An error bounded tangent estimator for digitized elliptic curves

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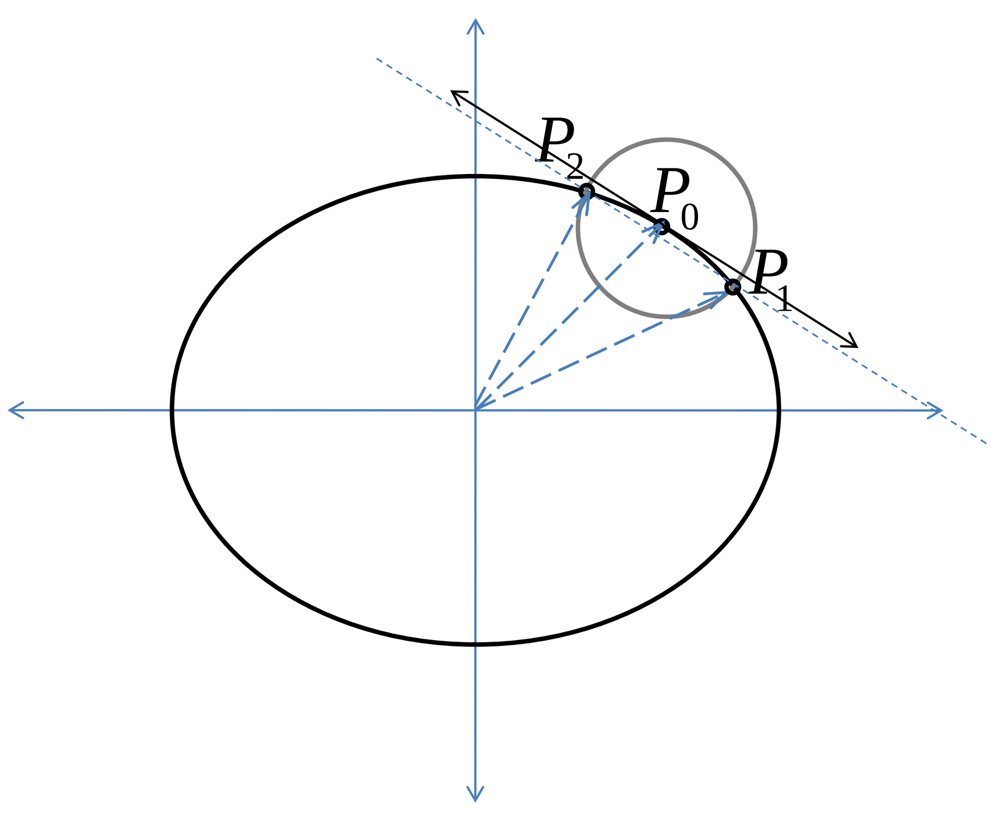
Introduction to the problem

1. Often, tangents need to be computed for digital curves
2. We propose an error bounded tangent estimator for digital curves
3. We calculate the error bounds as well
4. Presented error bound is for digital ellipses, though the technique is directly extensible to other digital curves as well

Contemporary methods

1. Fit a continuous function over the small region around the point of interest. Find the analytical derivative of the function and use this as the slope of the tangent
   - Restrictive in the choice of the nature of continuous function
   - Restrictive in the definition, shape, and dimension of the local region, etc.
   - Computationally intensive and afflicted by the quantization noise.
2. Use a Gaussian filter to smoothen the digital curve and obtain a smooth continuous curve. Use this Gaussian smoothened continuous curve for estimating the tangents
   - Similar issues as above
3. Consider a family of continuous curves of various types. Approximate the whole digital curve by one of the continuous curves in the family using a global optimization technique. Then compute the tangents on the curve chosen by optimization
   - Restricted to the curves in the family.
   - No guarantee of convergence of global optimization to the global minimum
   - Computation intensive
4. Approximate the digital curves using line segments. At the point of interest, find the maximal line segments passing through it. Compute a weighted convex combination of their slopes to find the orientation of the tangent.
   - parameter-free, has asymptotic convergence, and incorporates convexity property,
   - developed basically on heuristics, rather than analytic foundation.

