Efficient robust digital hyperplane fitting with bounded error

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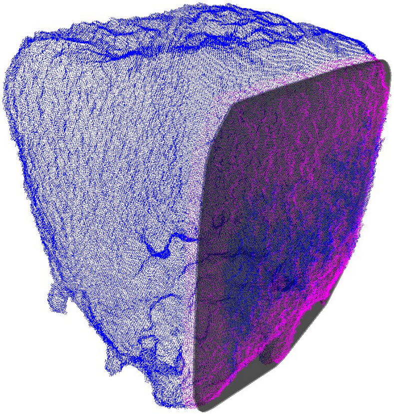
Our aim: hyperplane fitting



Example: 205001 discrete points in a 3D image generated from an electron nano-tomography image containing a cubical crystal.

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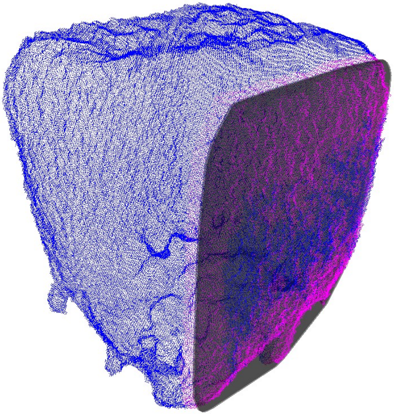

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Remark

As input contains a large number of points with many outliers, we need an **efficient** and **robust** fitting method.

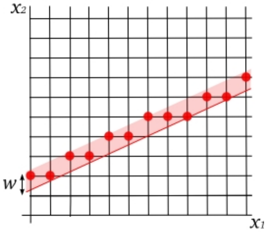
Digital hyperplane fitting

Problem

Given a finite set of N discrete points, seek the **maximum subset** whose elements are contained in a **digital hyperplane** defined by

$$\mathbf{D}(\mathbf{H}) = \{(x_1, x_2, \ldots, x_d) \in \mathbb{Z}^d : 0 \leq a_1x_1 + a_2x_2 + \ldots + a_dx_d + a_{d+1} < w\},$$

with the normalization $-1 \leq a_i \leq 1$ for $i = 1, 2 \ldots, d$ such that there exists at least one coefficient $a_i = 1$, where w is a constant width.



a digital line

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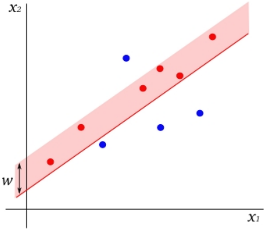

digital line fitting

Exact fitting and approximations

Contribution 1

The **optimal complexity** for finding an exact solution is $O(N^d)$ where d is the space dimension.

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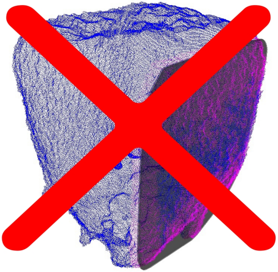
Number of operation $\approx 10^{16}$
Impossible! (10^7 seconds $\approx$ 100 days)



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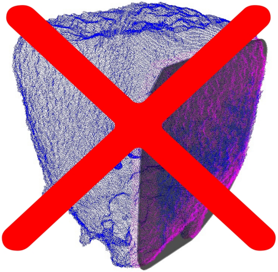


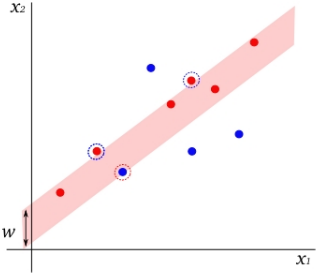
Contribution 2

Two **linear approximation methods** with different bounded errors are proposed.

Approximation 1

Bounded error in number of inliers

Given some $\varepsilon > 0$, we find a digital hyperplane that contains at least $(1-\varepsilon)n_{opt}$ **points**, where n_{opt} is the maximum possible number of points that belong to any digital hyperplane, assuming $n_{opt} = \Omega(N)$.



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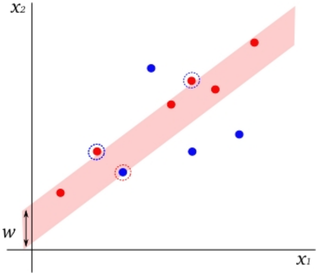
Runtime

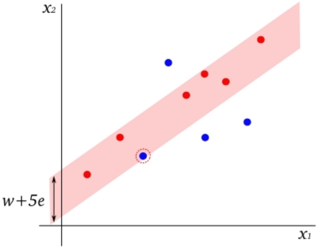
- $O(N + \varepsilon^{-4} \log N)$ for $d = 2$,
- $O(N(\varepsilon^{-2} \log N)^{d+1})$ for larger d.



Approximation 2

Bounded error in digital hyperplane width w

Given some $\varepsilon > 0$, we find a **digital hyperplane of width $w + 5\varepsilon$** that contains $n > n_{opt}$ points, where n_{opt} is the maximum number of points that belong to any digital hyperplane of width w in a grid $[0, \delta]^d$.



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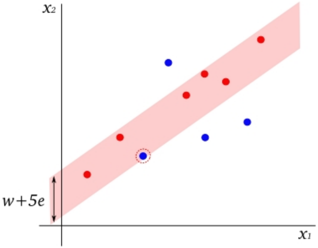
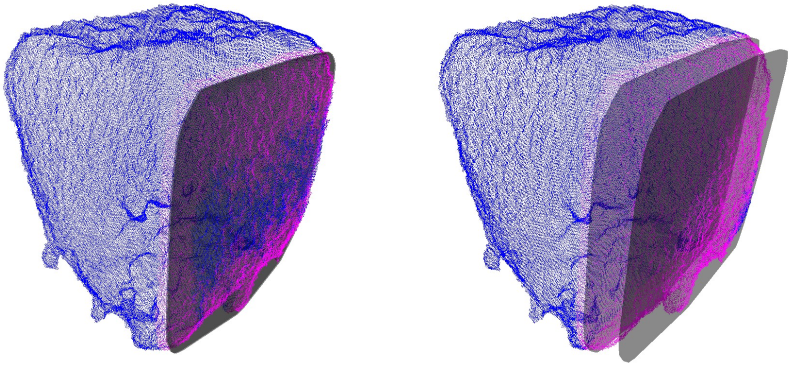
Runtime

$O(N + (\frac{\delta}{\varepsilon})^d \log^{O(1)}(\frac{\delta}{\varepsilon}))$



Results with Approximation 2

Digital plane fitting for a pre-processed 3D binary nano-tomography image containing 205001 discrete points is achieved in 12 seconds ($w = 1$, $\varepsilon = 4$).



$\varepsilon = 4$ for $w = 1$ (left) and $w = 25$ (right).