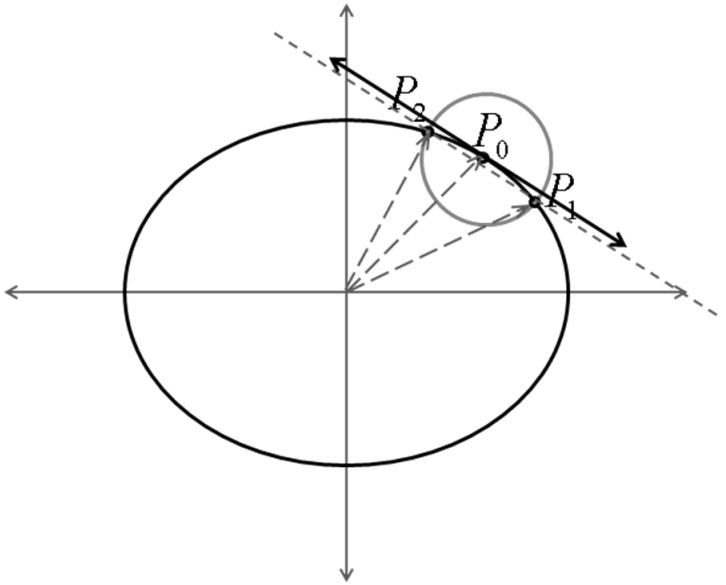
An Error Bounded Tangent Estimator for Digitized Elliptic Curves

- P_0 is the point at which we want to compute the tangent
- Make a circle of radius R (R is much smaller than the dimensions of elliptic curve)
- Get points P_1 and P_2
- Get the slope ‘m’ of line P_1P_2
- The estimated tangent is the line with slope ‘m’ but passing through P_0



There is a definite upper bound of the error

The upper bound

$$\partial\tilde{\phi}_{\max} = \max\left(\frac{1}{s^3}\left(\sin\tilde{\phi} \pm \cos\tilde{\phi}\right)\left(s^2 - s\left(\pm\cos\tilde{\phi} \pm \sin\tilde{\phi}\right) + \left(\pm\cos\tilde{\phi} \pm \sin\tilde{\phi}\right)^2 \right) \right)$$

$\tilde{\phi}$ = angle subtended by the actual tangent on the x-axis $\qquad s = |P_1P_2|$

Choice of R:

- angle subtended by the points P_1 and P_2 should be small

Total angle subtended: $2\Delta\theta$

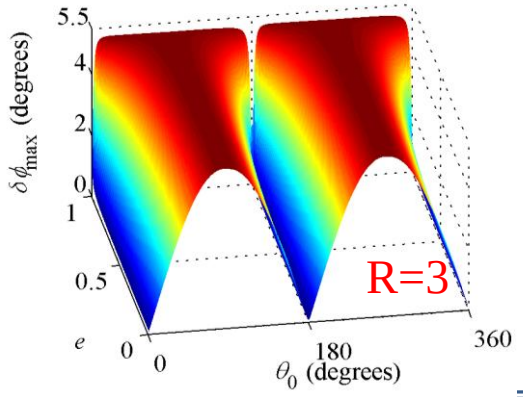
$$R \le 2b\sin\left(\Delta\theta_{\max}/2\right)$$

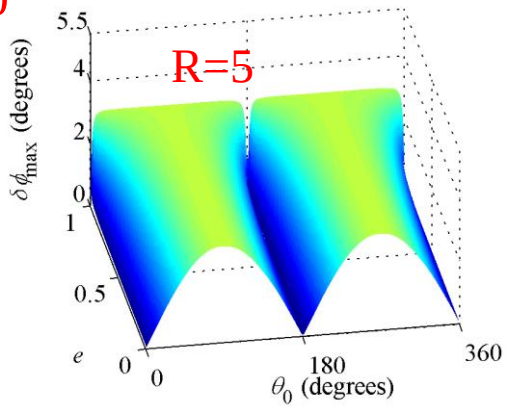
$$\Delta\theta = \pm 2\sin^{-1}\left(\sqrt{\frac{R^2}{4\left(a^2\sin^2\theta_0 + b^2\cos^2\theta_0\right)}} \right)$$

Example: $\Delta\theta_{\max} = (\pi/18)$ or $10°$

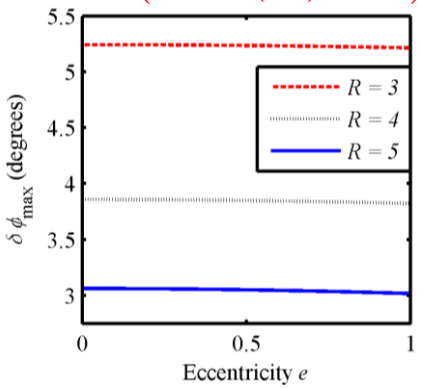
Then $R \le 0.1743b$

semi-minor axis $b = 30$

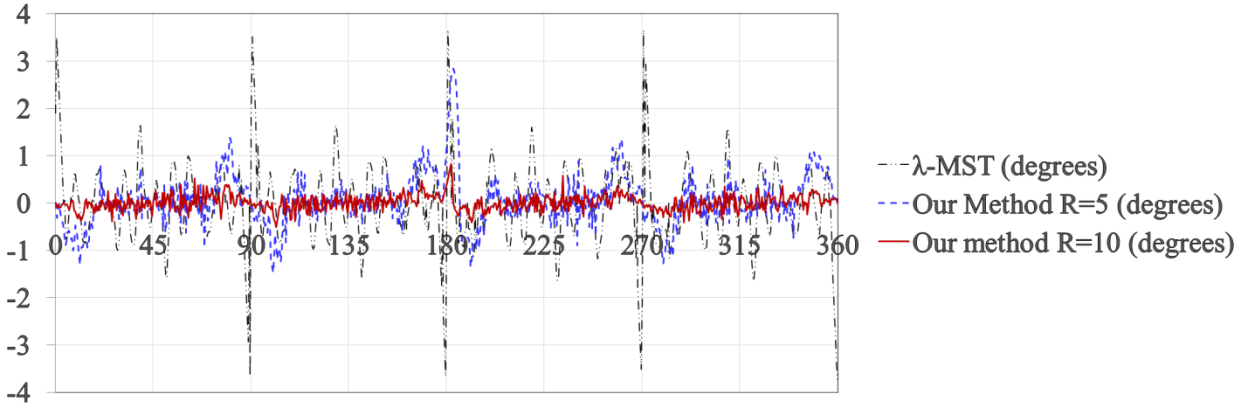




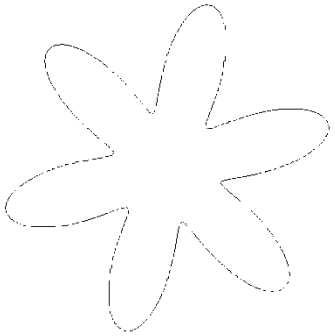
$\max\left(\partial\phi_{\max}(\theta_0); \forall\theta_0\right)$



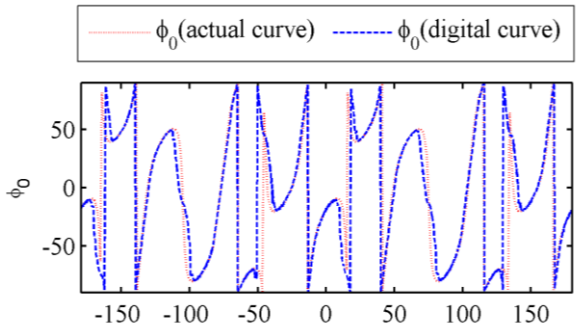
Examples



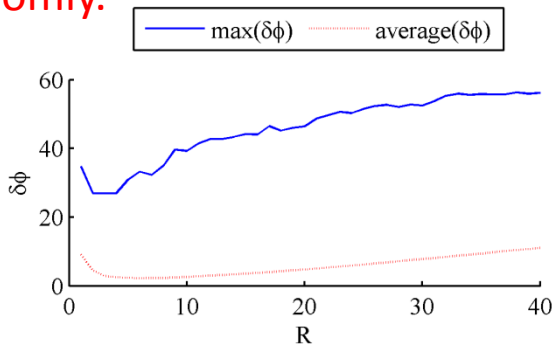
Average absolute error in the computation of tangents for 100 experiments with digitized circles of radius 100 and centers within 1 pixel region chosen randomly.



The digitized flower shape



The angle of the tangents on the actual curve and the digital curve (using *R*=20)



The error in the computation of the tangent due to digitization for various values of *R*

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