Circular Arc Reconstruction of Digital Contours with Chosen Hausdorff Error

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Abstract

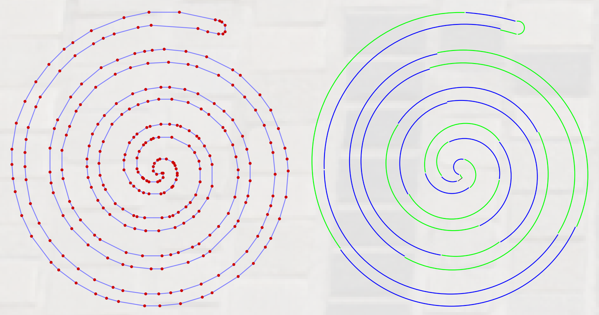
Instead of polygonalizing a contour, we propose to reconstruct the shape with circular arcs. To do so, we exploit the recent curvature estimators [1, 3]. From their curvature field, we introduce a new simple and efficient algorithm to approximate a digital shape with as few arcs as possible at a given scale, specified by a maximal admissible Hausdorff distance.

Keyword: Circular arc reconstruction, contour representation, curvature estimator.

1 Introduction

Objectives:

- To represent a contour more efficiently than polygonal contour.
- Less primitives with equivalent precision.
- To include a scale parameter.



$\delta_H = 2.018$ 288 segments — $\delta_H = 2.011$ 27 arcs, 1 segment

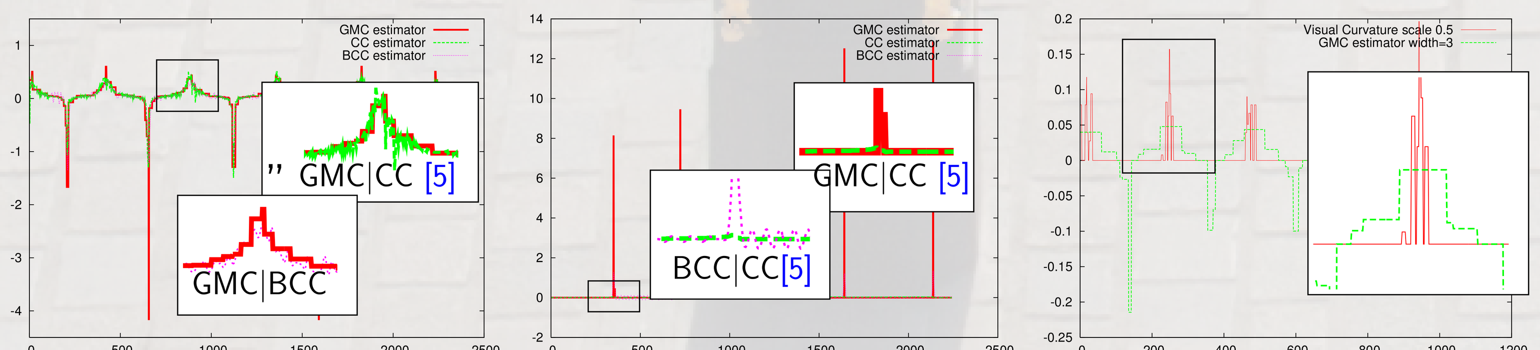
Main idea:

- Exploit recent and robust to noise curvature estimators [1, 3].
- Avoid the use of curvature post processing to reduce parameters [6].
- Use only one parameter associated to the scale.

2 Curvature estimators

- **Global Minimization Curvature [1] (GMC).**
  - Taking into account all the real shapes having the same digitization.
  - The shape which minimizes its squared curvature.
- **Binomial Convolution Curvature [3] (BCC).**
  - A discrete alternative to the Gaussian smoothing technique (scale controlled by m).
  - Successive convolutions of m binomial kernels.
- **Visual Curvature [2] (VC).**
  - Measure the number of extreme points from a height function.
  - Filter non-significant features at a given scale (only qualitative estimation).

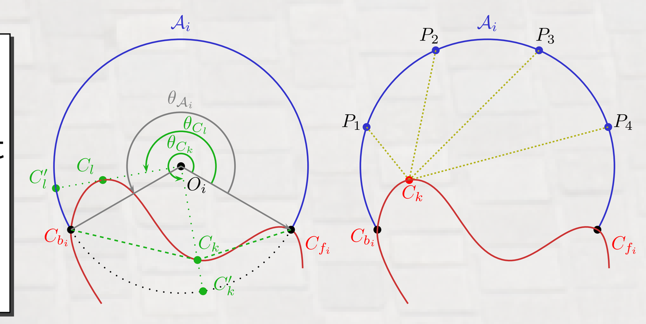
Illustration and comparison of curvature estimators:



3 Contour reconstruction with circular arcs

Main steps of the algorithm:

- Decompose curvature estimation to extract significant curved areas.
- Split/merge strategy (given maximal error E_{max})



Use an approximation of the hausdorff distance between the Arc $\mathcal{A}_i$ and the contour segment $\mathcal{C}_i$:

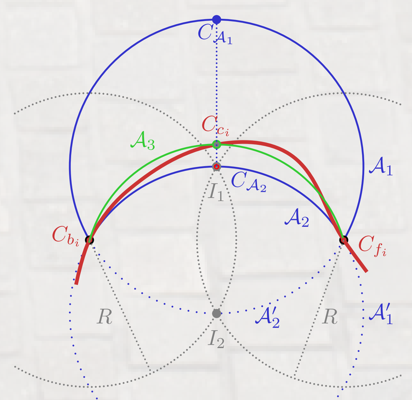
$$\delta_H(\mathcal{A}_i, \mathcal{C}_i) = \max\{\max_{b\in\mathcal{C}_i}\{\min_{a\in\mathcal{A}_i} d(a,b)\}, \max_{a\in\mathcal{A}_i}\{\min_{b\in\mathcal{C}_i} d(a,b)\}\}$$

Main algorithm

Data: $C = \{C_i\}_{i=0}^{n}$ digital curve, $\kappa = \{\kappa_i\}_{i=0}^{n}$ curvature estimation, float maxArcError;
Result: curve represented by a set of arcs and segments.
begin
  Decompose κ into a set of constant curvature interval S defined by: $\{(b_0, f_0), ..., (b_i, f_i), ..., (b_M, f_M)\}$.
  For each contour point C_i, store in $regionIndex[i]$ the index $k \in \{0, ..M\}$ of its region $S[k]$.
  Extract from S the set S_m containing all the regions which are a local maxima/minima.
  $S_{tmp} = S$;
  while $nbElements(S_{tmp})! = 0$ **do**
    S_{tmp} = SPLIT_REGIONS(S_{tmp}, S, $regionIndex$, maxArcError);
  // First extension from mini/maxima regions:
  while $nbElements(S_m)!=0$ **do**
    S_m = EXTEND_PLOT_REGIONS(S_m, S, regionIndex, κ, maxArcError);
  // Second extension from all others regions S_u:
  S_u= set of index of valid non maxima/minima regions of the current regions.
  while $nbElements(S_u)!=0$ **do**
    S_u = EXTEND_PLOT_REGIONS(S_u, S, regionIndex, κ, maxArcError);
  // Verify or change primitive for region which are better approximate
  // by a straight segment:
  checkBestPrimitive(S, tabRegionIndex, maxArcError);
end

Arc reconstruction: Several ways to reconstruct arcs between region extremities C_{b_i} and C_{f_i}.

1. Use curvature to determine and select one of the two possible solutions $C_{\mathcal{A}_1}$ and $C_{\mathcal{A}_2}$ (if they exist).
2. Use central contour point C_{c_i} to determine $\mathcal{A}_3$.

