A near-linear time guaranteed algorithm for digital curve simplification under the Fréchet distance

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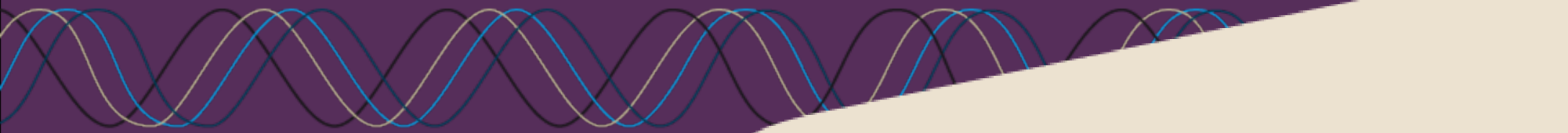

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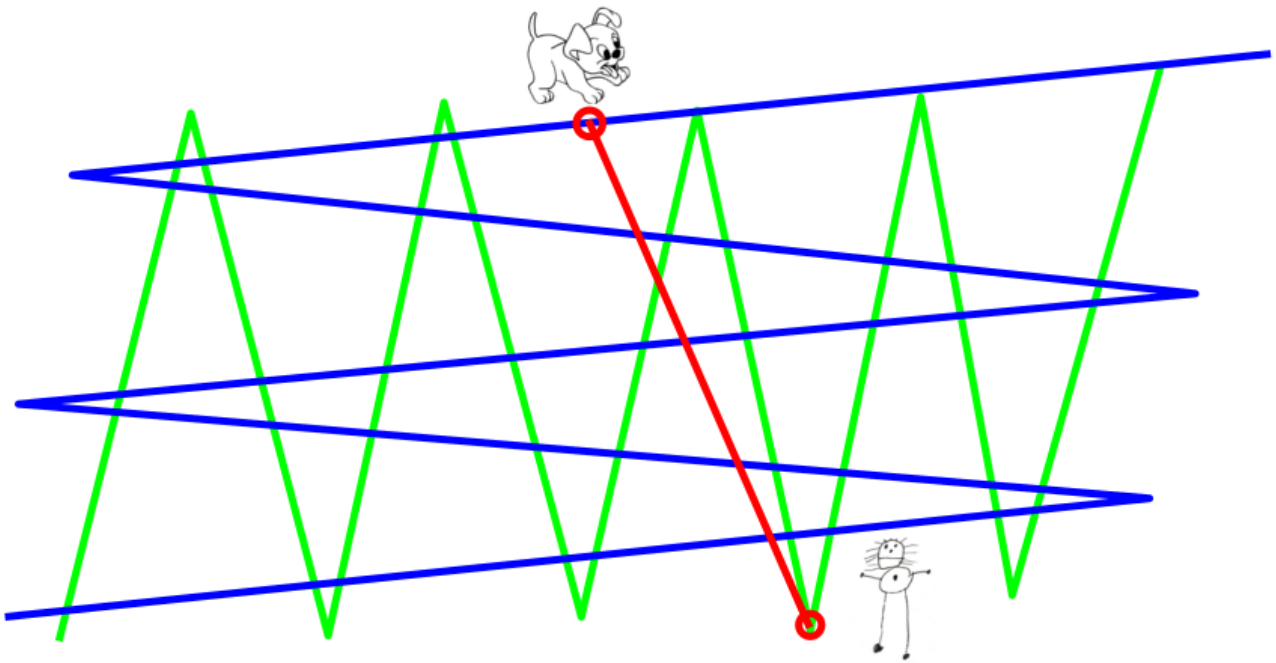
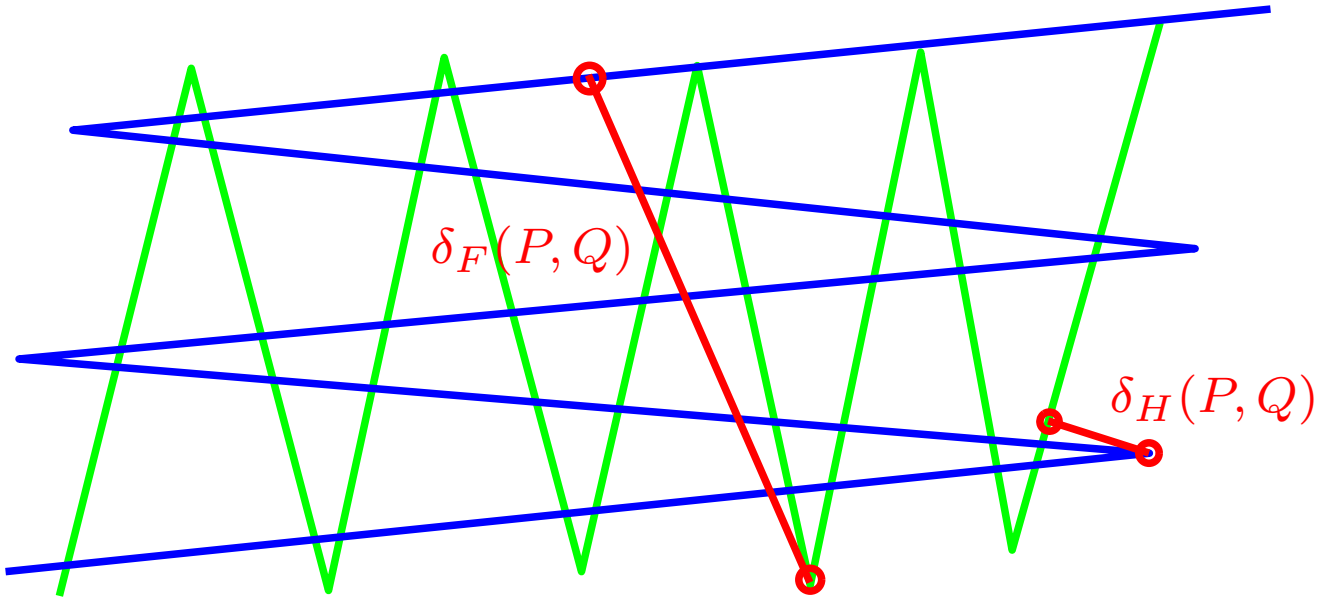
Fréchet Distance



