

INTRODUCTION TO THE NORMALIZING FLOW FRAMEWORK

- $p_X(\cdot)$ is to be estimated given $X = \{x_1, x_2, \dots, x_N\}$ [1, 2]

$$p_X(x) = p_U(f_\theta(x)) \left| \det \frac{\partial f_\theta}{\partial x}(x) \right|$$

- How? Transform observations so that they follow p_U .
- ..using a diffeomorphism $f_\theta : \mathbb{R}^D \rightarrow \mathbb{R}^D$
- f_θ is defined as a neural network $f_\theta(x) = f^K \circ f^{K-1} \circ \dots \circ f^1(x; \theta)$.
- learning = finding the optimal network parameters that transform X so that it follows $p_U(\cdot)$
- Framework is recastable to use in a supervised task [3]:

$$p_{Y|X}(y|x) = p_U(f_\theta(y|x)) \left| \det \frac{\partial f_\theta}{\partial x}(y|x) \right|,$$

with a maximum likelihood objective:

$$\arg \max_{\theta} \log \mathcal{N}(f_\theta(y|x)) + \sum_{k=1}^K \log \left| \det \frac{\partial f^k}{\partial z^k}(z^k|x; \theta) \right|,$$

where we set $z^0 = u, z^K = y, z^k = f^k(z^{k-1}|x) \forall k \in [1, K], p_U = \mathcal{N}$

FLOW LAYERS

Flow layers compose f_θ . They must be..

- expressible
- invertible
- cheap to compute (evaluating $\det \frac{\partial f_\theta}{\partial x}(x)$ can be a serious bottleneck!)

E.g. "Affine coupling" ($z^k = \{z_A^k, z_B^k\}$ a partition of z^k & f_θ^k is defined in terms of $f_{\theta,s}^k, f_{\theta,t}^k$

$$z^k = f_\theta^k(z^{k-1}|x) \implies \begin{cases} z_A^k = z_A^{k-1} \\ z_B^k = z_B^{k-1} \circ \exp(f_{\theta,s}^k(z_A^{k-1}|x)) + f_{\theta,t}^k(z_A^{k-1}|x) \end{cases} \quad (1)$$

Note that f_θ^k is (easily!) invertible, while $f_{\theta,s}^k, f_{\theta,t}^k$ can be arbitrarily complex and difficult to invert [4].

REFERENCES

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TEXT SR AND BINARIZATION AS NORMALIZING FLOWS

- Text Super-Resolution is cast as a supervised problem: Estimate HR image given LR image
- Likewise for binarization: Estimate the binarized image given the unprocessed image
- Results come as a probability density function of the output image given the input image
- Inference is performed by sampling:

$$y = f_\theta^{-1}(z|x), z \sim \mathcal{N}(0, \tau)$$



Figure 1: Binarization results: Original images and binarization results for different "temperatures" τ .

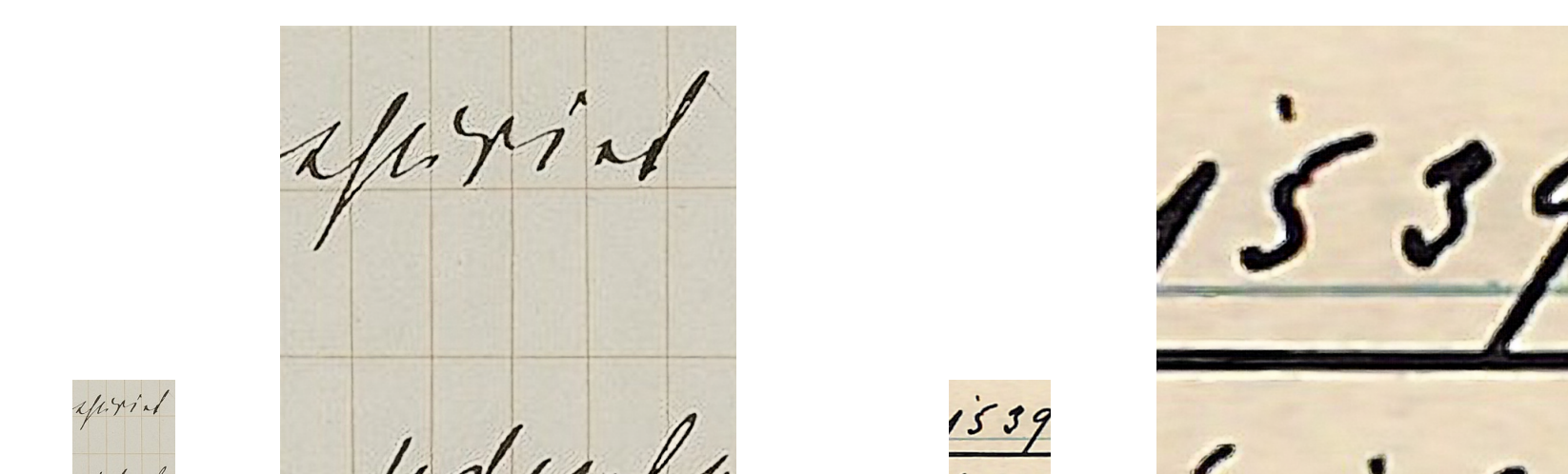


Figure 2: Super-resolution results: Original images and super-resolved images ($\tau=0.7$).

CONCLUSION

- A probabilistic setting combined with neural networks is an attractive option!
- Multiple outputs per input are possible, and we know which one is the most likely
- Probabilistic framework lends easily to nice extensions (e.g. combine with a task-specific prior)
- Future work: More research on appropriate flow models for document image processing