Arc segmentation in linear time

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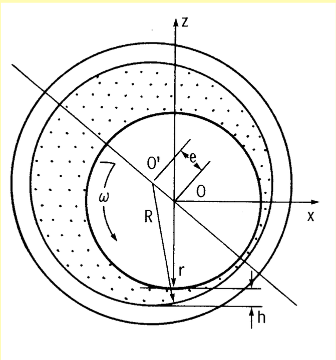
Contribution

We propose a linear algorithm based on discrete geometry approach for segmentation of a curve into digital arcs.

Motivation

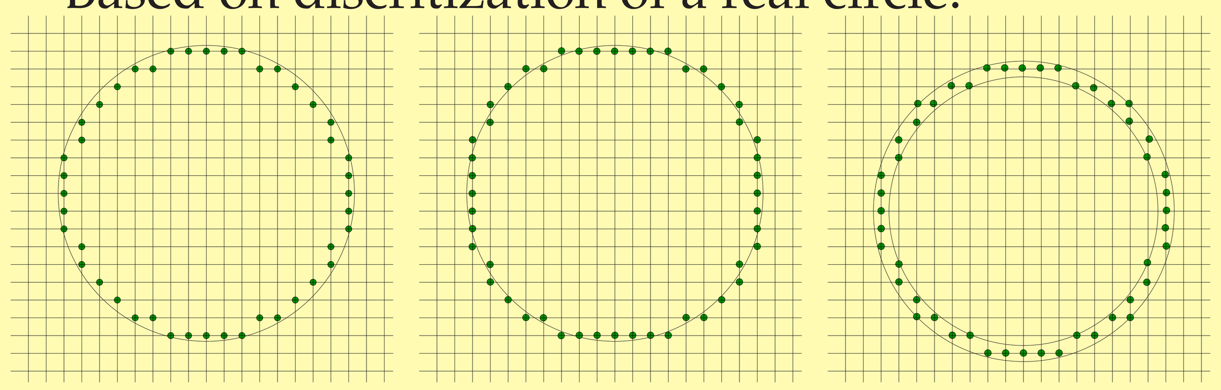
- Arc and circle are basic object in discrete geometry
- Arc and circle appear often in images
- Shape contains often digital arcs.

⇒ The study of these primitives is important.



Discrete circle

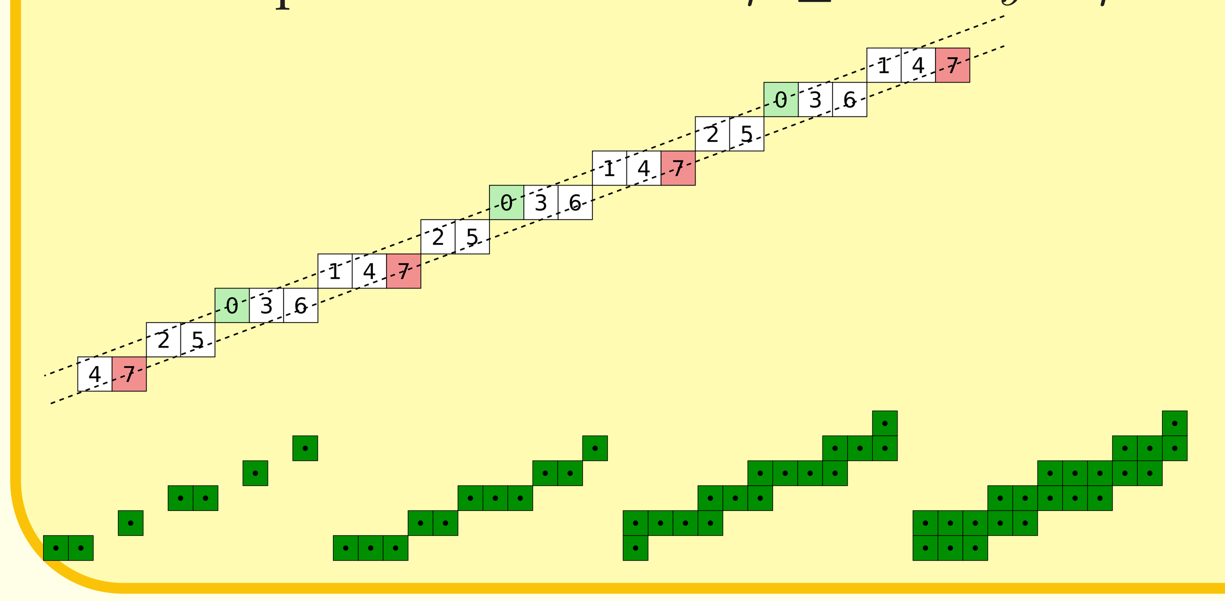
- Basic object in discrete geometry.
- Based on discritization of a real circle.