Concept of the proposed method



- P_0 is the point of interest
- Find points P_1 and P_2 which are on the intersection of the digital curve and a small circle of radius R centered at P_0
- Compute the line passing through P_0 with a slope equal to the slope of line $P_1 P_2$

- No need to know the geometric properties of the digital curve
- Geometrically, for continuous curves, the error in the computation of the tangent goes to zero for small value of R
- Only one parameter R: which if sufficiently low ensures accuracy over a very wide range

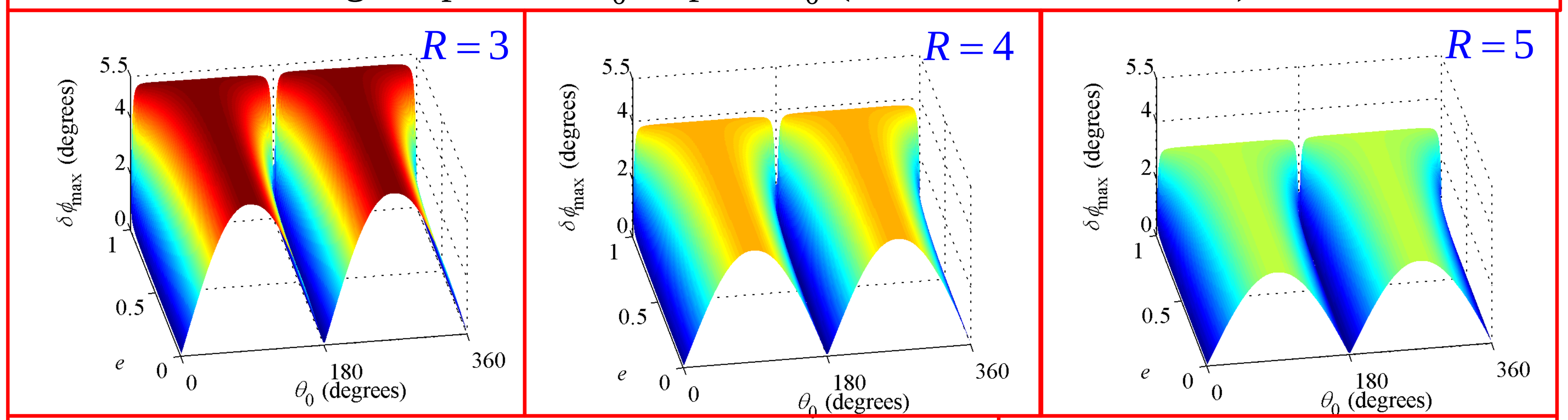
Guideline for choosing R: $R \le 2b \sin\left(\frac{\Delta\theta_{max}}{2}\right)$

Example: $\Delta\theta_{max} = (\pi/18)$ i.e. $10°$
$\Rightarrow \sin(\Delta\theta/2) = 0.0872$
Then $R \le 0.1743b$

