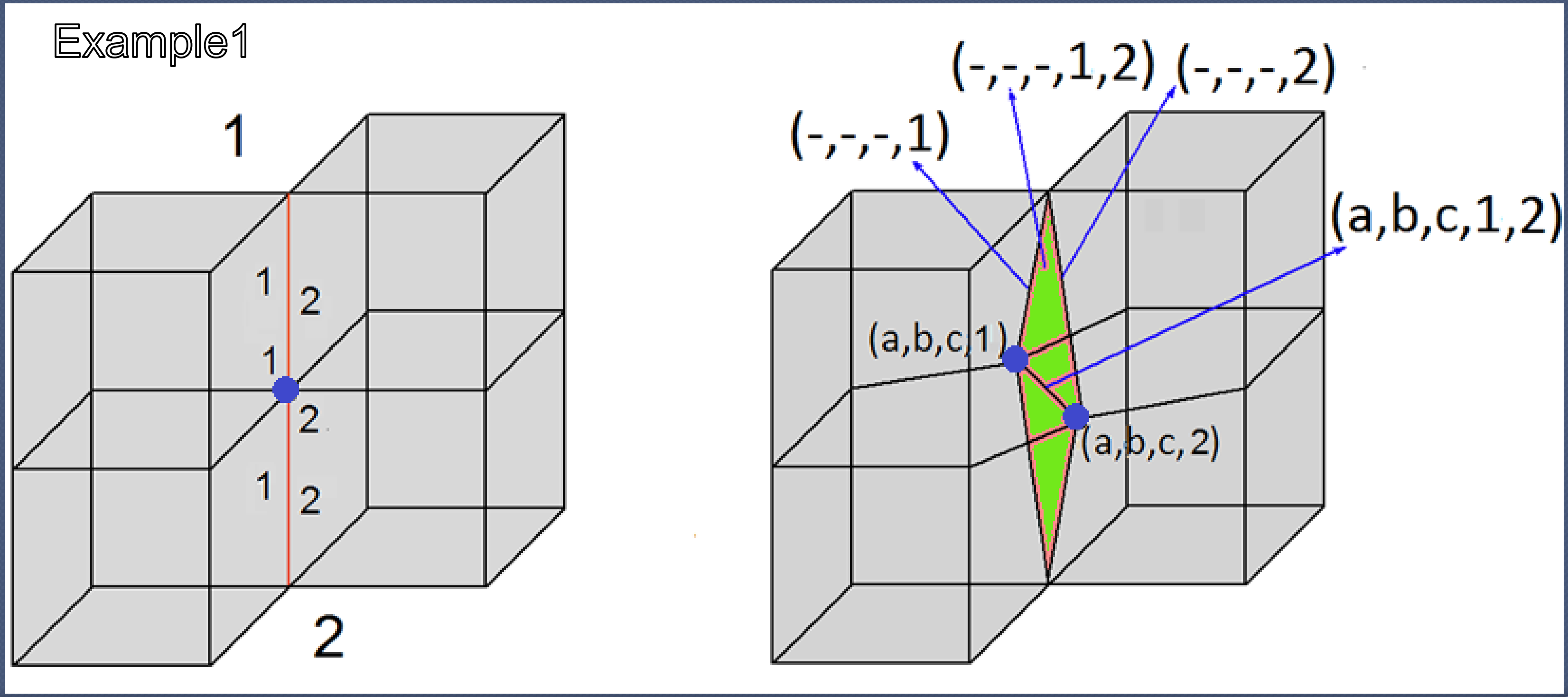

