MEASURES FOR SURFACE COMPARISON ON UNSTRUCTURED GRIDS WITH DIFFERENT DENSITY

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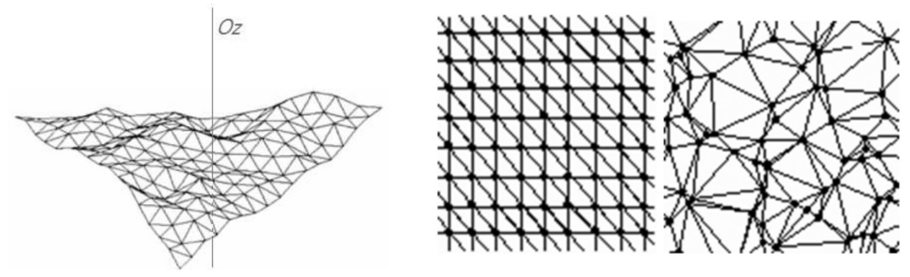
Abstract

We consider the problem of surface comparison given as spatial point clouds that can be explicitly projected onto a plane. This problem can be reduced to comparison of mesh functions of two variables given on different grids. A general case when both grids are unstructured and have different density is considered. A measure to compare such functions that allows to estimate difference on areas with nodes from both grids and an algorithm to compute it are proposed. Estimation for computational complexity of the algorithm is presented. Computing experiments on real data (3d face models) were carried out.

INTRODUCTION

With the rapid progress of modern 3d scanning technologies, objects' surfaces can be routinely acquired as discrete surface models. Spatial object's shape can be considered as a set of schlicht surfaces, i.e., such that can be uniquely projected onto a plane. In this work, new measures for comparison of such surfaces and effective methods to compute them are devised.

There are two main presentation methods for modelling of schlicht surfaces: definition on structured and unstructured grids. Using of structured grids has some essential disadvantages. Raw schlicht surface data acquired by 3d scanner can be considered as a discrete function defined at nodes of unstructured grid. In this case surface approximation quality is higher. At the same time it is required to introduce and design more complex measures and processing algorithms. In this work, we propose the approach, which saves initial nonregularity of grids.



Examples of schlicht surface (on the left), unstructured grid (in the middle) and uniform (structured) grid (on the right)

SURFACE COMPARISON PROBLEM

Problem Statement

Suppose two schlicht surfaces S_1, S_2 are given by functions F_1, F_2 at the nodes of grids G_1,G_2, respectively. It is required to introduce measures for comparison of surfaces S_1, S_2 and design an approach to compute them.

Suppose G_1 and G_2 are contained inside a certain general rectangle R, $\overline{F}_1$ and $\overline{F}_2$ are continuous on R analogs of functions F_1 and F_2, that are derived by interpolation. Denote by T_1 and T_2 the Delaunay triangulations constructed on grids G_1 and G_2, respectively. The Delaunay triangulation constructed on the union G of two grids G_1, G_2 is called *general Delaunay triangulation* and denoted by T.

Proposed Measures

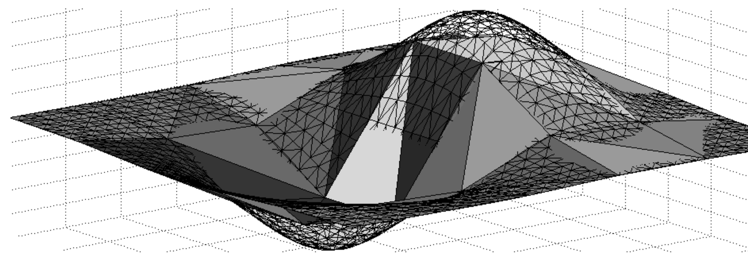
Consider a function $\mu(x,y)$ that defined weight of difference between two surfaces at a certain point with coordinates (x, y) in accordance with significance of function similarity in the region contained this point. Let A,B,C be nodes of triangle in grid G. By definition, put:

$$V_\mu(A,B,C,F_1,F_2)=\iint\limits_{\triangle ABC}\left|\hat{F}_1(x,y)-\hat{F}_2(x,y)\right|\mu(x,y)\,dxdy. \qquad (1)$$

In [2] the author proposed the following measure:

$$\rho_{V_\mu}(F_1,F_2)=\sum_{\triangle ABC\in T} V_\mu(A,B,C,F_1,F_2)/S_{\triangle ABC}, \qquad (2)$$

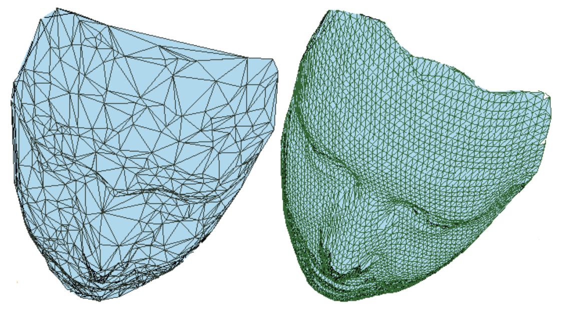
There are big interpolation errors in case of different density of two initial grids.



Let V be a value of V_μ for $\mu(x, y) \equiv 1$; suppose $\rho_V(F_1, F_2)$ is measure (2) for condition of $V_\mu = V$.

A new proposed measure allows to compute difference only on regions where nodes of both grids are concentrated:

$$\rho_{\delta V}(f_1,f_2)=\sum_{\substack{\triangle ABC\in T:\\ \triangle ABC\notin T_1,\\ \triangle ABC\notin T_2}} V(A,B,C,f_1,f_2)/S_{\triangle ABC}. \qquad (3)$$



In practise one can meet this case solving surface matching problem for surfaces acquired by two 3d scanners of different accuracy.

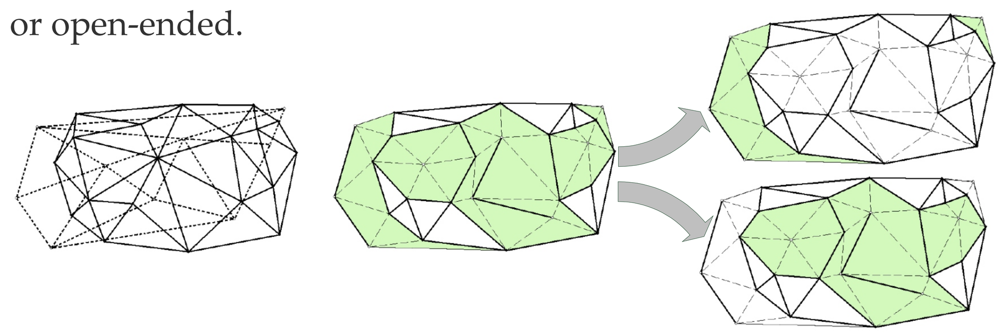
METHODS

During merging process of two Delaunay triangulations T_1 and T_2, some edges and triangles move to the united triangulation without changes and some of them are destroyed. So there are new edges and triangles, which connect nodes from different grids, in the general triangulation T.

We say that an edge or a triangle is called **interface** if it connects nodes from both of grids G_1, G_2. Measure (3) is calculated over interface triangles only.

Algorithm for Interface Triangles Extraction

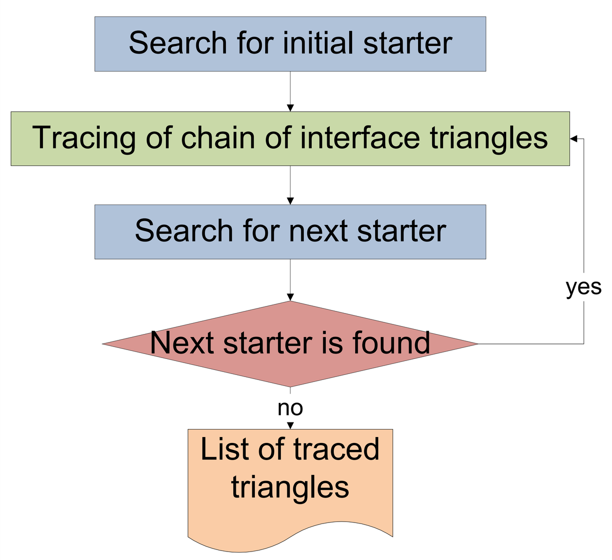
A set of interface triangles decomposes on several subsets. Each subset is a chain of triangles, which are pairwise incident by edges. This chain can be closed or open-ended.



Two triangulated grids (on the left); general Delaunay triangulation with fill interface triangles (in the middle); two open-ended chains and three closed chains of interface triangles (on the right)

We say that an interface edge of general triangulation T is called a **starter** if it belongs to a chain that is not traced yet. A starter initializes the process of chain tracing.

Hence the proposed *algorithm for interface triangles extraction* consists of the following stages:



Using an algorithm for merging non-separated Delaunay triangulations proposed by Mestetskiy, Tsarik in [3], the following theorem was proved:

Theorem 1. *Computational complexity of algorithm for interface triangles extraction is O(N), where N is a number of nodes in general grid.*

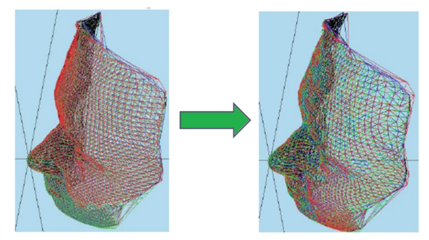
Computational Efficiency of Surface Comparison Algorithm

Theorem 2. *Localization of grid nodes in triangulation can be implemented using list of interface triangles in linear time.*

1 Construction of Delunay triangulations T_1, T_2 ~ $O(N_1 \log N_1) + O(N_2 \log N_2)$;
2 Construction of minimal spanning trees of triangulations using Cheriton, Tarjan algorithm ~ $O(N_1) + O(N_2)$;
3 Extraction of interface triangles (theorem 1) ~ $O(N)$;
4 Localization of each of grids G_1, G_2 in triangulation of the other grid (theorem 2) ~ $O(N)$;
5 Interpolation of function F_1 at nodes of G_2 and of function F_2 at nodes of G_1 ~ $O(N)$;
6 Computing measure (3) over all interface triangles ~ $O(N)$.

Theorem 3. *Computational complexity of algorithm for computing measure (3) is O(N logN). If Delaunay triangulations of initial grids are constructed during preprocessing stage then computational complexity of the algorithm is linear.*

Surface Mathing: search for best fitting



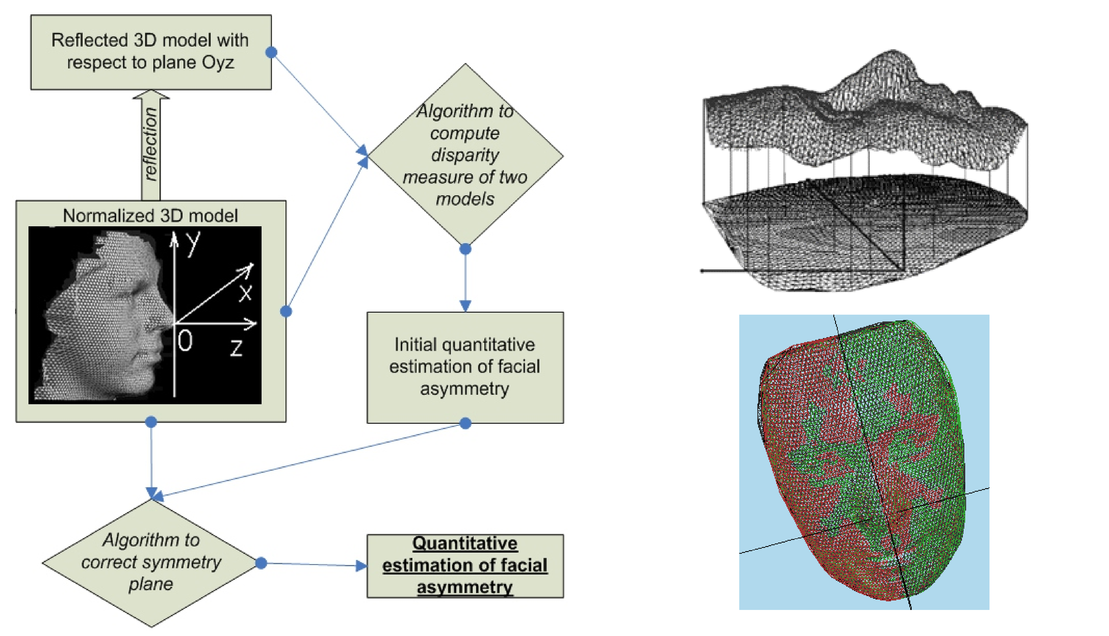
M — movement in space $\mathbb{R}^3$: composition of sequential rotations at angles $\alpha_M, \beta_M, \gamma_M$ around axes Ox, Oy, Oz respectively and parallel shift by vector $(\Delta x_M, \Delta y_M, \Delta z_M)$.

Optimization problem

$$\rho(G_1, M(G_2)) \to \inf_{\alpha_M,\beta_M,\gamma_M,\Delta x_M,\Delta y_M,\Delta z_M}$$

APPLICATIONS OF PROPOSED METHODS

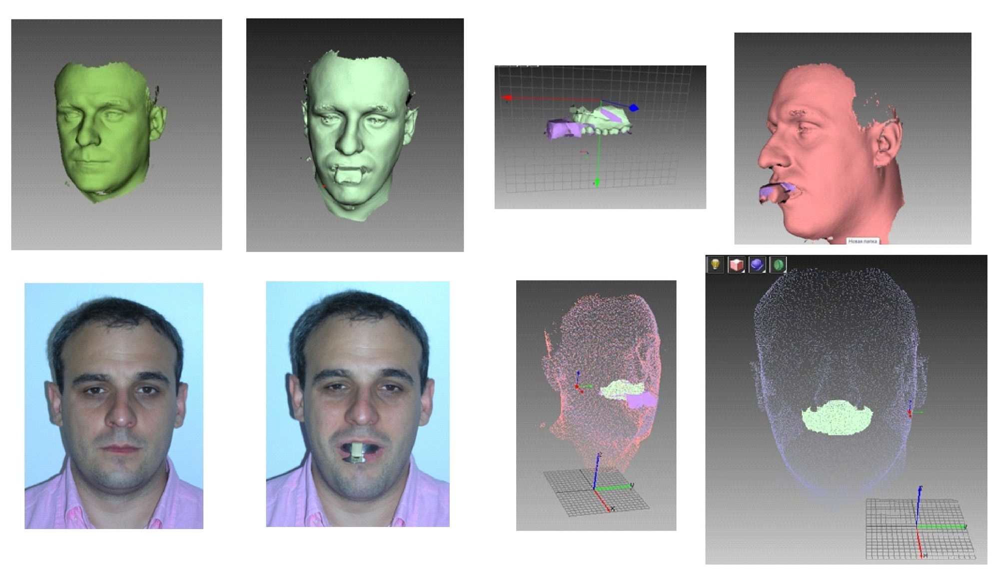
Quantitative estimation of facial asymmetry by 3d model



Face scanning using 3d camera Broadway by Artec Group Company

Scanning of a jaw plaster cast using dental scanner Laserdenta Open

Positioning of 3d jaw model in 3d head model using a reference object



EXPERIMENTS

Computational experiments were carried out on 3d face models acquired by 3d scanner Broadway designed by Artec Group Company [1]. The database consists of 48 models received by scanning of 8 different persons (6 different models for each person). Different models of one and the same person were used as surfaces for comparison.

Suppose S_1, S_2 are surfaces for comparison, S_2' is a reduced (simplified) second surface. S_2' is acquired from S_2 by uniform random thinning of the second grid. As the result of thinning 15% nodes of the second grid were removed.

Grids of S_1, S_2 are unstructured (nonuniform) but has approximately equal density. Hence we assume a value of measure (2) between them as adequate initial estimation. Grids of S_1 and S_2' are unstructured grids with different density.

Table 1. Value of measures (2) & (3) for surface comparison

Measure	Value (mm)	Comments
$\rho_V(S_1,S_2)$	7094	Adequate estimation for S_1 & S_2
$\rho_V(S_1,S_2')$	17963	$\rho_V(S_1,S_2') > \rho_V(S_1,S_2)$
$\rho_{\delta V}(S_1,S_2')$	10884	$\rho_{\delta V}(S_1,S_2') \approx \rho_V(S_1,S_2')$

Table 1 shows an example of measure values for comparison of surfaces from the database. We see that the measure (3) estimates difference between surfaces S_1 and S_2 more adequate than the measure (2).

CONCLUSIONS

New measure adapted for comparison problem of surfaces defined on unstructured grids with different density is introduced. An efficient algorithm for measure computing is proposed.

The measure allows only surface fragments that are represented by nodes of both grids. We call such fragments interface fragments. For efficiency of measure computing a new algorithm for interface triangles extraction is proposed. Computational complexity of the algorithm is presented.

Computing experiments for comparison of surfaces defined on grids with different density were carried out. As experimental estimations have shown, the introduced measure is adequate for such kind of source grids.

Literature cited

[1] Artec Group — 3D Scanning Technologies, http://www.artec-group.com.
[2] Dyshkant, N.: Disparity Measure Construction for Comparison of 3D Objects' Surfaces. In: Proceedings of the 2nd InternationalWorkshop on Image Mining, Theory and Applications (IMTA-2-2009), pp. 43–52. INSTICC Press, Lisbon (2009).
[3] Mestetskiy, L., Tsarik, E.: Delaunay triangulation: recursion without space division of vertices. In: 14th International Conference on Computer Graphics and Vision GraphiCon 2004, pp. 267–270. CMC MSU Press, Moscow (2004).

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