Introduction to Digital Level Layers

Yan Gerard, Laurent Provot and Fabien Feschet

Université d'Auvergne
Clermont-Ferrand 1
Campus des Cézeaux
63172 Aubière
France

ISIT - Bâtiment 3C
Faculté de médecine
28 place Henri Dunant
63 001 Clermont-Ferrand
France

yan.gerard@u-clermont1.fr, provot.research@gmail.com and feschet@acm.org

We have

A Euclidean shape
$\mathcal{S} = \{\mathbf{x} \in \mathbb{R}^d \mid f(\mathbf{x}) = 0\}$

Question ?

How to define a *digital primitive* wrt. $\mathcal{S}$?

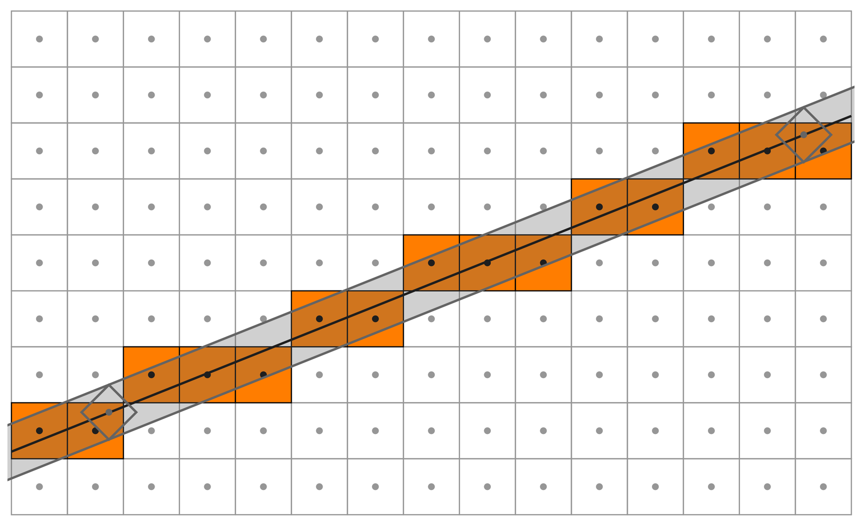
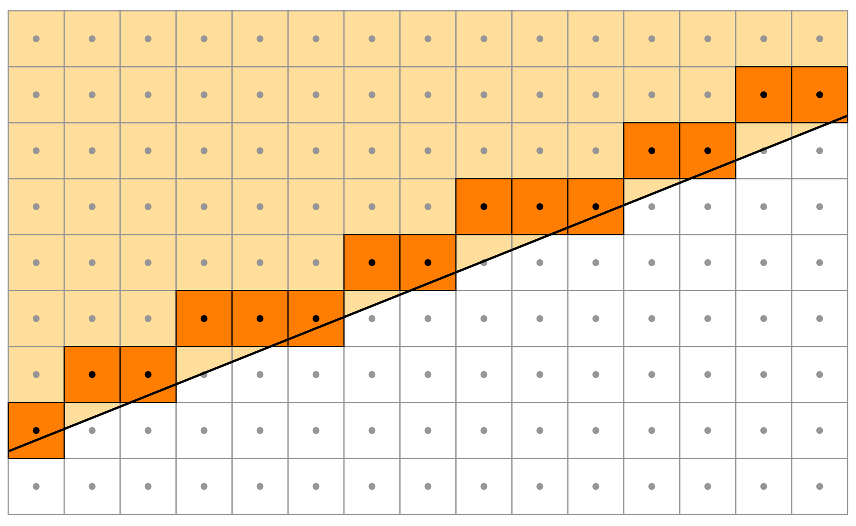
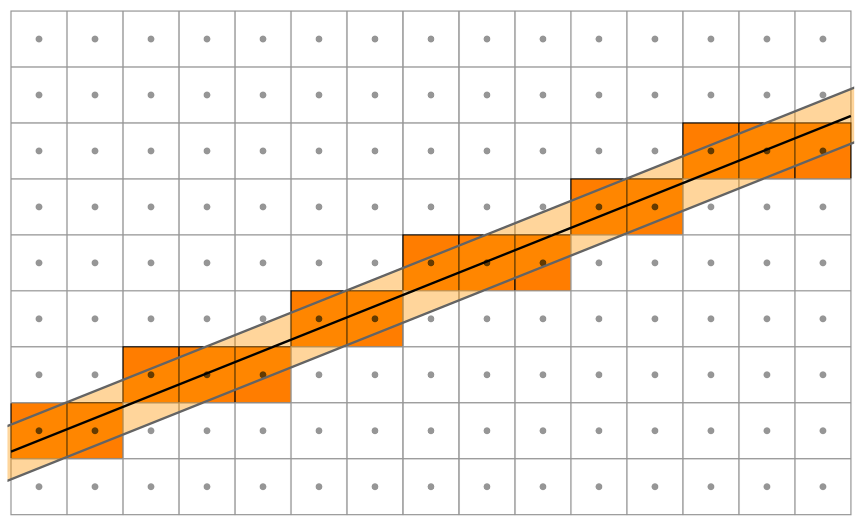
Morphological approach: $(\mathcal{S} + \diamond) \cap \mathbb{Z}^d$