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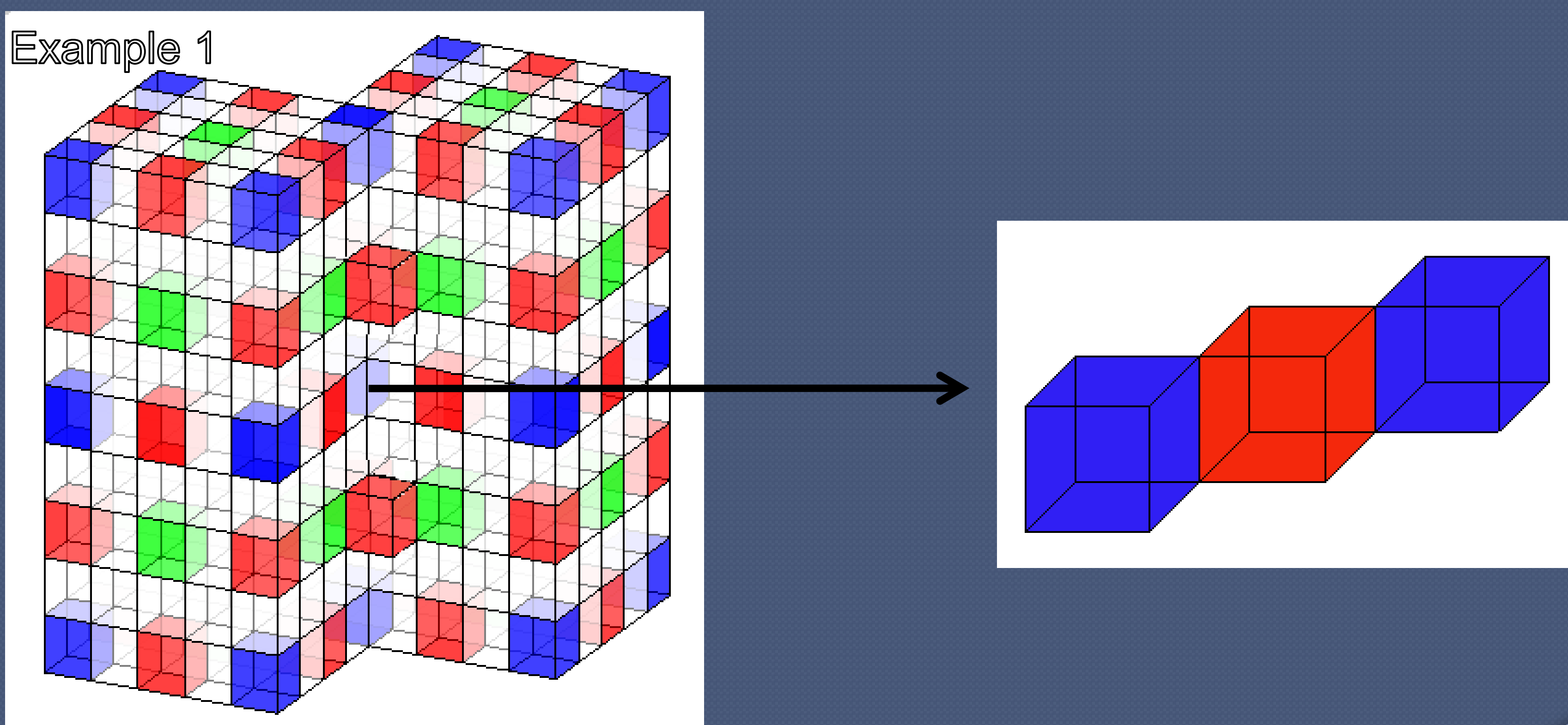

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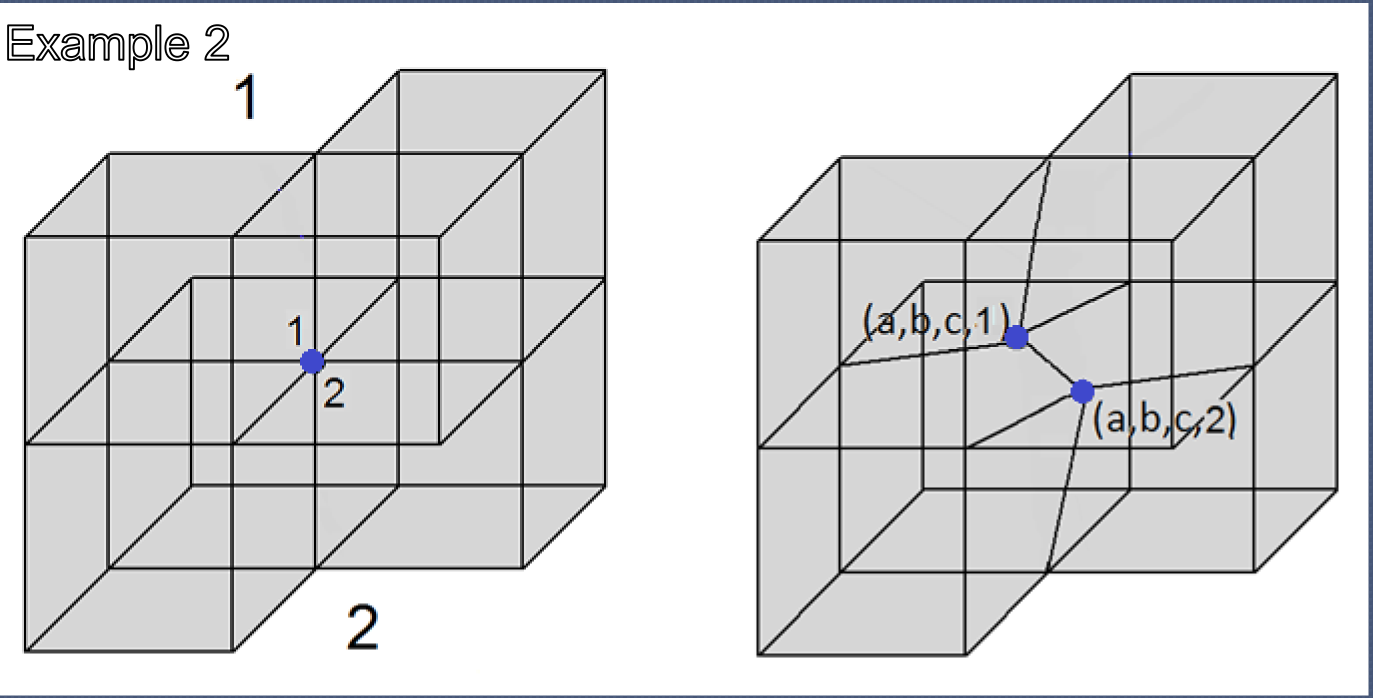

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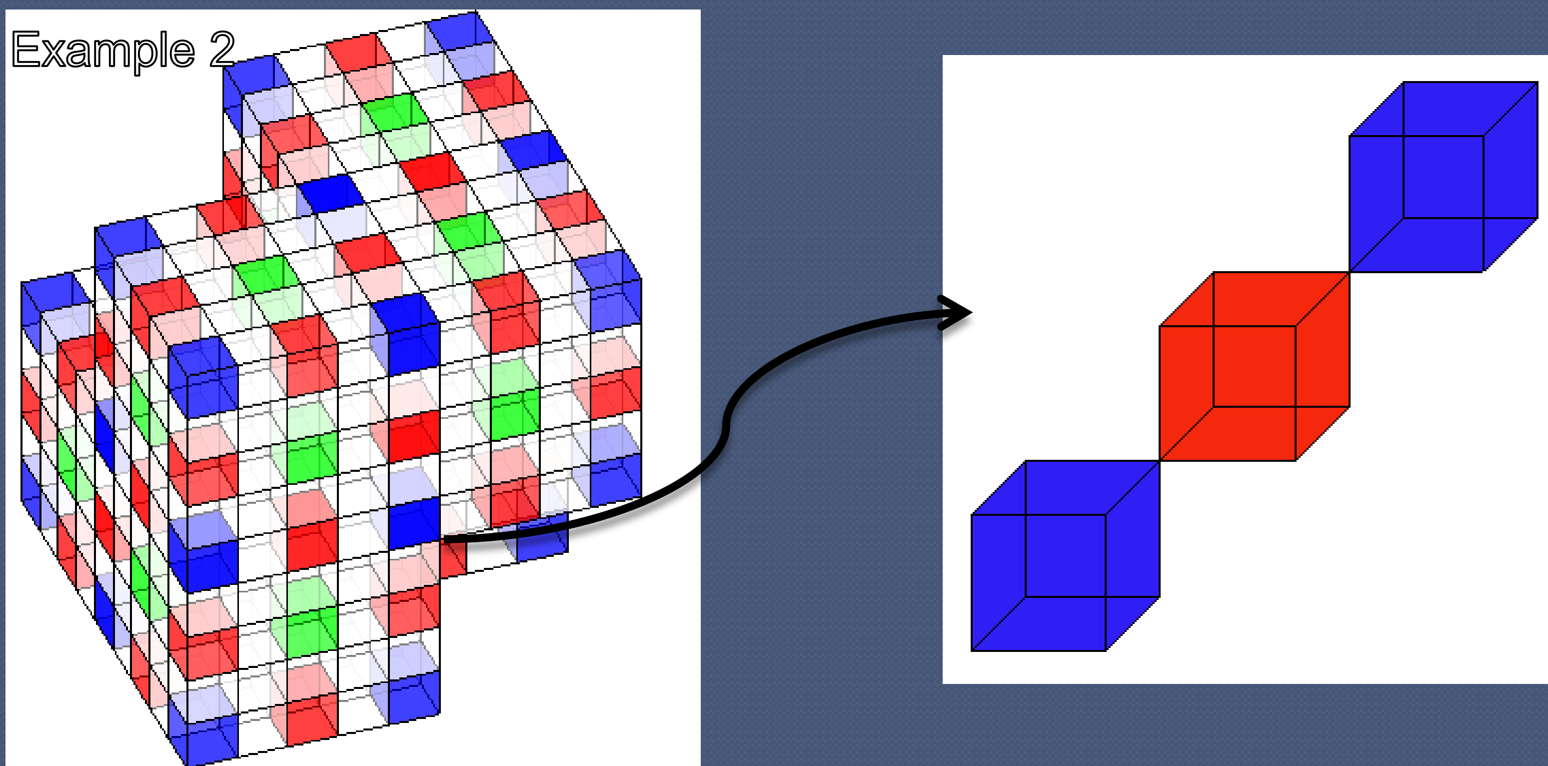

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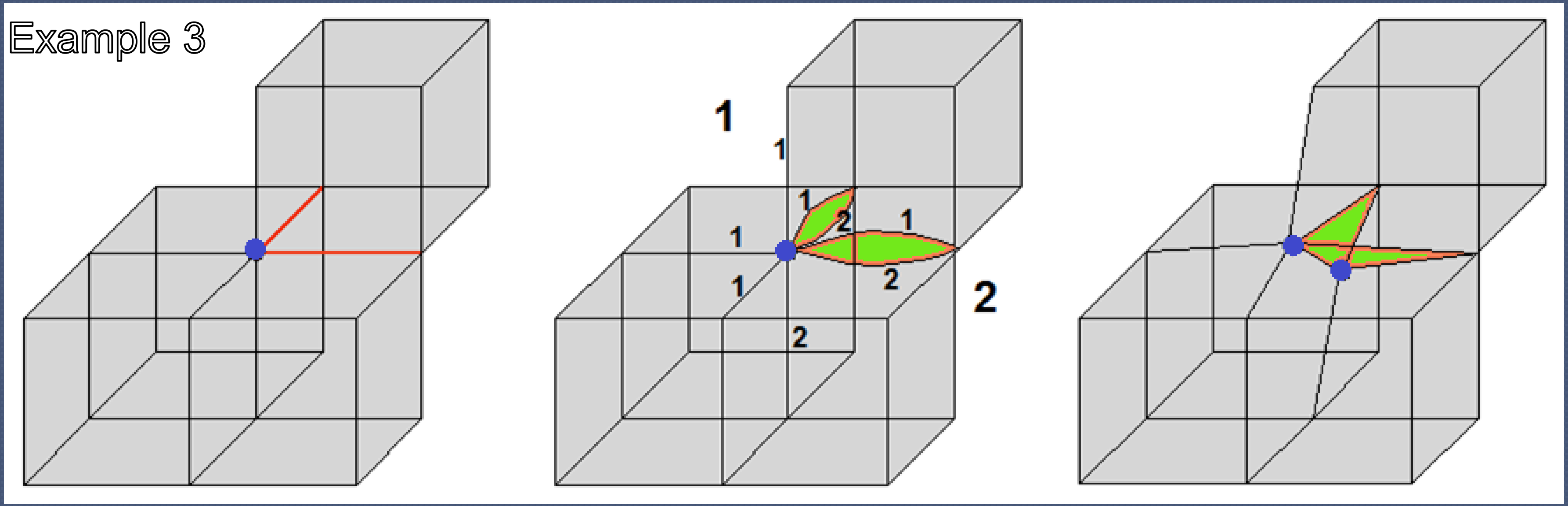