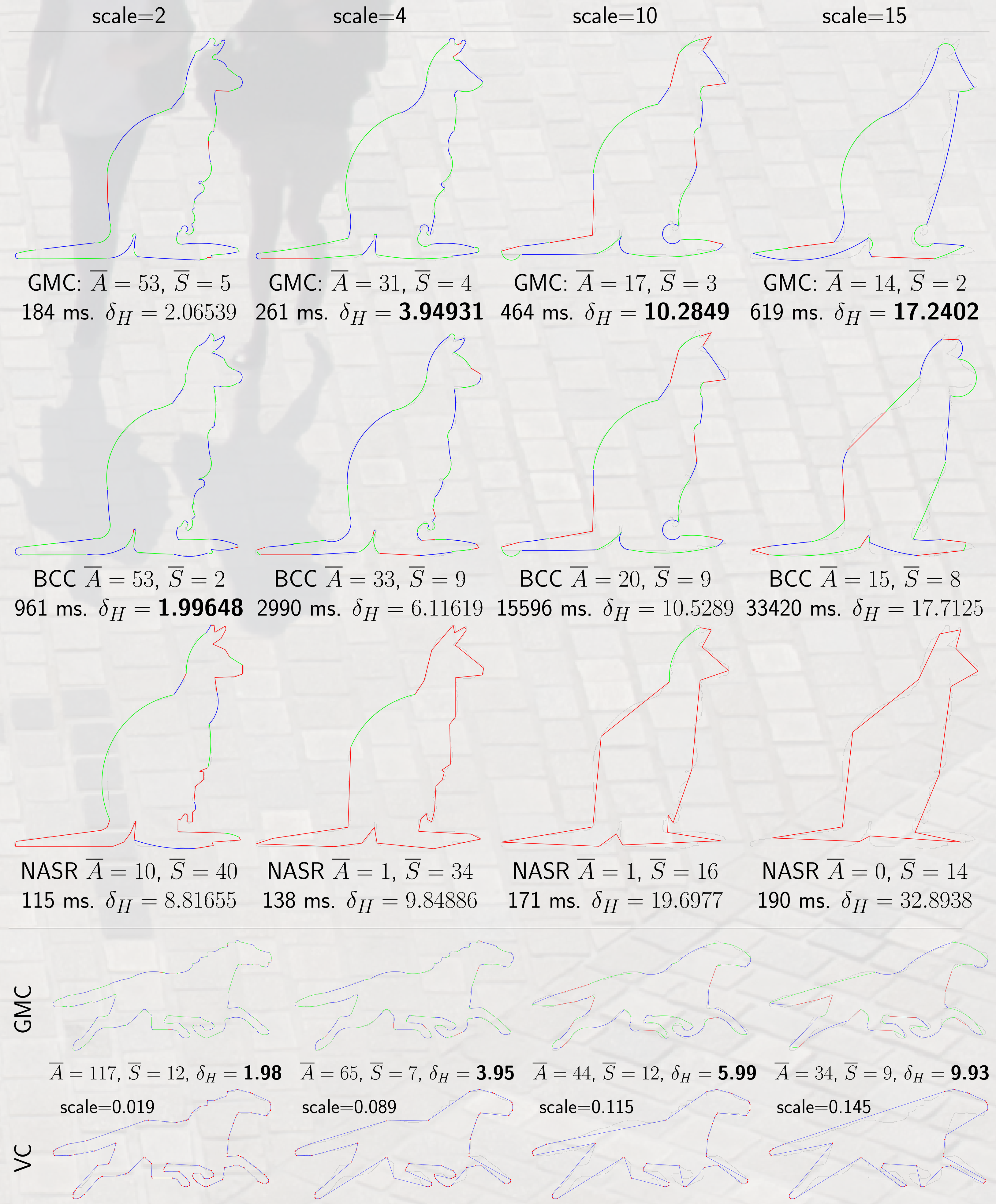


$\Rightarrow$ Solution 2 with arc $\mathcal{A}_3$ gives better results.

Merging process:

- Initiated from local minima/maxima (independent of the choice of the initial contour point).
- Performed only for plot regions with same curvature sign.
- Applied first to neighbor regions with nearest curvature values.

4 Experiments and comparisons with VC and NASR [4]



scale=2	scale=4	scale=10	scale=15
GMC: $\overline{A} = 53$, $\overline{S} = 5$ 184 ms. $\delta_H = 2.06539$	GMC: $\overline{A} = 31$, $\overline{S} = 4$ 261 ms. $\delta_H =$ **3.94931**	GMC: $\overline{A} = 17$, $\overline{S} = 3$ 464 ms. $\delta_H =$ **10.2849**	GMC: $\overline{A} = 14$, $\overline{S} = 2$ 619 ms. $\delta_H =$ **17.2402**
BCC $\overline{A} = 53$, $\overline{S} = 2$ 961 ms. $\delta_H =$ **1.99648**	BCC $\overline{A} = 33$, $\overline{S} = 9$ 2990 ms. $\delta_H = 6.11619$	BCC $\overline{A} = 20$, $\overline{S} = 9$ 15596 ms. $\delta_H = 10.5289$	BCC $\overline{A} = 15$, $\overline{S} = 8$ 33420 ms. $\delta_H = 17.7125$
NASR $\overline{A} = 10$, $\overline{S} = 40$ 115 ms. $\delta_H = 8.81655$	NASR $\overline{A} = 1$, $\overline{S} = 34$ 138 ms. $\delta_H = 9.84886$	NASR $\overline{A} = 1$, $\overline{S} = 16$ 171 ms. $\delta_H = 19.6977$	NASR $\overline{A} = 0$, $\overline{S} = 14$ 190 ms. $\delta_H = 32.8938$

GMC: $\overline{A} = 117$, $\overline{S} = 12$, $\delta_H =$ **1.98** | $\overline{A} = 65$, $\overline{S} = 7$, $\delta_H =$ **3.95** | $\overline{A} = 44$, $\overline{S} = 12$, $\delta_H =$ **5.99** | $\overline{A} = 34$, $\overline{S} = 9$, $\delta_H =$ **9.93**

VC: scale=0.019 $\overline{S} = 128$, $\delta_H = 11.58$ | scale=0.089 $\overline{S} = 72$, $\delta_H = 26.22$ | scale=0.115 $\overline{S} = 55$, $\delta_H = 35.44$ | scale=0.145 $\overline{S} = 33$, $\delta_H = 81.62$

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