Approximate Shortest Paths in Simple Polyhedra

Fajie Li
Huaqiao University
Reinhard Klette
The University of Auckland

April 7, 2011

Problem and Results

Problem: Connect two points p and q in a simple polyhedron Π by a Euclidean shortest path (ESP) that is contained in Π.

Results: an approximate $\kappa(\varepsilon) \cdot \mathcal{O}(M|V|)$ 3D ESP algorithm, not counting time for preprocessing.

Preprocessing time complexity: $\mathcal{O}(M|E| + |\mathcal{F}| + |V| \log |V|)$ for solving a 'fairly general' case of the 3D ESP problem.

V, E, $\mathcal{F}$, M: sets of vertices and edges of Π, the set of faces (triangles) of Π, and the maximal number of vertices of a so-called critical polygon.

$\kappa(\varepsilon) = (L_0 - L)/\varepsilon$ where L_0 is the length of an initial path and L is the true (i.e., optimum) path length.

Main Ideas and Conclusions

We randomly take a point in the closure of each critical polygon to identify an initial path from p to q. Then we enter a loop; in each iteration, we optimize locally the position of point p_1 by moving it within its critical polygon, then of p_2, ..., and finally of p_k. At the end of each iteration, we check the difference between the length of the current path to that of the previous one; if it is less than a given accuracy threshold $\varepsilon > 0$ then we stop. Otherwise, we go to the next iteration.

The given algorithm solves approximately three (previously known to be) NP-complete or NP-hard 3D ESP problems in time $\kappa(\varepsilon) \cdot \mathcal{O}(k)$, where k is the number of layers in a stack. The proposed approximation method has straightforward applications for ESP problems when analyzing polyhedral objects (e.g., in 3D imaging), of for 'flying' over a polyhedral terrain.