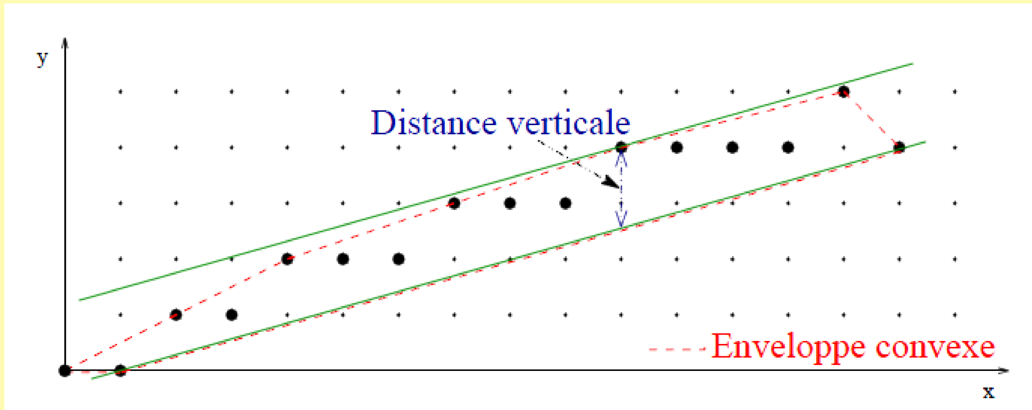
[Kim84] [Nakamura84] [Andres04]

Discrete line [Reveillès91]

A discrete line, noted as $D(a,b,\mu,\omega)$, $a,b,\mu,\omega \in \mathbb{Z}^2$, is a set of points that verifies: $\mu \leq ax - by < \mu + \omega$



Blurred segment [Debled et al. 06]



A blurred segment ν is a set of points that satisfies

- There exist a discrete line $D(a,b,\nu,\omega)$ that contains this set.
- $\frac{\omega - 1}{max(|a|,|b|)} \leq \nu$

Tagent space [Latecki00]

Input:

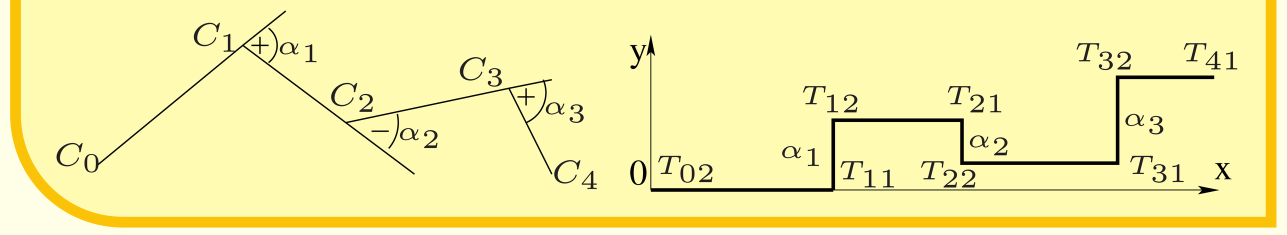
Suppose that $C = \{C_i\}_{i=0}^{n}$ is a polygonal curve

- $\alpha_i = \angle(\overrightarrow{C_{i-1}C_i}, \overrightarrow{C_iC_{i+1}})$
- l_i the length of segment C_iC_{i+1}.
- $\alpha_i > 0$ if C_{i+1} is on the right side of $\overrightarrow{C_{i-1}C_i}$, $\alpha_i <$ 0 otherwise.