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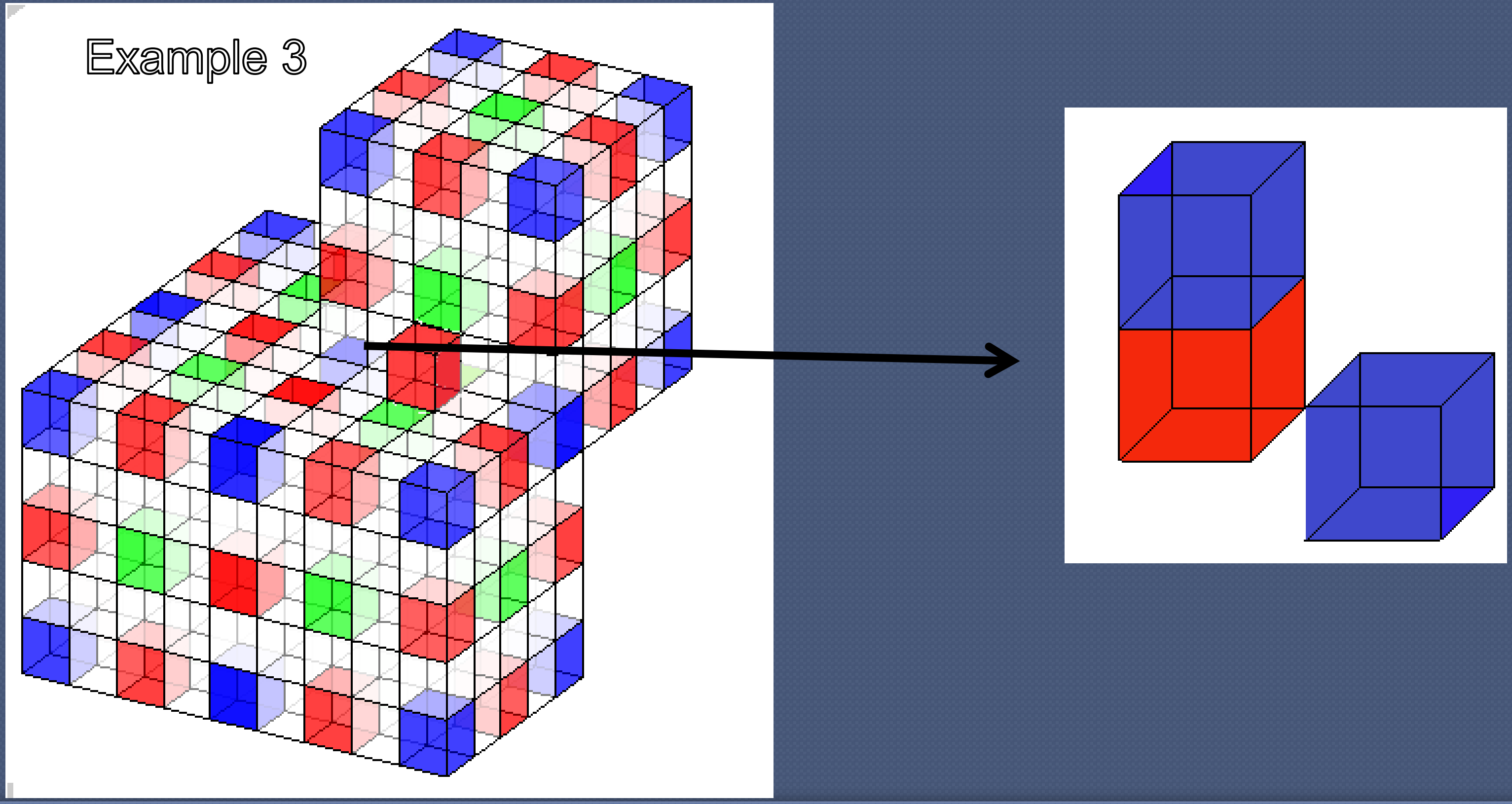

Future work:

Aims:

❑ To compute the homology of the foreground image as well as the background by computing the homology of the boundary surface;

❑ Geometrically control the representative (co)-cycles of homology generators;

❑ Deal with other 3D digital images: (6,26), (18,6) or (6,18) 3D images.

Thanks for your attention!

Questions