Topological approach: $\partial(\mathcal{S}^- \cap \mathbb{Z}^d)$

Algebraic approach: $\{\mathbf{x} \in \mathbb{Z}^d \mid h < f(\mathbf{x}) < h'\}$

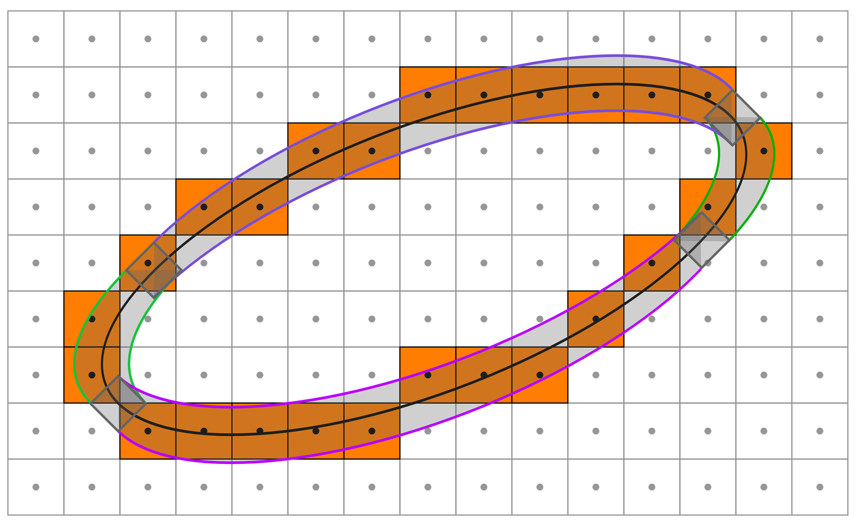
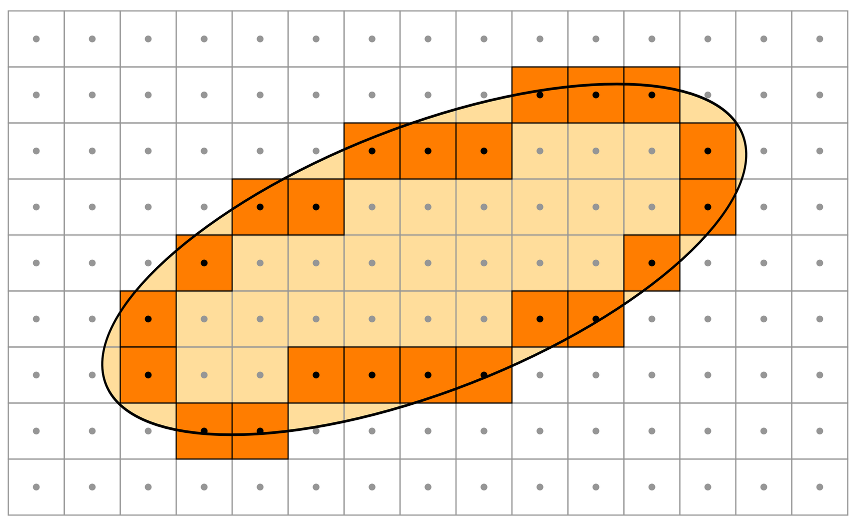
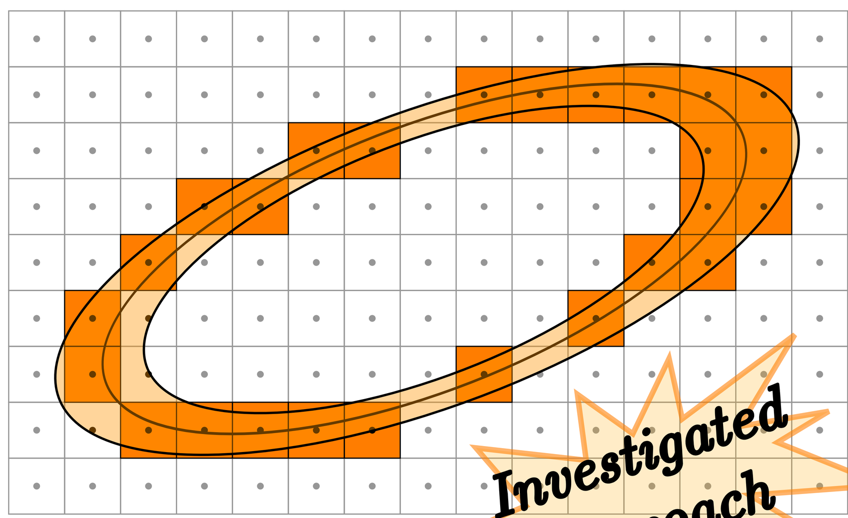
Linear case

Different definitions lead to *the same* digital primitive

 $\sim$  $\sim$ 

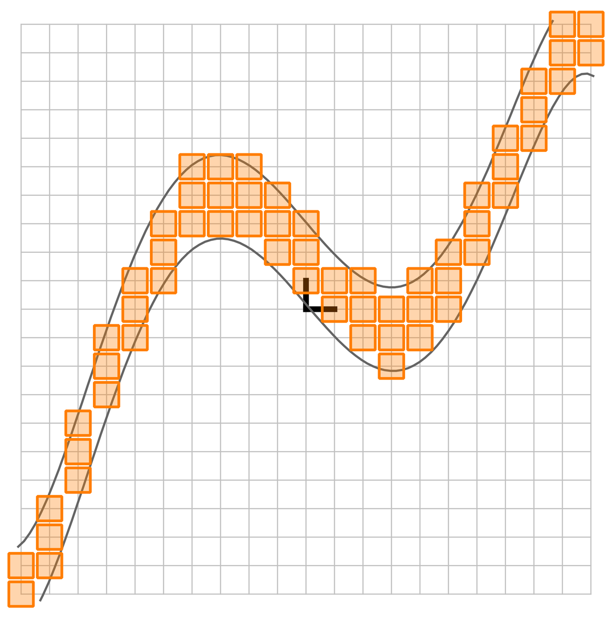
Non linear case

Different definitions lead to *different* digital primitives

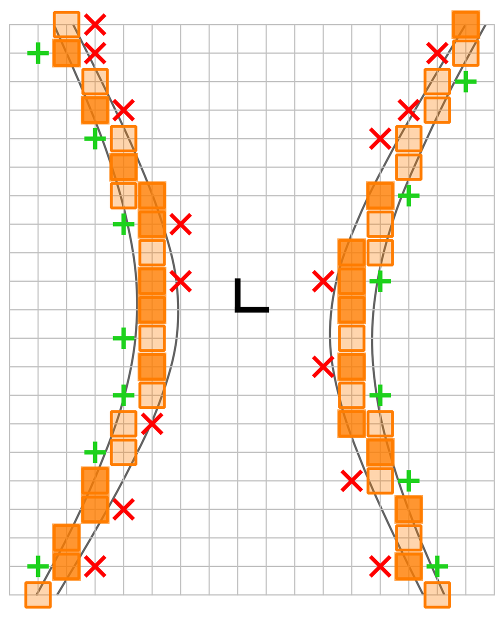
 $\neq$  $\neq$ 

Investigated approach

Digital Level Layers (DLL)



$6686400 \leq 5634x^5 + 4037x^4 - 1028960x^3 - 37211x^2 + 25533320x + 21746400y \leq 65632500$



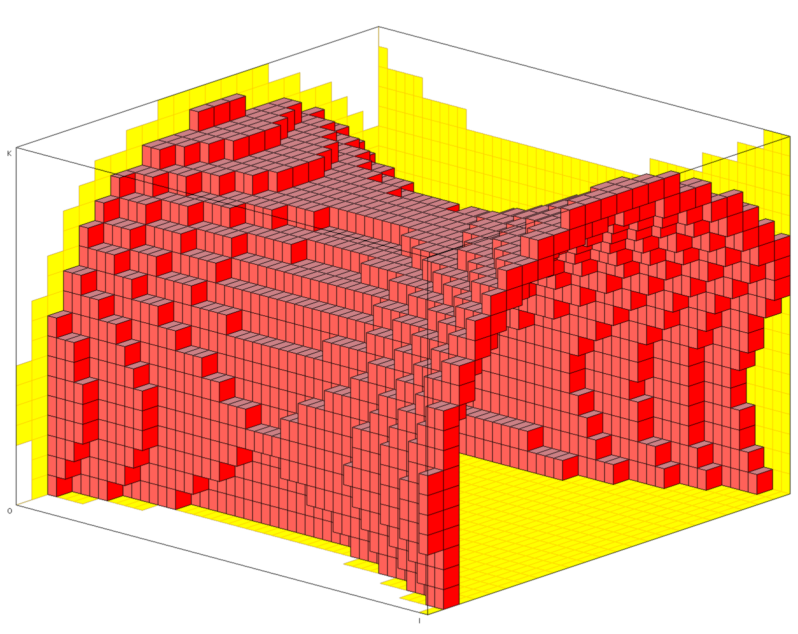
$-922728 \leq -54922x^2 + 5664xy + 20193y^2 + 66572x + 13791y \leq -372102$

Definition:

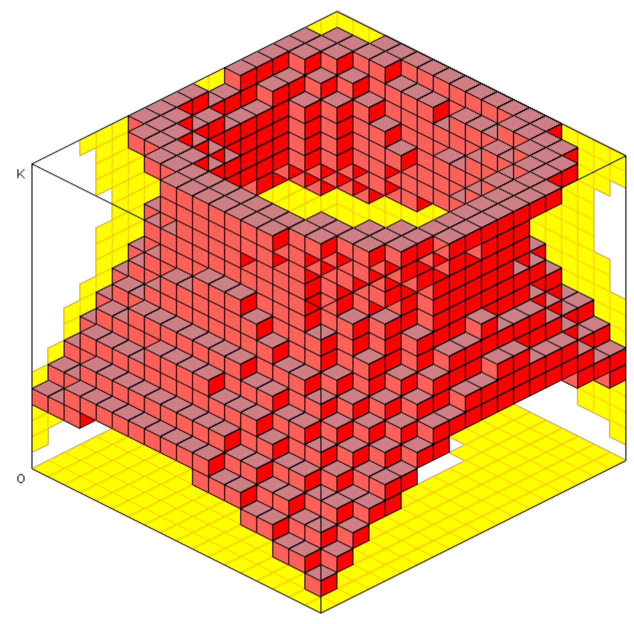
Subset of points $\mathbf{x} \in \mathbb{Z}^d$ such that:

$$h \leq f(\mathbf{x}) \leq h'$$

with $f : \mathbb{Z}^d \to \mathbb{R}$ and $h, h' \in \mathbb{R}$



$-1672341520 \leq -3132721x^2 + 24502323y^2 + 20172235z^2 + 127865587x - 593173497y - 166992955z \leq -1014598194$



$-19389174001 \leq 9706766352x + 34545685472y - 50669609424z - 1006860048x^2 - 1419723424y^2 + 1813925040z^2 + 346913568xz - 191023296yz + 213661758001 \leq 19389174001$

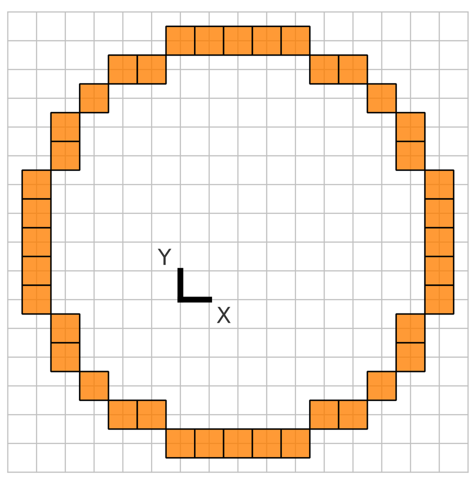
Recognition Problem

Input : A set of point $In \subset \mathbb{Z}^d$
Output : Characteristics of a "tight" DLL that contains In

Assume $f : \mathbb{Z}^d \to \mathbb{R}$ is given and belongs to a linear space $\mathbb{F}$ generated linearly by m functions $f_{i(1 \leq i \leq m)} : \mathbb{Z}^d \to \mathbb{R}$

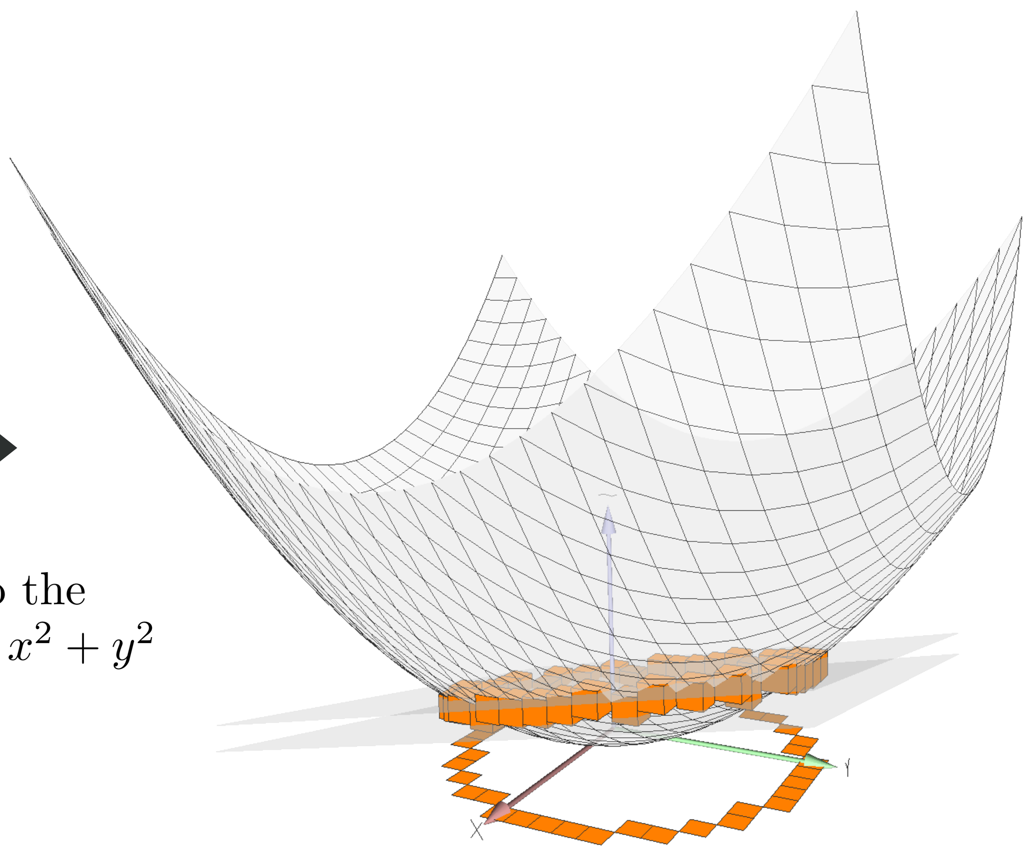
$$h \leq f(\mathbf{x}) \leq h' \quad \Leftrightarrow \quad h \leq \sum_{i=1}^{m} \lambda_i f_i(\mathbf{x}) \leq h'$$

The recognition problem in $\mathbb{Z}^d$ turns out to be linear in $\mathbb{R}^m$



Kernel Trick [ABR64]

Projection onto the paraboloid $z = x^2 + y^2$



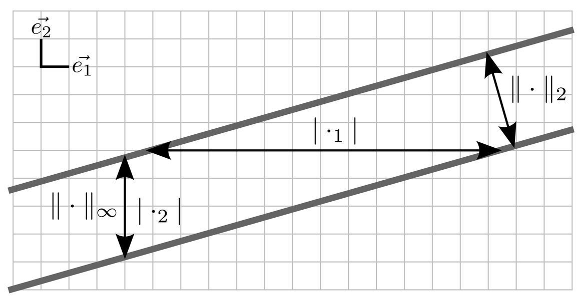
Control the width of the DLL

1. Explicitely: a bound δ is given

$\lambda = (\lambda_i)_{1 \leq i \leq m}$ and $y = \mathfrak{t}(\mathbf{x}) = (f_i(\mathbf{x}))_{1 \leq i \leq m}$

Introduce a notion of thickness through a function $\varphi(\cdot)$

Look for an affine strip $h \leq \lambda.y \leq h'$ that contains $\mathfrak{t}(In)$ and minimize $\varphi(\lambda)$



$\varphi(\lambda) = \|\lambda\|_\infty$ or $\varphi(\lambda) = \|\lambda_i\|$		$\varphi(\lambda) = \|\lambda\|_2$
Meggido [Meg84]	Chords Algo [GDRZ05]	Rotating Caliper [HT83]
Linear Programming	Computational Geometry	

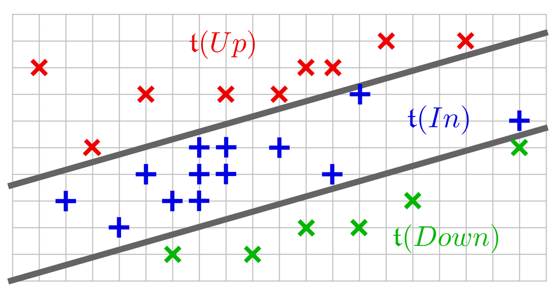
Check whether $\frac{h' - h}{\varphi(n)} < \delta$

2. Implicitely: two sets of outliers Up and $Down$ are given along with In

Look for an affine strip $h \leq \lambda.y \leq h'$ that contains $\mathfrak{t}(In)$

$\mathfrak{t}(Up)$ must lie above the strip : $h' < \lambda.y$
$\mathfrak{t}(Down)$ must lie below the strip : $\lambda.y < h$

Separation problem GJK Algo [GJK88]



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