Output:

We consider the transformation that associates C to a polygon of $\mathbb{R}^2$ constituted by segments $T_{i2}T_{(i+1)1}, T_{(i+1)1}T_{(i+1)2}$ for i from 0 to n-1 with

- $T_{02} = (0,0)$
- $T_{i1} = (T_{(i-1)2}.x + l_{i-1}, T_{(i-1)2}.y), 1 \leq i \leq n$,
- $T_{i2} = (T_{i1}.x, T_{i1}.y + \alpha_i), 1 \leq i \leq n-1$.



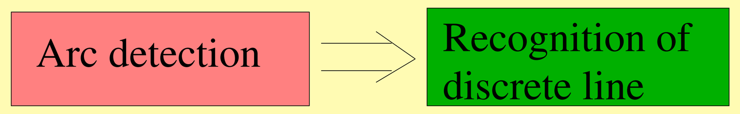
Arc in the tangent space

Proposition 1:

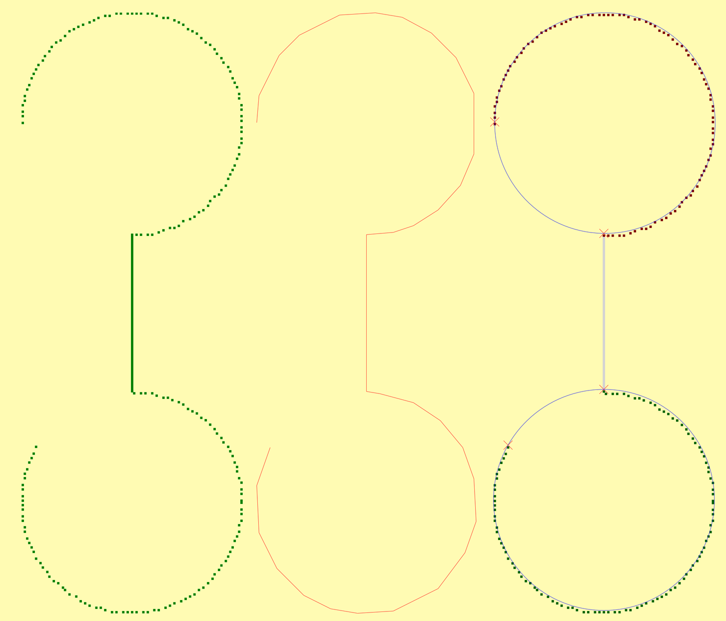
Let $C = \{C_i\}_{i=0}^{n}$ be a polygon, $\alpha_i = \angle(\overrightarrow{C_{i-1}C_i}, \overrightarrow{C_iC_{i+1}})$. The length of C_iC_{i+1} is l_i, for $i \in \{0,\dots,n-1\}$. The vertices of C are on a real arc of radius R with center O, $\angle C_iOC_{i+1} \leq \frac{\pi}{4}$ for $i \in \{1,\dots,n-1\}$. This results below is obtained.

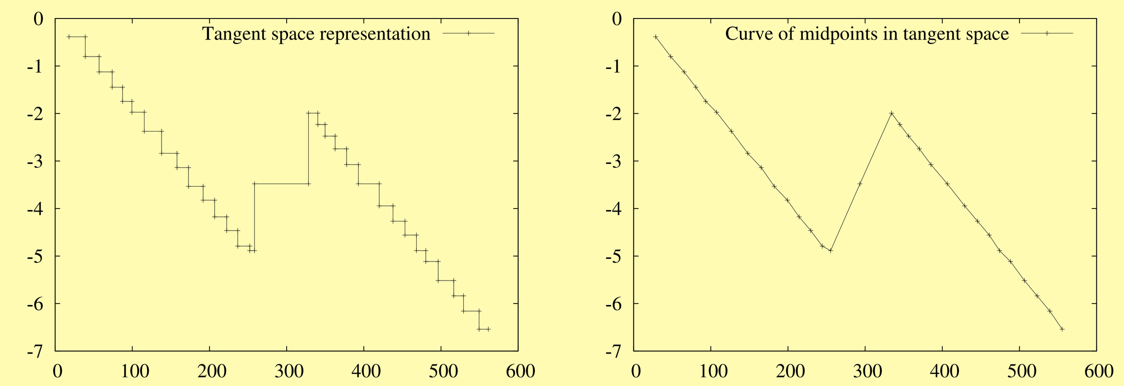
$$\frac{1}{R} < \frac{\alpha_i}{\frac{l_i + l_{i+1}}{2}} < \frac{1}{0.9742979R}$$