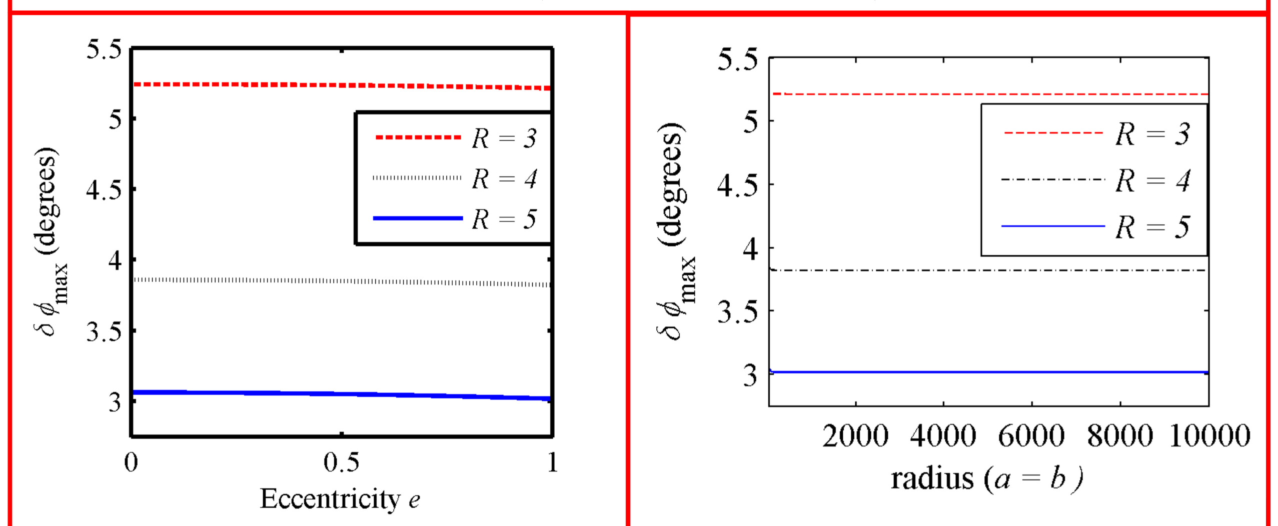
Error in computation of the slope using the proposed method: $\partial\phi = \tan^{-1}\left(\frac{\tilde{m}(\Delta x_2 - \Delta x_1) - (\Delta y_2 - \Delta y_1)}{(1+\tilde{m}^2)(x_2 - x_1) + (\Delta x_2 - \Delta x_1) + \tilde{m}(\Delta y_2 - \Delta y_1)}\right)$

Numerical results

$\partial\phi_{max}(\theta_0)$ for all various values of eccentricity e and angular position θ_0 of point P_0 (semi-minor axis $b = 30$)



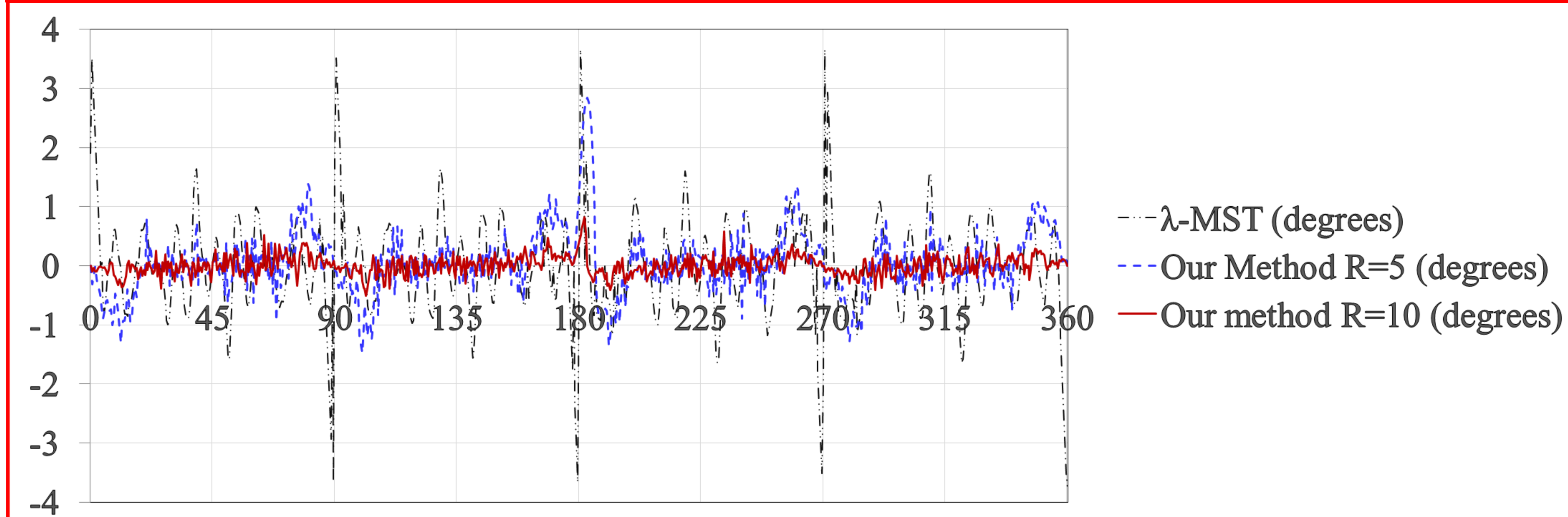
$\max(\partial\phi_{max}(\theta_0); \forall\theta_0)$



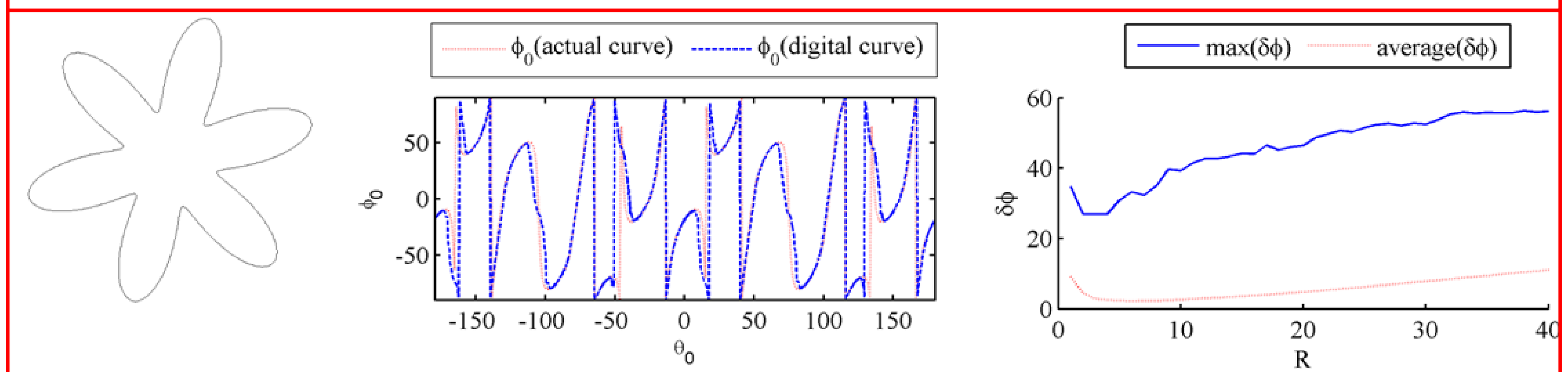
Ellipses with $b = 30$ — Circles

Error $< 5.5°$ for $R = 3$
Error $< 3.1°$ for $R = 5$

Why error decreases when R increases
Maximum error due to digitization: 0.5
Ratio of max. digitization error to R: $0.5/R$
$(0.5/R)_{R=3} > (0.5/R)_{R=4} > (0.5/R)_{R=5}$



Average absolute error in the computation of tangents for 100 experiments with digitized circles of radius 100 and centers within 1 pixel region chosen randomly. The result is compared with λ − MST estimator.

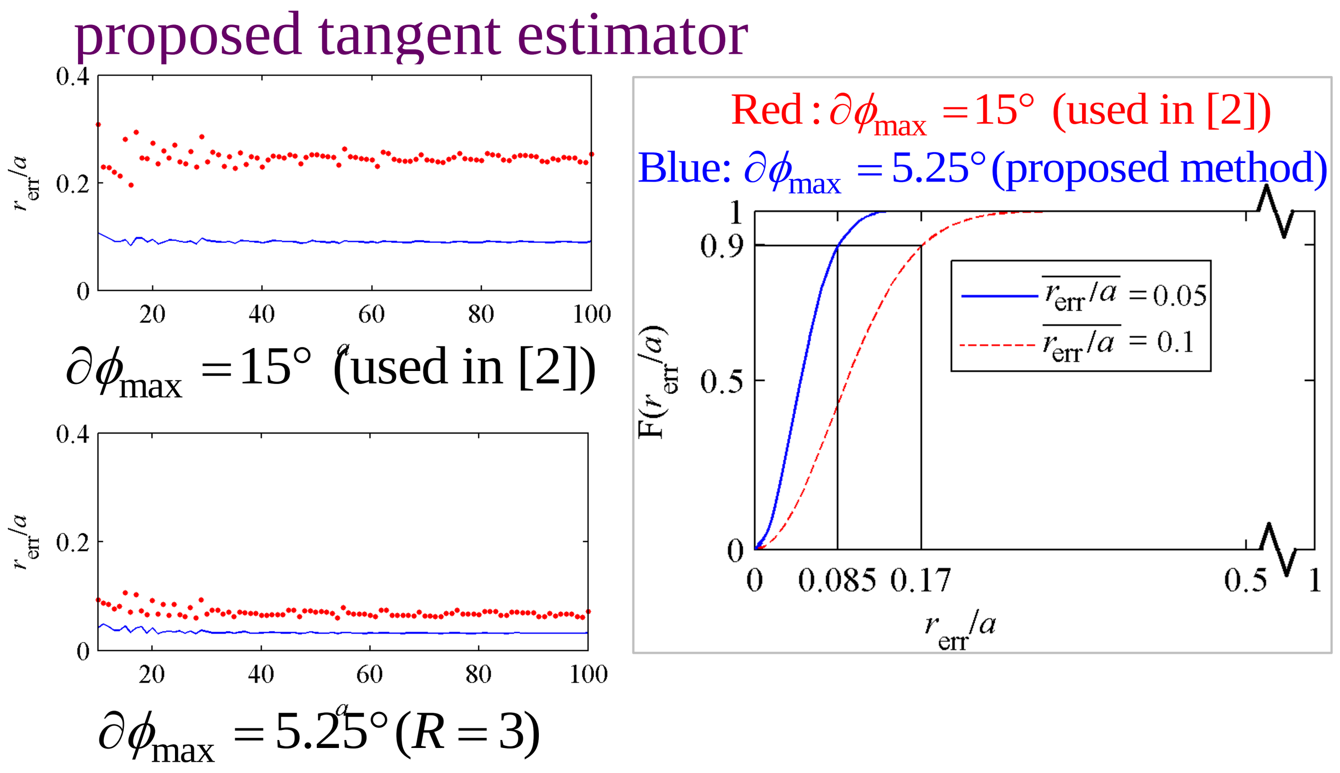


(a) The digitized flower shape represented by (22) (b) The angle of the tangents on the actual curve and the digital curve (using R=20) (c) The error in the computation of the tangent due to digitization for various values of R

Example of an analytical curve with inflexion points

Impact in practical applications

- A popular geometric method for ellipse detection (Yuen 1989 [1]) uses the computation of tangents at three points for finding the center of the ellipses
- It was shown recently that tangents are a major contributor to the error in the computation of centers using Yuen 's method [2]
- In the numerical results, error in tangents was considered to be $\partial\phi_{max} = 15°$, which is a reasonable estimate for the existing methods.
- We show that using the proposed tangent estimator, the error in ellipse detection can be reduced significantly
- We show that the reliability of the ellipse detection increases with the proposed tangent estimator



Probability density function of the relative error in Yuen's method [1] proposed in [2]

Earlier, relative error of 0.17 was considered to give 90% reliability.

Now, relative error of 0.085 gives 90% reliability

References: [1] H. K. Yuen, J. Illingworth, and J. Kittler, "Detecting partially occluded ellipses using the Hough transform," Image and Vision Computing, vol. 7, pp. 31-37, 1989. [2] D. K. Prasad and M. K. H. Leung, "Error analysis of geometric ellipse detection methods due to quantization," in Fourth Pacific-Rim Symposium on Image and Video Technology (PSIVT 2010), Singapore, 2010.