- no backtracking
- length of the smallest leash necessary for any man and dog speeds

Fréchet Distance



- Hausdorff or $L_{\{1,2,\infty\}}$ distances are not good measures of the similarity of curves.
- Fréchet distance takes into account the course of the curves.

Curve simplification problem

Find P' an ε-simplification of P

=

Find shortcuts $p_i p_j$ such that $error(i,j) = \delta_F(p_i p_j, P) \leq \varepsilon$

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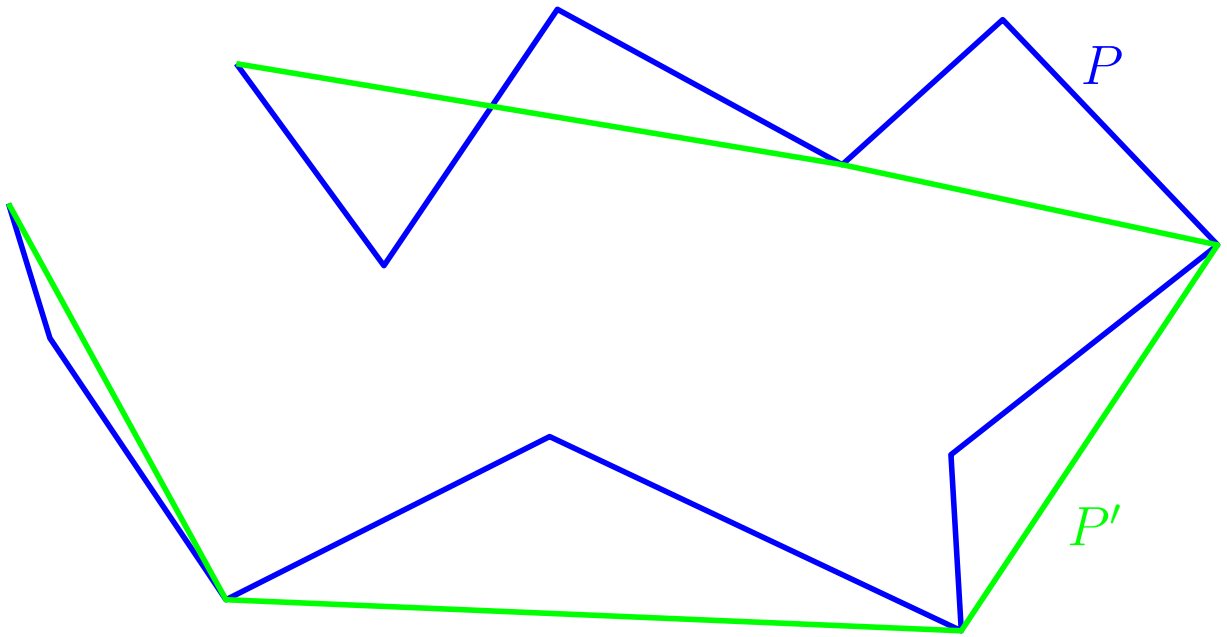
minimize the number of vertices of P'



Optimal algorithm in $\mathcal{O}(n^3)$

What are we looking for ?

Reasonable complexity

Previous work by Agarwal et al. :
$\mathcal{O}(n \log n)$ for any polygonal curve

Guarantee

Provable solution quality with respect to the optimal.

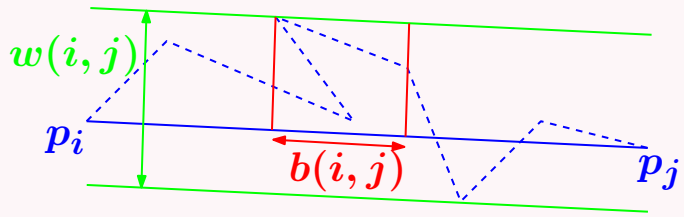
$\Downarrow$

Linear time algorithm with a guarantee for digital curves ?

Agarwal, P.K., et al. : Near-linear time approximation algorithms for curve simplification. Algorithmica (2005)

How do we achieve it ?

Approximated distance



$$f(w,b) \leq error \leq g(w,b)$$

Greedy algorithm

$i = 1, j = 2$
while $i < n$ **do**
  while $j < n$ *and* $\max(w(i,j), b(i,j)) \leq \frac{\varepsilon}{\sqrt{2}}$ **do**
    j=j+1
  create a new shortcut $p_i p_{j-1}$
  $i = j - 1, j = i + 1$

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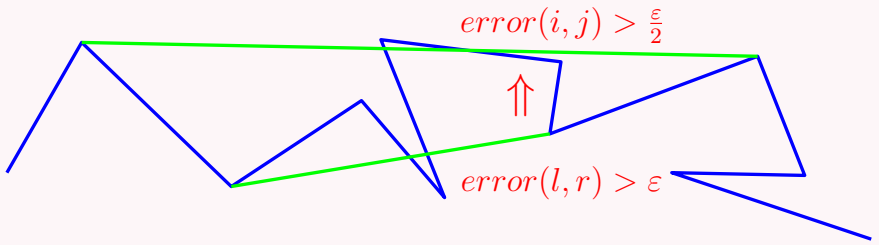
Nice local property



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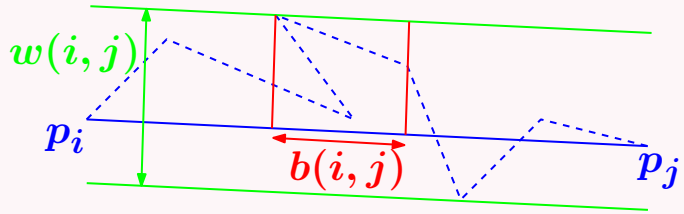
Guarantee !

Abam, M.A., et al. : Streaming algorithms for line simplification. In : SoCG '07

How do we achieve it ?

Approximated distance



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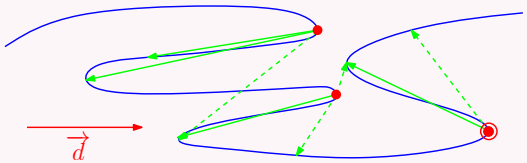
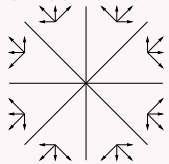
+

Digital curve specificities

Definition of occulters



In an octant, the occulters are the same.



The number of active occulters is **bounded by ε.**

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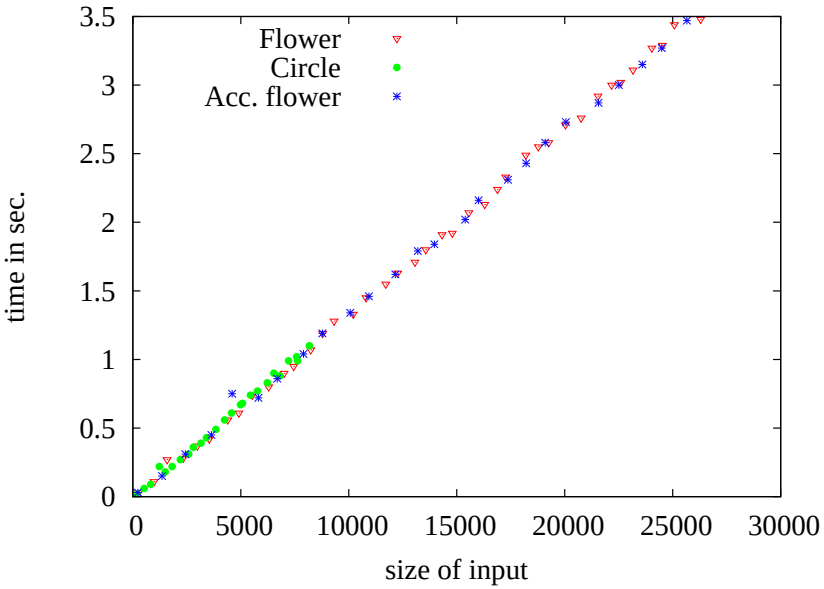
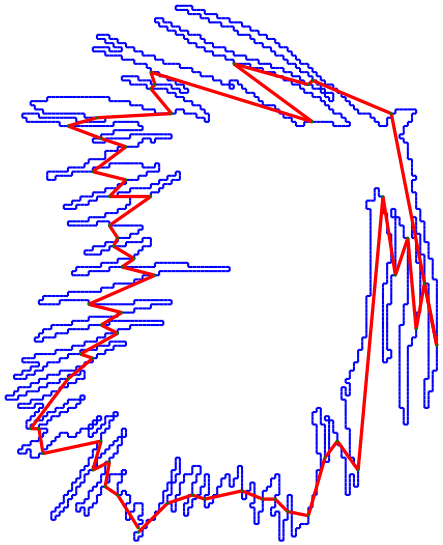
Near-linear time complexity !

Results



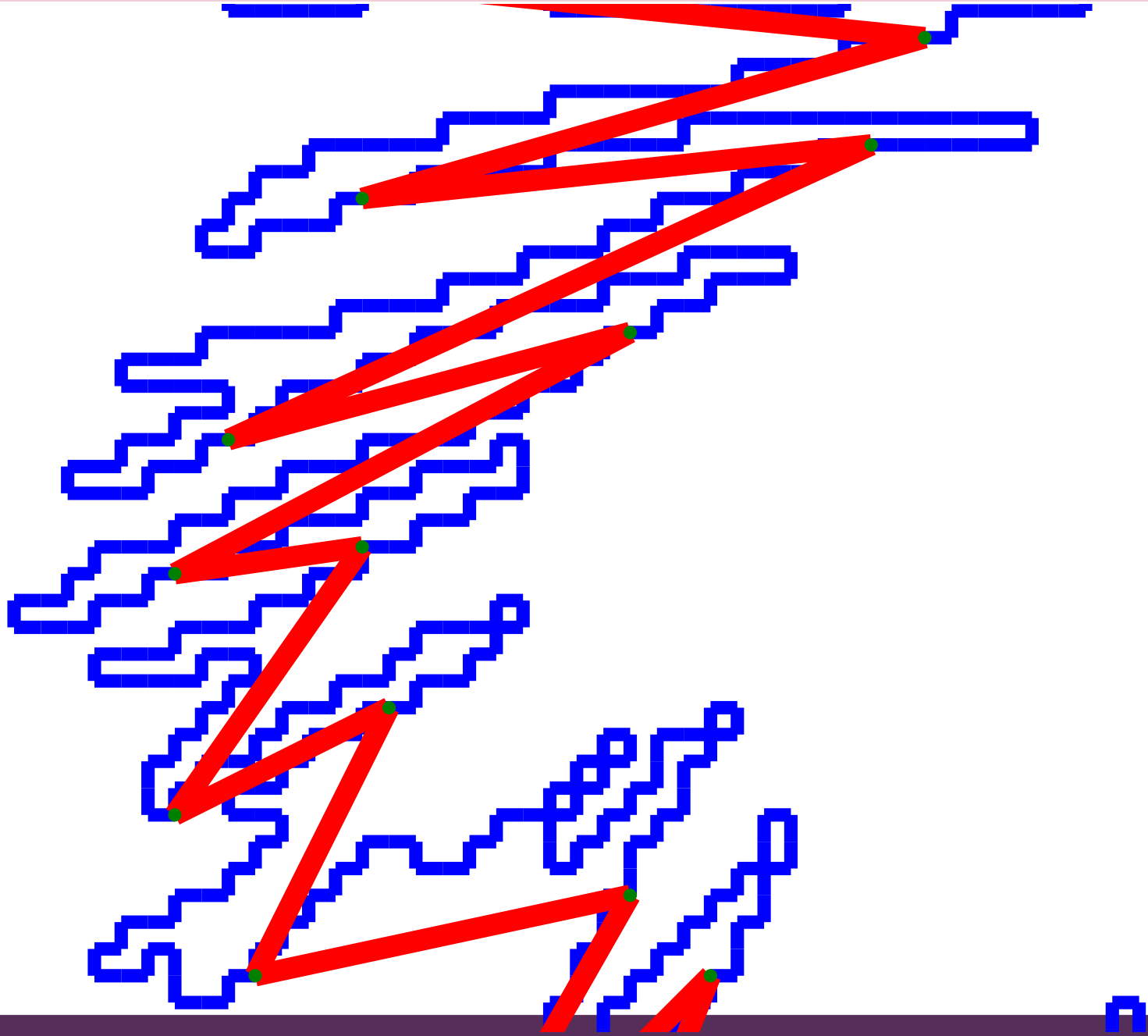
Runtime results for noisy synthetic shapes.

Fréchet distance
60 vertices



Width criterion
56 vertices

Results



Results