Interest of proposition 1



Example:





Deciding if a curve is an arc

1. Polygonalize the input digital curve by polygon P based on recognition of BS of width 1.
2. Transform P to $T(P)$ in the tangent space
3. Determine the midpoint set $MpC = \{M_i\}_{i=1}^{n-1}$ of horizontal segment of $T(P)$
4. Verify if MpC is a BS of width ϵ [Debled 06]

Arc segmentation

Main ideas:

1. Polygonalize the input curve
2. Transform the polygon to tangent space
3. Construct the curve of midpoints in the tangent space
4. Polygonalize the midpoint curve
   - Utilize parameter α to verify detected arcs

Study of quasi co-linear property

Convergence of radius of local circumcircles

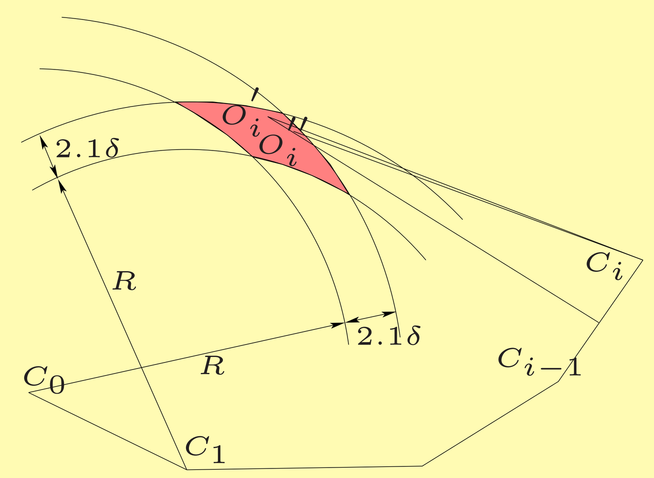
Proposition 2:

Let $C = \{C_i\}_{i=0}^{n}$ be a polygon, $\alpha_i = \angle(\overrightarrow{C_{i-1}C_i}, \overrightarrow{C_iC_{i+1}})$. The length of C_iC_{i+1} is l_i, for $i \in \{0,\dots,n-1\}$. We denote O_i, R_i, H_i respectively the center and the radius of circumcirle that passes to 3 points C_{i-1}, C_i, C_{i+1}, the projection of O_i on C_iC_{i+1}, suppose that $R_i - OH_i \leq h$ for $i \in \{1,\dots,n-1\}$. This results below is obtained.
$R_i\alpha_i \geq \frac{l_{i-1}+l_i}{2} \geq R_i\alpha_i - 0.3377h\alpha_i$

Convergence of centers of local circumcircles

Proposition 2:

Let us consider a sequence of points $\{C\}_{i=0}^{n}$. We denote $O_i^{'}$ (resp. $O_i^{''}$) and $R_i^{'}$ (resp. $R_i^{''}$) are the center and radius of circumcircle that passes to 3 points C_0 (resp. C_1), C_i, C_{i+1}. There exist R and δ satisfied $R, \delta \in \mathcal{R}, 0 \leq R_i - R \leq \delta, i = 1,\dots,n-1$. Suppose that $\angle C_kC_jC_{j+1} > \frac{\pi}{2}$ for $k \in \{0,1\}, k < j < n$. Therefore, we have this property $0 \leq R_i^{'} - R \leq \delta, 0 \leq R_i^{''} - R \leq \delta$, for $1 \leq i \leq n-1$.



Experimentation

Experimental results: