Properties and Applications of the Simplified Generalized Perpendicular Bisector

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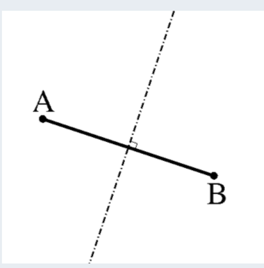
1. What is the Simplified Generalized Perpendicular Bisector?

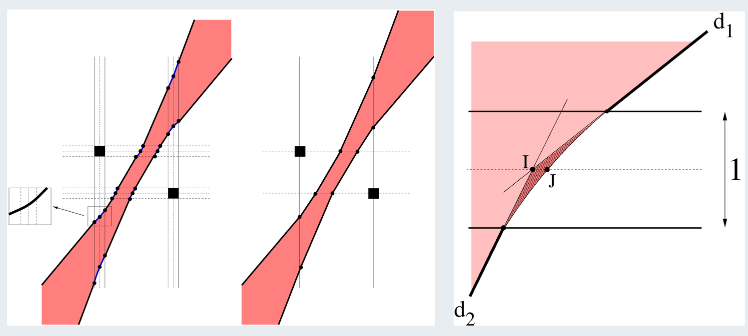
The Perpendicular Bisector (PB)



The PB between two points A and B is the set of points that are at equal distance of both points.

The Generalized Perpendicular Bisector (GPB) and the Simplified GPB (SGPB)

The GPB between two regions S_1 and S_2 is the set of the PB of every couple of points that belongs to S_1 and S_2.



For computational purposes, in the GPB, the parabolic pieces have been dropped by extending the straight lines (*i.e.* changing the distance definition). This defines the SGPB.

2. Characterization of the points belonging to the GPB

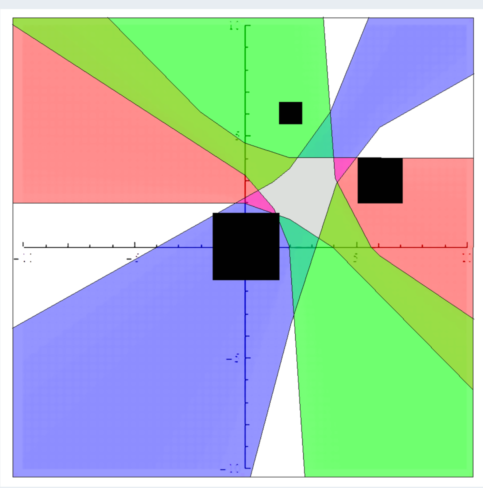
- $\mathcal{S}_1$ and $\mathcal{S}_2$: two bounded connected regions;
- $d_{i_{min}}(X) = \min_{Y \in \mathcal{S}_i}(d(X,Y))$;
- $d_{i_{max}}(X) = \max_{Y \in \mathcal{S}_i}(d(X,Y))$ where d is the usual Euclidean distance.

Every Euclidean point $X \in \mathbb{R}^n$ such that:

$$[d_{1_{min}}(X), d_{1_{max}}(X)] \bigcap [d_{2_{min}}(X), d_{2_{max}}(X)] \neq \emptyset \qquad (1)$$

belongs to the GPB of $\mathcal{S}_1$ and $\mathcal{S}_2$.

3. GPB and adaptative pixels (pixels of different sizes)



Proposition The boundary of 2D-Simplified Generalized Perpendicular Bisector between two pixels $P_1 = (x_1, y_1)$ of size λ_1 and $P_2 = (x_2, y_2)$ of size λ_2 is composed of at most 10 line segments and half-lines.

4. Simplified Generalized Circumcenter (SGC)

The SGC of a set of n finite and connected regions $\mathcal{S} = (S_i)_{i \in [1,n]}$ is defined as the intersection of the SGPB of every two regions of the set:

$$SGC(\mathcal{S}) = \bigcap_{i,j \in [1,n], i<j} (SGPB(S_i, S_j)).$$

Property Each point of the SGC corresponds to the center of at least one circle that intersects all the adaptive pixels.

5. Dual

Proposition The dual of a SGPB is a convex polygon of at most 8 vertices and 8 edges. At most two vertices may be at the infinite (the dual polygon edges are vertical (determination in O(1)).

Proposition All the straight lines crossing the duals of all the SGPB of every pair of adaptative pixels P_i and P_j is the dual of the Simplified Generalized Circumcenter.

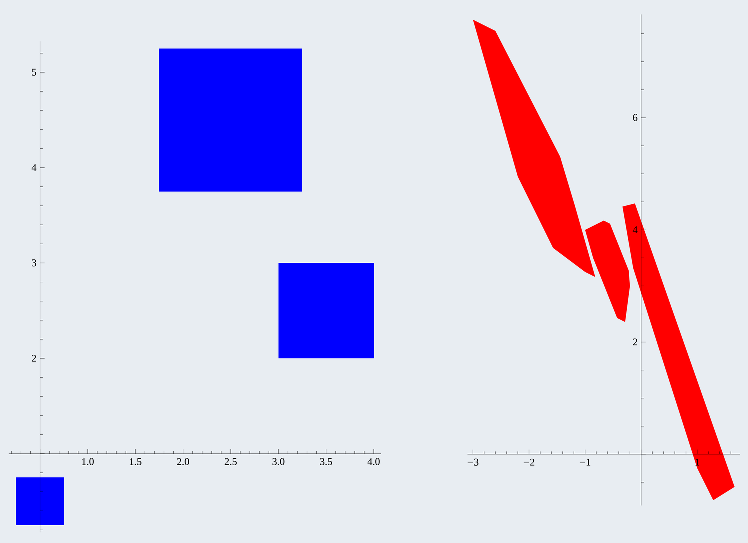


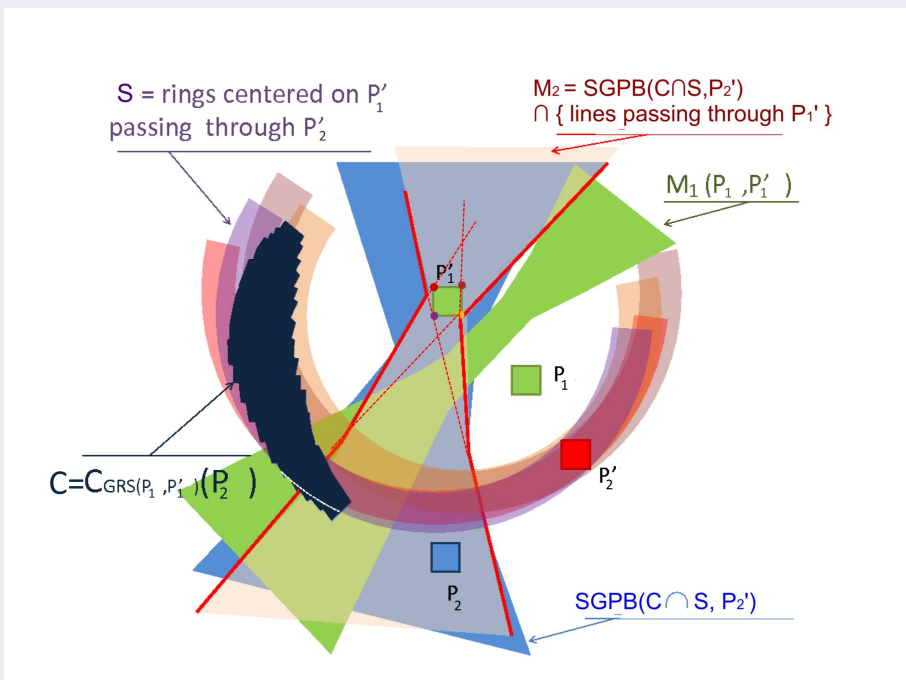
Figure: The dual of the three SGPB corresponding to three pixels of different sizes.

6. Illustration of the rotation reconstruction using the SGPB

Fontijne's rotation estimation algorithm:

- ⋆ **reconstruction of a rotation**;
- ⋆ from n points p_i and their images p'_i;
- ⋆ incremental determination of PB;

1. Construction of the PB Δ_1 between p_1 and p'_1;
2. Construction of the PB Δ_2 between the image of p_2 by Δ_1 and p'_2.



Application of the SGPB:

- ⋆ Points $p_i, p'_i \Rightarrow$ pixels P_i, P'_i
- ⋆ PB $\Rightarrow$ $SGPB$
- ⋆ **"visualization" of the error generated by the rotation reconstruction method of Fontijne.**

7. Application to noisy circle recognition

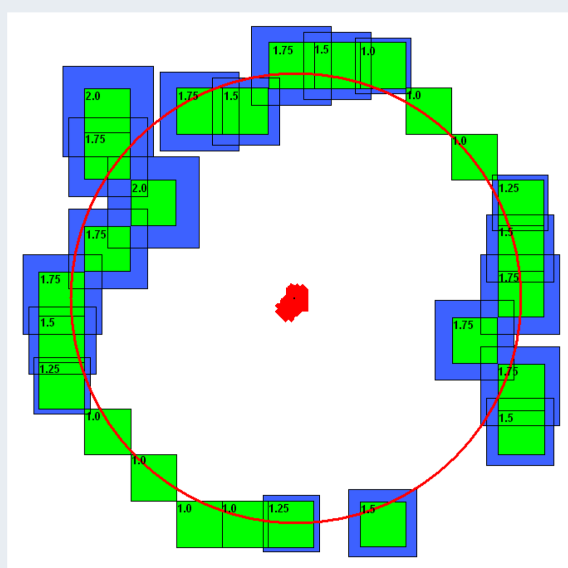


Figure: A Bresenham circle of radius 5 with misplaced and missing pixels.

- Increasing of the size of each pixel according to a local noise estimator;
- Computation of the $SGPB$ of each couple of pixels (with the new sizes).

$\Rightarrow$ Intersection = set of possible circle centers (the SGC.)

8. Conclusion and perspectives

- Conclusion:
  - Definition of the SGPB between two pixels of different sizes;
  - Study of the dual of the SGPB;
  - Application to exhaustive parameter estimation of noisy circles;
  - Reconstruction of the noisy rotations using the SGPB.
- Perspectives:
  - Link between the SGPB and other discrete bisectors.
  - Investigations in higher dimension.